Mathematics Education and Life at Times of Crisis

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WHERE ARE THE CHILDREN? AN ANALYSIS OF NEWS MEDIA REPORTING ON MATHEMATICS EDUCATION

Yasmine Abtahi & Richard Barwell

University of Ottawa, Canada

While analysis of how children are constructed in mathematics education research discourses dates back to the 1980s, there has been much less analysis of the construction of children in media reporting about mathematics education. In this paper, we report our analysis of a corpus of Canadian newspaper reports on mathematics education, focusing on the underlying construction of (Canadian) children. We show that children are generally constructed in broadly negative terms: they lack desirable mathematical knowledge and perform less well than children elsewhere. We seek to explain these findings with reference to the general framing of mathematics education in the corpus in terms of a choice between discovery learning or going back to basics.

INTRODUCTION

There have been numerous critiques of the way children are constructed in the discourse of educational research, including the discourse of research in mathematics education. Most notably, Walkerdine (1988), drawing on broadly a Foucauldian perspective, argued that progressive discourses construct children as developing a particular kind of innate rationality. More recently, Valero and Knijnik (2015) have argued that mathematics education research contributes to a discourse that requires children to "become the desired rational, Modern, self-regulated, neoliberal beings, who are to run current societies adequately" (p. 38). These discourses, then, produce particular kinds of children, particularly when refracted through curriculum and government policy:

> A child-centered pedagogy and its baggage of liberating the individual, reasoning subject actually constructs each child as a "subject." This "subject," where the word starts out as meaning something like "an active agent", turns into a grown person who is "subject" to the power properties of a governing state that uses "reasoning" to manipulate and control the now-understood "subject" in its rational thinking and behavior. (Appelbaum, 2014, p. 17)

Children are thus constructed as active learning beings in ways that simultaneously requires them to become particular kinds of beings.

Research examining the construction of children (and of teachers) in mathematics education research and policy is well established in the literature. There is, however, less work looking at the discourses of mathematics education circulating in the wider public domain and, more specifically, at the construction of children within these discourses. Our own recent research on Canadian news media discourses of mathematics education had shown how news reporting constructs mathematics education in moral terms. In this reporting, one type of approach to teaching mathematics ('discovery learning', itself a construction of the news discourse) is associated with a general decline in standards, skills and national prestige (Barwell & Abtahi, 2015). In conducting this work, we began to pay attention to how children are portrayed. For example, Canadian news reporting has included claims such as:

- "Canada's student performance, formerly well above the OECD average is now considerably less so";
- "Canadian students are getting weaker, not better, particularly in math";
- "our students are doing decidedly worse in math than they did a decade ago".

Claims like these seem to construct children rather negatively. We therefore conducted a detailed analysis of how children and other actors were constructed. In this paper, we report some of this work and relate our findings about the construction of children in news reporting about mathematics education with previous work on the construction of children in educational and policy discourses.

LITERATURE REVIEW

News stories about general education, and mathematics education in particular, are common and mediate public images of the education system and its different stakeholders, such as governments, teachers, parents and children (Leder & Forgasz, 2010). Press coverage of educational concerns, including the position of children, is not neutral. News stories always involve choices about what to underline and what to leave out (Camara & Shaw, 2012). What is reported in news media shapes and is shaped by the wider social context. Media reporting has the capacity to influence public opinion and the way people make sense of and discuss current issues (Scheufele & Tewksbury, 2007). The reporting of a particular news story over time can create a storyline that comes to provide meaning for that issue (Gamson & Modigliani, 1987). News media discourses are likely to have a widespread impact on how children are constructed within mathematics education by the general public.

One of the stories that we have been following in three Canadian newspapers is about the so-called "math wars". This story involves disagreement about the benefits and harms of 'discovery learning', which is consistently contrasted with a 'back to basics' approach. Several authors who have examined the math wars (see Schoenfeld, 2004, for a review) argued that the underlying debate has gone on for over a century. Wright (2012) discussed some of the criticism of Jo Boaler's research, arguing that the strongly entrenched positions are due to a clash of ideologies. In particular, the arguments about approaches to teaching mathematics are related to an underlying ideological clash between progressive and utilitarian approaches. Utilitarian approaches emphasise the acquisition of mathematical skills that will be of benefit in the labour market and the economy in general. These studies provide valuable background to the debates about mathematics education occurring in news media, although they are reactions to the media debate, rather than analyses of the news reporting itself.

Analyses of media reporting have noted the particular salience of the outcomes of international comparisons, including the resulting 'PISA shock', in which national scores prompt widespread public discussion of mathematics education (Pons, 2012). The construction of children in such debates has, however, received less attention (Gutiérrez & Dixon-Román, 2011). In one study, Lange and Meaney (2014) looked at Australian news coverage of national testing, and showed how children were constructed as commodities, "with mathematics achievement being the value that can be added to them" (p. 337). In our readings of the news reporting of the "math wars", we have noticed a similar negativity, as we have noted already.

THEORETICAL FRAMEWORK: REGIMES OF TRUTH

For this research, we drew on Foucault's theory of discourse and power. From Foucault's work, we have drawn specifically on his idea of the 'regime of truth', which stemmed from his notion of discontinuity. Foucault (1980) studied discontinuity in historical events –for example, discontinuity in science of medicine or psychiatry– to show how the certain gradual transformation of a type of discourse could profoundly transform ways of speaking and seeing, whole ensembles of practices and, ultimately, truth and knowledge. In particular, in relation to changes in medical discourse over a period of only 30-years, he says: "These are not simply new discoveries, there is a whole new 'regime' in discourse and forms of knowledge" (Foucault, 1977, p. 112). He argued that such changes in discourse and in what counts as 'truth' were neither a change of content nor a change of theory; they reflect, instead, a question of "what power" governs statements and the formation of the discourse to then produce "what truth". Truth in Foucault's view is not beyond power. He explains:

> truth isn't the reward of free spirits, the child of protracted solitude, nor

> the privilege of those who have succeeded in liberating themselves. Truth is a thing of this world: it is produced only by virtue of multiple forms of constraint. And it induces regular effects of power (Foucault, 1977, p.131)

This view of truth led Foucault to then see how each society has its own general politics of truth: 'the regime of truth', understood as the types of discourse which society accepts and makes function as true. For example, the work of Walkerdine uncovered the regime of truth produced by progressive approaches to mathematics education, including the idea of children as rational beings.

Our analysis of news media discourses has so far examined the moral dimension of a possible regime of truth, organised around a framing in which there are just two ways of teaching mathematics ('discovery learning' and 'back to basics'), one good and one bad (Barwell & Abtahi, 2015). News framing is often organised in terms of pairs of concepts, which readers come to see as logically connected (Price & Tewksbury, 1997). Such framings, if they become widely taken up, may become part of a general regime of truth.

In looking at news reporting, we must, of course, ask what power governs its production. News is, generally, a commercial business. News reporting must respond both to the journalistic requirement to inform, and the commercial requirement to attract readers. News, then, is not simply 'news'; it is carefully produced to appeal to an audience and both reproduces and shapes the way current issues are understood. Foucault's view about the interrelationships between the regimes of truth and discursive formation, such as ones journalists use to frame the news, provide a way to view the distribution of power within the dominant discourse of the newspaper reports to see how Canadian children are constructed in news reporting about mathematics education, in what senses they are constructed as actors, are acted upon, and whether they are ever constructed in a position of power.

METHOD

We examined three national Canadian print publications: the Globe and Mail, the National Post (both daily newspapers) and Macleans (a weekly publication), to represent a range of national news coverage. We collected all news articles on issues related to mathematics education within a six-month period (September 2013–March 2014). In total, we found 53 articles: 39 in the Globe and Mail, 11 in the National Post and 3 in Macleans. We began by looking at two categories, which we called actors and acted-ons. Actors consist of anyone portrayed in news reporting as doing or influencing mathematics education in some active way. Actors included teachers, government ministers, university mathematics educators, parents and sometimes students. Acted-ons consisted of anyone who was portrayed as subject to mathematics education in some way. Acted-ons included teachers (e.g., subject to the curriculum), parents (e.g., subject to government policy) and students (e.g., subject to teaching).

For this paper, we focus on our analysis of the construction of children. In this analysis, first we extracted from our corpus all utterances that included the words ‘student(s)’, ‘children’ and ‘child’. We found a total of 499 mentions in 348 statements/paragraphs. We then extracted all clauses in which any role (actor or acted-on) was attributed to the children, resulting in 233 such clauses. Then we looked at story lines across the entire corpus of extracted data. We looked to see how the reports are framed in such a way as to present particular images of children and their actions, the power they posses (if any) and the power(s) that are acting upon them. That is, we looked at how children were portrayed as actors and as acted-ons, and the narratives within which these portrayals were produced. For example, statements such as ‘the education system is to raise student achievement in numeracy’ (Globe and Mail, 3 December, 2013) [1] or ‘Students assigned to high value-added teachers are more likely to go to university and earn higher incomes’ both position students as acted-ons, constructing them as passive subjects of the education system. Along the way, we also noted the images or ways of thinking that were being denied or repressed in these representations. During all our analysis, we were aware that the immediate context of the construction of children in the news was situated within a broader political context.

CHILDREN AS ACTORS AND ACTED-ONS

Our critical reading of the articles revealed a predominant narrative about how the news media portrayed Canadian children, in relation to the educational system, in general, and to the learning of mathematics and their consequent and rather poor mathematical performance, in particular.

Children’s actions and reactions are framed within the broader politicised discourse of “discovery learning” and “back to basics” we have previously reported (Barwell & Abtahi, 2015). In the news media discourse, mathematics education was framed as a war zone; with the newly reformed mathematics education (i.e. discovery learning) on one side and the push for back to basics on the other in a ‘battle that’s been brewing for years’ (The Globe and Mail, 10 January 2014) [8]:

> The battleground is fractured and the sides aren’t clearly drawn, but at the centre of the debate over so-called discovery learning is this question: should the teacher be a stage on the stage, a guide at the side, or both?

On both sides, however, *children* were viewed as being in need of help; help from the education systems, teachers, parents and so on.

In general, we noticed an overarching storyline carrying an implicit judgment: it is good to perform well in mathematics – for example look at Germany, Korea, Poland and Singapore. The justification for such claims was often connected to the broader economic prosperity of the children (i.e., their success in future jobs) and of the nation as a whole. For example one report noted: “student performance in math matters [...] both for academic success and future job prospects” while not performing well would negatively affect “the future of Canadian students and the prosperity of the country” (The Globe and Mail 3 December 2013) [9]. The reports then go further to elaborate what “performing well in mathematics” means: for example, students should develop confidence and deep conceptual understanding, know basic skills and have memorized mathematics facts (The Globe and Mail, 28 October, 2013) [2].

To explain how Canadian children can perform well in mathematics, they were occasionally portrayed as actors and, much more often, as acted-ons. In the following sections, we elaborated on these two positionings.

Children as actors in relation to mathematics and its learning

Of 233 clauses that attributed some role to the children, only 64 clauses portrayed children as actors in relation to mathematics. Moreover, when children were positioned as actors, they were generally constructed as poor performers of mathematics. Other than in the eyes of the education ministers and with the exception, at times, of students of the province of Québec (The Globe and Mail, 6 December 2013) [3], Canadian students are constructed as not performing well: “they struggle”, “they fail”, “their abilities decline”, “they get weaker” “they worsen” and finally “they lose ground”. Children are also constructed as not knowing how to do mathematics, as in the following examples:

- “this [a word problem about apples] appears to be a fairly straightforward, multi-step operation, right? Not so fast. This is the type of math question that a great many Ontario Grade 6 students had trouble with according to the most recent EQAO provincial tests” (The Globe and Mail, 19 February 2014) [4]
- “the ability to work with such things as fractions, rates and how to convert one measurement into another is where students commonly fall down” (National Post 11, November 2013) [5]
- “Grade 9 students still using their fingers to calculate six times seven” (The Globe and Mail, 12 January 2014) [6].

Students are occasionally portrayed in more successful terms, although such instances were a minority, and often referred to students from elsewhere in the world or to particular sub-groups of students within Canada: for example, “Ontario student are among better performers”.

Moreover, in many instances, positive portrayals were still couched in relatively negative terms. In the following examples, some good performance is contrasted with more wide-spread poor performance:

- "30 per cent of Asian students performed at the "high achieving" level in math, compared with 16 per cent of Canadian students" The Globe and Mail, 9 January 2014) [7].
- "Quebec students performed heads above students from the rest of Canada" [2]
- "Canada's student performance, formerly well above the OECD average is now considerably less so" [7]

Thus, Canadian students do worse than Asian students, or only do well in Quebec, or are doing worse than previously.

Overall, then, children are constructed as actors relatively rarely (as compared with other actors in mathematics education) and this agency is generally constructed in negative terms of what children cannot do, or are failing to do, or should be able to do in mathematics, often in comparison with other, better-performing students.

Children as acted-ons in relation to mathematics and its learning

Of 233 clauses that attributed role to children, 169 clauses portrayed children as acted-ons. Instead of being active actors, both in the school system and in their own learning, children were more often viewed as acted-ons who needed to be worked on by different means and for different reasons. As noted in one report "The great value of education is to teach [children] skills they need to successfully find, consume, think about and apply it in their lives". That is: for students to become "Engaged", "Ethical", and "Entrepreneurial". They need to be prepared "for the world of dizzying change", or for "the real world". The role of the educational systems is thus portrayed as being able "to transfer expertise and real-world skills to students", who are constructed as passive subjects who must be moulded into suitable forms in order to survive in the world. This construction of children builds in both what must be done and *why* it must be done: children must be prepared for the future for their own good.

More specifically, in relation to mathematics education, the education system is there to "raise student achievement in numeracy" to "produce top students", "to give them better foundational math skills", "to enhance their mathematical learning", "to boost their outcome" and "finally to light the spark in their students".

In our corpus of news reports, then, the education system is constructed as acting on students to be able to consume, think and apply. They do so by developing curricula to be implemented by teachers, and by testing students' performance. Things do not always work out so nicely, however. Sometimes the curriculum allows open-ended student investigation, or encourages students to work with physical materials or to use complicated stuff (i.e. mathematical processes), all of which are frequently portrayed in a negative light. Some other useful materials, according to the back to basics frame, are constructed as absent from the curriculum, such as some basic skills, including memorisation of basic facts and standard arithmetic methods such as: “addition with a carry, subtraction with a borrow, long multiplication and long division” (The Globe and Mail, 12 January 2014) [6].

Children in the math wars

We have shown how children are constructed in largely negative terms in our corpus of newspaper reports, whether as actors or acted-ons. What might explain this portrayal? After all, we do not see such reports as portrayals of truth, but rather as producing a regime of truth. One explanation comes from the morality frame we have previously identified in our corpus (Barwell & Abtahi, 2015). Morality frames have several components, including a problem, a cause and a treatment (Entman, 1993). The most widespread problem with mathematics education presented in our corpus is that individual, national mathematical (and economic) performance is in decline. The cause is generally assumed to be the use of ‘discovery learning’ and the treatment is generally proposed as a return to tried and tested methods –back to basics. The moral dimension of news frames is apparent in reporting that seeks to “personalise the news, dramatise or emotionalise it, in order to capture and retain audience interest” (Semetko & Valkenburg, 2000, p. 96). That is, a moral dimensions responds to the commercial need to sell papers. The moral dimension of the framing of mathematics education in our corpus appears in the form of a narrative of national decline (in mathematics and economically), as well as through accounts of the suffering of students or their parents faced with present or future failure, or discovery learning methods.

This framing helps us to understand why children should be portrayed in such negative terms. News frames need first and foremost to identify a problem. It is difficult to think of how a problem can be constructed in mathematics education that does not involve a negative portrayal of children, particularly when the most salient indicators appear to be performance in tests or parental reports of their children’s struggles. If performance in tests is constructed as excellent, there is not much of a news story (unless some problematic dimension can be found).

DISCUSSION AND CONCLUSIONS

We have argued that news reporting of mathematics education in Canada builds a regime of truth in which children are necessarily constructed in negative terms: as performing poorly in mathematics and as passive subjects of trendy teaching methods, as promoted by misguided mathematics teacher educators and the mathematics curriculum. Indeed, this construction of children is a necessary consequence of the way newspaper reporting is organised –of its politics. To recap, news reporting needs a problem, a cause, a remedy and a moral judgment. Hence, in reporting mathematics education, Newspapers identify children's performance as the problem (despite, we may add, Canadian children being, in general terms, among the most mathematically well-educated in the world). In this case, children are the victims, a passive, negative construction. The cause of the problem is the 'discovery learning' approach and the solution is to go 'back to basics'. On either side of this dichotomy, children are constructed as being subjected to mathematics teaching. It is others who decide how children should encounter and learn mathematics: ministries of education, business leaders, teacher educators, parents or, in some cases, teachers. Moreover, children's poor performance, and its related cause, is morally bad. As a result, children come to signify or represent moral decline.

Unlike the construction of children identified in mathematics education research discourse, news reporting generally uses discourses antithetical to the construction of the progressive child. There is little sign of any liberation of children through mathematics education, or even of the benefits of developing mathematically rational thought. Instead, the utilitarian approaches noted by Wright are prevalent, emphasising economic performance, individual career opportunities and mathematical performance, rather than mathematical rationality.

The regime of truth we have started to uncover is problematic, because news reporting does not simply produce the regime of truth, it also recursively reflects what is circulating in wider society. That is, the negative portrayal of Canadian children –that seems to be a deeply entrenched structural feature of reporting on mathematics education– not only produces a regime of truth, but also reflects the beliefs of the general public. As mathematics educators, it is not clear to us how to respond to this regime of truth. We would therefore like to open up a space to think about and reflect on the type of discourses within which the MES community constructs its own regime(s) of truth with regards to children and their capacities. We ask if the acceptable truth(s) with which the community of mathematics educators views the capabilities of children clashes with those created by the news media. If a transformation of public discourse could ultimately transform the portrayal of children and their learning, do we not have an ethical responsibility to make the discourses of our community be heard?

NOTES

1. We're on it: education ministers respond to math results. By Johnson, J. The Globe and Mail, 3 December 2013.
2. Kids can't teach themselves math. By Mighton, J. The Globe and Mail, 28 October 2013.
3. Quebec might hold the formula to better nationwide math scores. By Peritz, I. The Globe and Mail, 6 December 2013
4. Why doing math in multiple ways helps kids. By Rodrigues, B. The Globe and Mail, 19 February 2014
5. CEO of Ontario's student testing agency worried by five-year decline in provincial math standard. By MacDonald, M. National Post, 11 November 2013
6. Stop bashing teachers. It's up to all of us to improve kids' math skills. By Vanberbutgh, I. The Globe and Mail, 12 January 2014.
7. Canadian education: The math just doesn't add up. The Globe and Mail, 9 January 2014.
8. Canada's fall in math-education ranking sets off alarm bells. The Globe and Mail. Dec. 03 2013
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TENSIONS AND DILEMMAS AS SOURCE OF COHERENCE

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As a context for professional development, Lesson study (LS) has been used widely and differently around the world. Here, we look at LS as a community of practice (CoP) in which teachers and researchers work together on a specific object of learning. We highlight two ethical dilemmas that emerged in one cycle of LS in a project in South Africa, in an attempt to explore the workings of that CoP. Both dilemmas were sparked by mathematical errors and discussed in the reflections after the lessons in which they occurred. The first was the teacher's dilemma: whether to address the learner errors that came up, or follow the produced joint-plan. The second dilemma was the observing teachers and researchers' dilemma: whether to interrupt a lesson in the face of an error made by the teacher on the board, or leave this for reflective discussion. While these dilemmas could become threats to the CoP we show that they created opportunities for strengthening the CoP and its coherence.

INTRODUCTION

Lesson study (LS) is seen as a professional learning community that is structured around lesson planning, teaching and reflection where a group of teachers, researchers or educators work together to plan a lesson on a selected topic, teach and reflect on it. One teacher will volunteer to teach the first lesson and the rest of the group will observe. After the reflection on the taught lesson, a new plan will be set to teach the same topic by another teacher with another group of learners. This planning–teaching–reflecting–reteaching–reflecting is called one cycle of the LS. Some LS cycles might teach the same lesson more than twice.

LS itself has different traditions and forms such as the Japanese (Fernandez, 2002), the Chinese (Gu & Gu, 2016; Huang & Shimizu, 2016) and Learning Study in Sweden (Marton, Runesson, & Tsui, 2004). LS has been extensively practiced and researched in different contexts around the world. In the Wits Maths Connect Secondary (WMCS) project in South Africa we have developed a LS process adapted to our context. We conduct LS to support ongoing professional learning of teachers who previously participated in Mathematics for Teaching courses within the WMCS (see below). We have adapted and shaped LS activity to suit our contextual conditions, and have shown that notwithstanding its different forms and functions, the WMCS LS is a learning space for both teachers and researchers (Alshwaikh & Adler, 2017), thus confirming research findings elsewhere (e.g. Lewis, 2016). LS is a productive space where knowledge and practice are co-produced by researchers, teachers and learners working together.

In this article, we explore on the functioning of our LS focusing on the team of teachers and researchers as a community of practice. We bring ethical dilemmas encountered during one cycle in LS into focus, and how these tensions between community participants played out in the CoP.

STUDY BACKGROUND AND RESEARCH QUESTIONS

Education in post-apartheid South Africa has been focused on overturning the damages and inequalities of apartheid education, and in the first decade emphasis was on opening access and equity (Venkat, Adler, Rollnick, Setati, & Vhurumuku, 2009). In the past decade, given increasing evidence that access had increased dramatically, this was not coupled with equitable quality in the system. Low learner performance particularly in poor communities and schools brought increased attention on teachers' knowledge and practice (For a detailed analysis of the SA mathematics education context see Adler and Pillay (2017)). The WMCS project emerged in this latter decade and has worked in its first phase (2010-2015) to support teachers in improving their mathematical knowledge for teaching (MKT) and their teaching practice too. WMCS offers a 16-day MKT course (Transition Mathematics –TM) at the University of the Witwatersrand, focused mainly on mathematical content, with some but less time on teaching mathematics, and then follow up LS in some school clusters.

The WMCS has adapted LS as a medium for teacher professional development. We conduct LS to support ongoing in-school professional learning of teachers who previously participated in the WMCS TM courses. One of our adaptations is the theoretical lens on teaching that we bring into this work. We have developed a socio-cultural framework for describing and interpreting shifts in teaching practice, called mathematical discourse in instruction –MDI. MDI has been developed over time within the project, and is grounded in the South African education context (e.g. Adler & Ronda, 2015; Adler & Venkat, 2014). The MDI framework provides a set of analytic tools to look closely at the practice of teaching mathematics and analyse what mathematics is offered for learners, how teachers teach and how learners participate in their own learning. As a socio-cultural framework, MDI privileges the development of scientific concepts (Vygotsky, 1978) in school mathematics, and thus views mathematics as specialised connected and coherent knowledge.

In a nutshell, MDI focuses on four elements of a mathematics lesson. (1) The object of learning or lesson goal, what students are expected to know and be able to do at the end of a lesson. (2) Exemplification refers to a sequence of examples, tasks and representations teachers use to bring the object of learning into focus. (3) Explanatory communication focuses on the word use and justifications and substantiations of mathematics as specialised knowledge. Finally, (4) learner participation focuses on what learners do and say regards the mathematics they are learning.

In our LS work with teachers in their schools, we use MDI as a structuring device to guide lesson planning and reflection. In order to communicate the MDI, we redescribe it as a Mathematical Teaching Framework (MTF). As Figure 1 shows, the MTF mirrors the components of the MDI framework and is intended, as already noted, to assist teachers in planning and as a reflection tool on the mathematical 'quality' of their teaching in Lesson Study.

Lesson goal:		
Exemplification Examples, tasks and representations	**Learner Participation** Doing maths and talking maths	**Explanatory communication** Word use and justifications

Figure 1: WMCS Mathematics Teaching Framework

While MDI in the form of the MTF provides analytic resources for working on and researching the improvement of mathematics teaching, our LS research needs further theoretical and analytic resources for investigating the functioning of the CoP, and how social relations are implicated in the developing mathematical work in the LS. To this end, we look at the team conducting LS as a CoP in which teachers and researchers work together on a specific object of learning. The team shares teaching as a practice, and as they engage with a specific task with a specific goal, the MDI and MTF become their shared repertoire (Wenger, 1998). While engaging in LS, the CoP creates its own 'enabling engagement' as a dynamic of inclusion as Wenger (1998, p. 74) describes:

> Whatever it takes to make mutual engagement possible is an essential component of any practice. (...) Being included in what matters is a requirement for being engaged in a community's practice, just as engagement is what defines belonging.

According to Wenger, mutual engagement is the first characteristic of a practice to be considered as a source of coherence of a community. In this article, we explore how a particular LS CoP faced dilemmas that emerged through answering a key question: How do CoP members act and react when they encounter conflicting ethical responsibilities in a LS, and with what consequences for the CoPs functioning?

With this as our question, and given space constraints we background the MDI/MTF analysis in this paper, and focus in on the CoP and its functioning. Emerging from the incidents discussed below, we found we needed to extend the membership of our CoP and bring learners into view. Learners' learning is the object of the CoP. Thus, while the teachers and researchers are mutually engaged with a lesson, their responsibilities in the moments of teaching extend to their learners' learning. We flag this up here, as learners were significantly constitutive of the dilemmas as these emerged.

DATA AND METHOD

Our LS process follows the mainstream method of LS (e.g. Yoshida, 2012) where teachers select a topic, plan, teach, reflect, reteach and reflect again. In this specific cycle, teachers chose to work on "simplifying algebraic expressions with brackets in different positions" with classes of Grade 10 learners. We were constrained both by the time teachers have available for this work, and the practical constraint that this needed to take place in the afternoon after school. Our LS cycle took place one afternoon a week for three consecutive weeks with the planning session in the first week. In the second week, a class of learners remain after school to participate in a teaching session, with reflection and re-planning meeting following the lesson. A repeat process then occurred in the following and third week.

In this particular cycle, four teachers from three different schools in Johannesburg and three researchers from the WMCS met in May 2016 for planning the first teaching. The almost a 1-1 relation between researchers and teachers was unusual. A practical reason for this is that in this LS cycle, the first author, who is relatively new in the project, was learning about LS, and so supported by the second author, project director, and another graduate student who had participated in a previous cycle. On the other hand, two teachers were in an advanced stage of their teaching and of their engagement with the WMCS. The other two teachers were relatively new to LS work.

We audio-recorded the planning discussion in the first week, and video recorded the lessons and reflection discussions. All these recordings have been transcribed and we draw on this data to describe two critical incidents.

Data analysis

In order to explore how a CoP in LS would act and react when they encountered conflicting responsibilities, it was important for us to examine these in relation to different participants and practices in the CoP: teachers and teaching, learners and learning, researchers and researching and the community functioning (in this article we take out the research and researching component from focus). In particular, we looked how the incident we describe below were related to these four dimensions of the CoP in order to judge the quality of the decisions made by teachers or researchers during the two incidents and how such decisions contributed to the coherence of the community based on our goals in WMCS using MDI/MTF tool. For example, to look at the quality of a decision made by a teacher and its impact on teachers and teaching we looked at the examples and tasks as well as explanatory communication and then make a general evaluation such as appropriate or not appropriate decision. Table 1 shows the analytical tool only for three components that we deal with here.

Table 1: Analytic tool and its components

	Indicators	Incident 1	Incident 2
Teachers & Teaching	Examples Explanatory communication		
Learners & learning	Learner participation		
Community	Joint plan and commitment Shared values		

TWO CRITICAL INCIDENTS AND DILEMMAS

Using the MTF tool, a joint lesson plan was produced by the end of the planning session through a discussion of the object of learning or lesson goal and then the related exemplification, explanatory talk and learner participation (Figure 2). Given the lesson goal (see below), the focus of the discussion was on how to bring the position of the brackets into focus through examples and tasks, and then connecting these with teacher's explanations and how to engage learners. In general, we agreed to start with numerical examples to exemplify the process of changing brackets before moving to algebraic expression. Briefly, the principles of variation amidst invariance (Watson & Mason, 2006) to bring particular aspects of mathematical objects into focus can be seen in the selection and sequencing of examples in the lesson plan. At the end of the meeting, it was agreed that T1 would teach the first lesson, and T2 the second revised lesson. After that meeting, T1 set a WhatsApp group including all relevant people, to facilitate communication between all in the LS group.

Lesson goal: Learners can simplify expressions with brackets when these are in different positions.

Pre-test assessment

Introduction/Introducing the lesson: Calculate the following:

1. $4+3(4+5)=$
2. $(4+3)4+5=$
3. $(4+3)(4+5)=$

Activity 1: Simplify the following

1. $x+3(x+5)=$
2. $(x+3)x+5=$

Activity 2: Simplify

3. $(x+3)(x+5)=$
4. $(x+3)+(x+5)=$

Activity 3: Simplify

5. $(x-3x)+5=$
6. $(x-3)x+5)=$
7. $x(-3x+5)=$
8. $x-(3x+5)=$

Activity 4: Simplify

9. $x-8(x+6)=$
10. $(x-8)x+6=$
11. $(x-3)(x+3)=$
12. $(x-3)-(x+3)$

Activity 5 (Post-Test): Simplify

1. $2p-(4+p)=$
2. $2p(-4+p)=$
3. $(2+p)+(-4+p)=$

Figure 2: Examples and activities as suggested in planning meeting-P1

Now we describe two incidents, one occurring in each of the two teaching sessions. The rationale behind selecting them is that they were identified as critical during the reflection sessions followed each teaching. By critical we mean an incident that raised a tension or a dilemma for/in the CoP. We start each incident by providing a general summary of what happened followed by detailed account of that dilemma, highlight the ethical responsibilities and the consequences for the coherence of CoP.

Dilemma 1: Learners' errors and responsibilities for teaching

The reflection session on teaching 1 began with the teacher having the opportunity to talk first about what she wished to discuss. Here is an excerpt from her reflection: [italics and bold are our highlights]

> One thing that I just thought like the like terms, the I felt that at some stage I needed … because *some maybe they* [learners] *don't know what are like terms*, so I didn't know whether I was supposed to talk about that one, address what is it … in detail. So I wasn't sure on that one because *I was just then focusing on the, the objective*. Because *you might find out that someone* [learner] *here they don't even know what like terms are*. (…)
> Remember number five in activity three … the main aim even though it ended up initially what *we agreed upon is that we are just investigating the position of the bracket;* did it make any difference? So they all [learners] answered that yes it will make a difference. But then I didn't know how, and that's why I had to ask them for solutions in the activity. But *we did not agree on that one [in our planning?].*

On one hand, T1 expected that there were some learners who do not know some prerequisite concepts for the current lesson. On the other hand, she felt committed to the shared plan that was produced in the planning session. This situation left T1 in dilemma: whether or not to keep teaching according the shared plan or addressing learners' errors. She also expressed her hesitance in tackling learners' errors because of the plan of the lesson, stressing that she "used exactly the same plan".

Teacher 1 paid deliberate attention to the position of brackets as planned, drawing learners' attention to what was similar and different about the different examples. After doing the introductory activity with the learners, she moved to the following activity and asked learners to solve them individually and later to share their responses with their partners. Then she started to discuss activity 1 stressing that she will continue as she did in the introduction by asking "*what is the same in these two questions and what is different?*" In this discussion, T1 also drew learners' attention to the anticipated error of conjoining variable and constant when solving question 1 in activity 1 [$\chi+3(\chi+5)=$] and warned learners that these were not correct "mathematically" as variables and numbers "are not like terms". This preventive step, however, did not prevent errors (see below). As she expected, when she discussed question number 4 in activity 2 [$(\chi+3)+(\chi+5)=$)] one learner gave an answer of 8χ with justification of the conjoining error ("*χ plus three which is three χ*").

T1 also mentioned the use of the distributive law in the introduction and invited learners to make use of it when needed as in question number 2 in activity 1 [$(\chi+3)\chi+5=$]. Later, she invited learners to work on questions three and four in activity 2. Later when T1 asked learners in a whole class discussion how they got the answers, the majority expressed that they used FOIL strategy, which is an acronym for a procedure used to multiply two terms with brackets: First, Outside, Inside, Last. In her discussion of question number 4 in activity 1 [$(\chi+3)+(\chi+5)=$], she asked for answers from different learners and wrote them on the board. The answers were 8χ, $2\chi + 8$ and $\chi^2+8\chi+15$, respectively. When asked how the answe 8χ r was obtained, a learner explained: "I said χ, χ plus three which is three χ times χ. (..) Plus χ which is five χ, [and that gives us] Eight χ." When T1 asked what other learners thought about that answer, only two learners publicly "disagreed" with the solution. T1 moved to discuss why conjoining is wrong by referring to the solution of a previous question to show that 8χ is not correct.

Comment

We consider the decision made by T1 as appropriate for the teacher and teaching since the examples were carefully planned to suit the expected capabilities of learners by teachers. As T1 commented above, she was not sure that the majority of learners (e.g. *someone*) were having problems. It is very common in South African schools to meet learners who face challenges in the current lesson because of lack of prerequisite knowledge. As a rule of thumb, we think that teachers' judgement as to whether to address learners' mathematical errors must be based on the number of learners who face challenges, and that is a subjective judgement. If teachers sense that the majority of learners are struggling with the current topic then it would be appropriate to address such challenges. If it is only a few learners, then it would be appropriate to proceed with the lesson and later support learners who face challenges.

Having said that, we ask what might be expected consequences of such judgements and decisions made by T1 on learners and learning. There is no clear answer here. For some learners and learning such decision is appropriate because they feel competent to proceed in the current lesson and they may feel neglected if the teacher addresses what they already know. On the other hand, such a decision could further alienate learners who do not know the prerequisite knowledge.

As for the community of practice, we think that T1 made an appropriate decision in following the joint plan. Such decision, we argue, reflected the responsibility and accountability that T1 felt towards the community, as well as enabled the different participants of the CoP to feel the value of the shared practice in planning and thus created a sense of coherence to the community. As a whole, we think that the decision made by T1 was suitable decision within the context of the LS discussed and it created opportunities for coherence within the CoP. See Table 2 for a summary of the consequences discussed here.

Dilemma 2: Teacher's error and consequential responsibilities

Despite the persistent errors of some learners in the lesson, T1 felt that the lesson was easy for learners –even though she did not know the reason. The other teachers agreed with her. As a result, the CoP discussion changed activity 3 to the following:

Activity 3: insert bracket(s) in the expressions on the left side so that the two sides are equal

1. $\chi - 3\chi + 5 = -3\chi^2 + 5\chi$
2. $\chi - 3\chi + 5 = -2\chi^2 - 5$
3. $\chi - 3\chi + 5 = \chi^2 - 3\chi + 5$

In re-teaching the lesson, Teacher 2, included this activity. He stressed the use of the distributive law many times in the introductory activity and activities 1 and 2 (Figure 2). When he solved the first question in activity 1 with learners he also raised the issue of the conjoining error, and he extended his discussion about like- and different-terms. When T2 introduced activity 3, learners complained that that activity was difficult. To exemplify what was required, T2 inserted a spontaneous (unplanned) example. As he worked on this new example, however, he introduced a mathematical error. A dilemma was now faced by the observing researchers and some of the teachers (as not all noticed the error). Do we interrupt the lesson and if so how?

The example T2 added was $3\chi+2\chi-3=-9\chi-6\chi$. T2 showed learners that he would get a different result if he placed the brackets in a different position between 2 and χ $[(3\chi+2)\chi-3]$. Then he inserted brackets around $3\chi+2\chi$ suggesting the solution as $(3\chi+2\chi)-3=-9\chi-6\chi$.

In the reflection on this incident, when researchers raised a question as to the placing of the brackets in this example, T2 did not see immediately what we were referring to. While reflecting on this example, T2 mentioned that he wanted learners to see (-3) as an integer until one of the researchers asked him what he would do if the sign in front of 3 was plus [as $(3\chi+2\chi)+3=-9\chi-6\chi$], he said "oh, okay, no!". Another teacher in the group commented that she now felt 'guilty' towards her own learners as she has been making the same mistake in her teaching. Thus it seems that some teachers in SA may share this misconception with these two teachers.

Nature of the dilemma

The dilemma in this episode is different from the previous episode. Here the teacher himself makes a mathematical error. In this case, the observers were not sure whether or not to interrupt the lesson and if so how. As researchers, although we felt ethical responsibilities towards teaching, learners and learning and the functioning of the CoP, our main concern was the teacher himself. Although we looked at each other wondering what to do, none of the researchers interrupted the teacher, and this was thus left for the discussion and reflection after the lesson. When we raised this issue with T2 during reflection, he was clear that we should have interrupted, but done so by asking a question of him that would have given him the opportunity to think it through and deal with it in the lesson:

> for example, … you can ask … is it not that negative three, is it not subtracted from the expression? And then after thinking well what I'm thinking this is an integer it's multiplying into that bracket. But there's a negative sign and that number is in is subtracted. It could have given me some way to think.

When we commented that we were not sure about the consequences of interrupting with a question especially with respect to his relation with learners (losing confidence, for example), he commented:

> If you stand up and you disagree with me like face to face then that's when they will see there's a problem and now you've created an impression to them. But you don't make it as if you've seen, you make it as if you can't see, you're asking.

What was interesting in T2's comments is his confidence and openness to working with the error during the lesson and so for the learners too. He stands as a good example for participation in a professional learning community.

While our decision was appropriate for the teacher, we think that it was not appropriate for teaching and learners and learning. Even though the teacher has a strong mathematical knowledge, making an error while teaching may have consequences on learning and learners such as building misconceptions. As for the CoP, we think that such decision provides a safe space for teachers not to be judged and to share their thinking aloud.

To sum up, we argue that the decision made the researchers was appropriate one for the CoP, its shared values, responsibility and, hence, its coherence. At the same time the decision is questionable for its consequences on learners and learning. Table 2 shows the summary of the consequences discussed in both incidents.

Table 2: Summary of the consequences of researchers' decision in incident 2

	Incident 1	Incident 2
Teachers & Teaching	Appropriate decision.	Appropriate for teacher Not Appropriate for teaching.
Learners & learning	Arguably Appropriate decision.	Not Appropriate.
Community	Appropriate for the coherence of the community.	Appropriate for the coherence of the community.

DISCUSSION AND CONCLUDING REMARKS

In a previous article (Alshwaikh & Adler, 2017) we have looked at two incidents from two lessons in a LS cycle and through these focused on three different areas of learning: mathematics teaching, mathematical knowledge and doing lesson study itself. In the article, we turned our lens towards the dilemmas created by those two incidents and the consequent ethical responsibilities. We identified four of such responsibilities each of which towards teachers and teaching, learners and learning, researchers and researching and the CoP in LS.

The responsibilities towards teachers and teaching were raised when a teacher faced a dilemma between addressing the not unusual errors learners in SA make or following the joint plan produced by the team in this cycle of LS. The teacher made an appropriate decision by keeping her track in following on the plan and leaving the errors out of focus despite the fact this decision was not easy for her. We, observers and researchers, felt similar way when Teacher 2 did the mathematical error. We did not know if we should interfere or not because of the consequences we imagined. The suggested clue by the teacher himself helped us how to approach similar incidents when we conducted the second cycle of LS in which we talked to the teachers in private raising questions about specific issues without interrupting the lesson.

As for learners and our issues about the mathematics they learn with some mistakes, we again felt that tit was wise not to interfere. One simple reason is that such incidents occur in teaching as other professions. We are 'there' once or twice and teachers are with them all the year if not more and teachers can find their way to correct mistakes when happen. All what we need, we decided, is to talk to teachers in a professional way in our community.

The responsibilities towards the community were manifested in Teacher's 1 sense of accountability towards the community whether to follow the joint plan or learners' errors, and with us as researchers when Teacher 2 made the mathematical error. Both aspects raise a question about the mechanism of a professional CoP in LS, a needed research in LS to understand the instructional improvement (Lewis, Perry & Murata, 2006). Both teachers and researchers in the two lessons made good decision by not interrupting the plan of the lesson or the lesson itself and left the controversial issues for discussion and reflection. This created a 'safe' space for participants of the community in which they felt something like "let us leave it to the community and see." There is a sense of mutual accountability in both cases. "These relations of accountability" according to Wenger (1998)

> include what matters and what does not, what is important and why it is important, what to do and not to do, what to pay attention to and what to ignore, what to talk about and what to leave unsaid, what to justify and what to take for granted, what to display and what to withhold, when actions and artifacts are good enough and when they need improvement or refinement. (p. 81)

Thus, instead of seeing dilemmas as threat to the community, which they might be in some cases, this shared safe space and sense of accountability, we argue, maybe seen as a source of coherence for the community of practice. In Alshwaikh and Adler (2017) we suggested the need for agreement on code of practice in doing LS clarifying the roles of different participants. While we agree that formation of community of practice is a process and that participants come to know the "code" of negotiation and mutual accountability, etc. through practice we think that there is a need to make the issue of dilemmas and accountability public during that process without undermining the agency of the participants.

A final comment is the relation between understanding the mechanism of a community of practise in LS and instructional improvement as highlighted by Lewis, Perry and Murata (2006). This article can be seen as a response to their call by offering some insight on that mechanism as it happened in our experience.

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LOVE AND BULLYING IN MATHEMATICAL CONVERSATIONS

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In this paper we show the ways love and bullying appear in mathematical communications. We developed an analytic frame that distinguished between responsiveness and dismissiveness, and that identified whether communication acts opened dialogue or closed off the voices of others. The frame helped us identify what authority was used to close dialogue, and how dialogue was opened up as well. The findings allowed us to illustrate how responsiveness and opening up dialogue are central to love and mathematically productive, but also to problematize this argument.

INTRODUCTION

While we were reading and analysing transcripts in a context of Canadian 15-year olds doing a mathematical investigation, we became captivated by the very different ways the students used their language to either support their learning and their relations with each other in a loving and caring way or to bully or be mean to each other. The mean interactions raised for us questions about the way mathematics was intertwined with the bullying. This drew our attention to similar intertwining of love and care with mathematics. These relationships between mathematics and ways of interacting are fundamental to students' learning experience, and are enacted through communication acts in mathematical conversations.

We claim that one cannot understand communication about mathematical processes without understanding that these acts are also part of repertoires for other discourses that are intertwined with mathematics. This is true for students trying to understand each other, students trying to understand teachers, teachers trying to understand students, and researchers trying to understand students and teachers. Most relevant to this article, communication acts in a mathematics classroom can be seen as moving mathematics forward (or not) but also as an articulation of love or of bullying. We say these things are 'intertwined' (Andersson & Wagner, 2016) because it is not that a mathematics discourse supports bullying or vice-versa. One does not necessarily dominate over the other –they are intertwined. The mathematics gives a context in which I can bully or love someone, and the bullying or love can give a motive for mathematical moves.

We begin with attention to the literature's treatment of the oftignored concepts of love and bullying mathematics contexts. This follows with examples from our research, in which we identify the communication acts and positioning of students in relation to the mathematical and relational moves.

LOVE

Conceptualizations of love comprise a variety of different feelings, states, emotions and attitudes. These legitimate conceptualizations may range from pleasure and care to interpersonal affection. In this article, we understand love as the virtue that represents kindness and compassion as "the unselfish loyal and benevolent concern for the good of another" (http://www.merriam-webster.com/dictionary/love). We see love as attentive to the concerns of others, and responsive to them. Hence, love is an expression of positive sentiment. Because antonyms are contingent on perspective (Visser, 2000), we identify two possible antonyms for the kind of love we identify –*hate/antipathy*, as in “I hate mathematics,” or *apathy* as in “Mathematics is boring.”

When we browsed through the (prominent and English) mathematics education research journals with a search on the words *love* and *mathematics* together, rather few articles showed up. In those we found, *love* was used in a variety of ways, most often to describe relationships with mathematics and/or with mathematics education:

> Mathematics is joy and exasperation. It is beauty and power. As mathematicians we love elegant solutions and the way that the same mathematics can be applied to different contexts. (Wood, 2011, p.5)

Hossain, Mendick and Adler (2013) showed that a student teacher’s choice of teaching mathematics was about her love for the subject: “after all the subjects I looked at, I thought that I loved maths the best” (p. 45). The teacher’s desire to teach was inseparable from her desire to teach mathematics.

We problematize the idea of loving mathematics or school mathematics with reference to Wagner and Herbel-Eisenmann’s (2009), use of positioning theory. They note that there is no actual presence of mathematics in any situation; it is only ever present as mediated through people and texts. Thus we ask what or whom people love when they say they love mathematics? Love or attachment to some mathematical processes may not go hand in hand with loving people with whom one interacts mathematically.

Esmonde and Langer-Osuna (2013), in the context of a year ten heterogeneous mathematics classroom involved in student-led work showed how the figured worlds of friendship and romance allowed disadvantaged students to position themselves more strongly in a mathematical figured world. In one of their examples, a student (Dawn) positioned herself more powerfully in the two friendships and romance discourses and hence engaged further in mathematical practices. Romance and friendship discourses may share commonalities with love but are not exactly the same.

The literature on affect in mathematics education also connects to love. For example, Debellis and Goldin (2006, p. 137-138) argued that "[i]ntimate mathematical *experiences* include emotional feelings of warmth, excitement, amusement, affection, sexuality, time suspension, deep satisfaction, 'being special', love, or aesthetic appreciation accompanying understanding." They also pointed to the fact that individuals' accounts of loved ones –for example, 'a parent being proud of me'– point to experiences that are "more than merely enjoyable or otherwise positive; they *build a bond* between the personal knowledge constructed and the mathematical content" (p. 138). In other words, these researchers argued that emotional feelings are important for the students' loving relationship with mathematics and/or mathematics education (or unloving relationships) and that these emotions may originate in love, care and appreciations for significant others.

We also found the idea of *pedagogical love* exemplified by the Finish teacher Sirpa in an account written by Kaasila (2007). Sirpa's thoughts about her teachings and relationships to her students "is captured in her use of the utterance of pedagogical love: 'I want to be a teacher who really cares how she acts. Pedagogical love –the word feels suitably descriptive. It includes caring for oneself as a teacher and as a human being" (p.380). This reminds us of the idea of *fostering by* in the ways Ms. Bradley acted in her classrooms, as described by Bonner (2014, p.395):

> Ms. Bradley's classroom was highly structured and disciplined, focusing on high expectations and success through "tough love." When a student did not have his or her homework, for example, Ms. Bradley would take the student in the hallway to call his or her parent or guardian. Furthermore, if a student was not participating in the group chants or problem-solving activities, Ms. Bradley would "call him [or her] out and take him [or her] to church," meaning she would stop the lesson and "preach" about the decisions students were making and the importance of academic success.

Long (2008) suggested that student mistakes provide another context for examining care. She argued that teachers may respond to students' mistakes in two different ways, with a tension between either caring for the student or caring for the idea of mathematics. We see this tension as analogous with the tension identified by Bartell (2013, p.140) as "a tension in negotiating mathematical and social justice goals." We reflect that pedagogical love seems to suggest that the acts of love are separate from the disciplinary work "I teach mathematics and I also show love to my students." In contrast, we argue that we can love (or show love) through the way we do (or communicate) our mathematics. And by contrast, we can bully using mathematics.

BULLYING

Intimidation and humiliation are typically present in bullying situations, among school children, among adults or between adults and children. Bullying may include threat, physical assault, or what we noticed in our data –verbal harassment and sarcasm. We follow the definition of bullying by Juvonen and Graham (2014, p. 161) who described bullying in contexts involving inequalities such as when a "physically stronger or socially more prominent person (ab)uses her/his power to threaten, demean, or belittle another". The authors emphasised that this specific power imbalance distinguishes bullying from conflict and that repetition is not a required component for bullying. Thus bullying can be seen as an antonym for loving.

A large body of research shows that bullying significantly impacts students' achievement in general (c.f. Nakamoto & Schwartz, 2009). Bullying has been described as one important reason for not pursuing mathematics (Sullivan, Tobias & Mcdonough, 2016) As an example of this, we also found Lutovac and Kaasila's (2014) narratives of a pre-service teacher (Reija): "Reija's self-confidence and participation in math classes worsened in secondary school because her classmates bullied her: 'I tried to be as invisible as I could'" (p. 136). Reija did not pursue her mathematics education.

Some mathematics educators have described mathematics itself as bullying. In the foreword to a book by Davis (1996), Pirie wrote:

> Let us construe mathematics not as a human endeavor, but as itself a living being. [...] It is saying "here is a way of being, and in this being lies a (but not the) potential for growth and change." How might such a re-envisioning of mathematics affect the classroom? How much less frightening might it seem to children? What if they were encouraged to resist its bullying and to see its problems as living possibilities and not as mandated chores? (p. xiv)

Finally, we point to research that describes tense relationships among students when doing group work. Kurth, Anderson and Palincsar, (2002) showed that students may fail to learn during group work due to the fact that they need to achieve intersubjectivity in relation to both knowledge assimilation and obligation within the group. These researchers concluded that "Even when [teachers] cannot monitor directly what is happening in collaborative groups, [they] still have considerable influence over how students construct the floor, both in terms of who holds the floor and what topics the students consider 'on task.'" (p. 310). Andersson (2011) came to similar conclusions. In a critical mathematics education context, three students were working together and experiencing some tension –not to the point of bullying though. One of them commented on the classroom blog: “This was really meaningful and it was good to take personal responsibility for planning and for our own labor. But this is new; we have to practice this way of working” (Andersson & Valero, 2015, p.212). The problem for this group was not the mathematical task at hand, it was how to collaborate when exploring mathematics.

Summing up, the research on bullying indicates that bullying occurs in interactions –hence through the use of language. This is specifically important to consider for teachers when students do collaborative work in mathematics as it not only impacts students’ wellbeing but also their achievement –even if they love the subject.

POSITIONINGS

Our analysis in this article will focus on communication acts in group work. We try to identify relationships of love and bullying on the basis of particular communication acts. Our conceptual frame comes from a recursive process starting with the data that prompted our attention to the phemomena, connecting that to literature and conceptual frames we knew, finding difficulties, modifying the frames, trying again, etcetera. We emphasise that the development of the frame was intertwined with the analysis. Figure 1 gives an overview of the frame we developed.

The conceptual frame for identifying authority structures refined by Wagner and Herbel-Eisenmann (2014) helps us see how students make space for each other in their interactions, which relates to the idea of being responsive to each other or not. We identified three different ways of opening space or acknowledging that others have autonomy. First, *explicit invitations,* in which someone asks for another’s point of view. In another authority structure, someone contributes an idea but identifies that it is a possibility or that someone else might think differently. We call this *spoken as possibility*. Finally, explicit reference to choices, which we call *identifying decision*, demonstrates awareness of potential alternative possibilities.

Following our reading and reflection on literature describing love and bullying, we use the distinction of *responsive versus dismissive*. This distinction appears in the horizontal axis of Figure 1.

		Responsive (R)	Dismissive (D)
Opening space (heteroglossia) (inviting)	Spoken as possibility (SP)		
	Invitation (I)		
	Identifying decision (ID)		
Closing space (monoglossia) (restricting)	Personal authority (PA)		
	Discourse as authority DA)		
	Spoken as shared (SS)		

Figure 1: Conceptual Frame

The excerpts below come from the same class, to emphasize that loving and bullying are likely present in every class (which is not the same as saying they all belong in the classroom). We selected example excerpts that show the relationships clearly and other excerpts that demonstrate complexity. We think it is important to choose episodes that push at the boundaries of conceptual frames.

INTERACTIONS THAT SUGGEST LOVE

This first excerpt is chosen as a relatively clear indication of love among the four students in the group. We include in the right hand column of the transcripts our coding using the acronyms from Figure 1.

A15	Nadia	Do you have to write the question down?	DA/ID-R
A16	Rachel	I don't know.	SP-R
A17	Andrea	I wouldn't, I'd say that just one block has no red.	ID-R
A18	Nadia	Um, how many have one red face?	DA -D
A19	Andrea	So, it would be like... wait...	SS-R
A20	Emma	[points] This one...	SS-R
A21	Andrea	This one. [*spoken in rapid succession*]	SS-R
A22	Rachel	Two. [....*rapid succession*]	SS-R
A23	Nadia	Two, three. [... *rapid succession*]	SS-R
A24	Andrea	Four. [... *rapid succession*]	SS-R
A25	Nadia	Two, four, six. [... *rapid succession*]	SS-R
A26	Andrea	Six. [... *rapid succession*]	SS-R
A27	Nadia	Yeah. [... *rapid succession*]	SS-R
A28	Andrea	It would be six. [... *rapid succession*]	SS-R
A29	Andrea	because there was... Is it only six?	SS-R SP-R

A30	Rachel	We'll figure it out at the end. We'll see if we've accounted for twenty-seven blocks.	SS-R
A31	Andrea	Okay.	ID-R
A32	Rachel	So, two red faces ...	DA-D/R
A33	Nadia	two red faces, okay, so...	SS-R

We first consider how the group participants open and close space for each other's views. Nadia (turn A15) appealed to the authority of the discourse (DA) asking if they "have to" write. Perhaps this is what they were accustomed to from their usual mathematics classes. However she also posed this as a question and thus identified the group's latitude to make a decision (ID), which opens a space for her partners' ideas. Rachel's matter of fact response "I don't know" (turn A16) indicated her awareness of possibilities (SP). And Andrea kept the space open (turn A17) using the modal verb *wouldn't* to indicate her choice (ID). Significantly, this open space, already communicated by three of the students in succession, leads into Andrea's opening of mathematical possibility, with a conjecture –"I [would] say...". Nadia continued the open approach to the mathematical exploration by inviting a decision (ID) (turn A18). This led into a rapid series of observations, which we coded as spoken-as-shared (SS) because the students were trying to answer their question. While this structure is generally a closed approach, its appearance in response to a group's question may also be interpreted as honouring her opening up question. Eventually (turn A29), Andrea again asked a question identifying possibility (SP), and Sara responded with an expression of confidence in the group, immediately followed by a means for checking their work. Andrea's response (A31) indicates the expectation that agreement is necessary (ID), and Rachel, returned the group back to following the instructions and personal authority (PA) of the teacher.

In this environment of openness to each other, we coded the students as responsive to each other (R), except when students brought the groups' attention back to a question on the instruction sheet (turns A18 and A32). However, even these moves that apparently ignore (or dismiss) what comes before (D), may in fact be tacit recognitions that the group was ready to move on, in which case one could claim that even these moves to follow the instructions demonstrated responsiveness to the group's progression.

The responsiveness is an indication of sensitivity to each other, and even the moments of returned attention to the task given by the teacher may be deemed as sensitivity to the group's desire to move forward mathematically. These are indicators of care for the other, or what we call love. Other indicators of this love include the use of the pronoun *we*, indicating a sense of solidarity, and the intense series of observations in turns A21 to A28, in which the four students were huddled close together, with their hands in close proximity, pointing to parts of the cube model. We reiterate that the mathematical work is also productive within the open and caring interaction in this group.

INTERACTION THAT SUGGEST BULLYING

We pick up the conversation a little further along when a group of boys is engaged with the mathematical task.

C35	Caleb	For the eight corners, you're gonna have three sides facing. One, two, three. So, eight times three?	SS-D I-R
C36	Rob	Fifty-four.	SS-R
C37	Dennis	[*Laughs*] No! You're a dummy.	SS-R
C38	Tim	[*Interrupting*] Twenty-four.	SS-R
C39	Caleb	So, red faces. We'd have to have eight because there's eight corners, three sides exposing. There. One, two, three, ... eight corners. One, two, three, four, five, six, seven, ... For that you just write beside it, "eight."	DA-D SS-D PA-D

In turn C35 Caleb was responsive to the task set by the teacher and to Rob's move to turn attention to that task. He was responsive to the personal authority (PA) of one or the other. He spoke as if the others would make the same conclusion he made (SS) –"you're gonna have", which closed the conversation space, but then he opened it up by inflecting "eight times three" as a question, inviting response (I). Rob was responsive, and answered with no sentence, just a number (SS) in turn C36. Unfortunately, his answer was incorrect. Dennis responded with ridicule, and Tim responded with the correct answer. Caleb (turn C39) ignored the wrong answer and the responses to that wrong answer, turning attention again to the discourse as authority –"we'd have to have..."– and continued speaking as if there were no options (SS). He finally instructed the others what to do, as if he had the personal authority to do so (PA).

This interaction is a clear instance of responsiveness not being loving –as Dennis responds with ridicule to Rob's mistake. We could instead say he is dismissive of Rob's worth. Following this, Caleb dismissed his group mates' interaction, ignoring it really. This move may have been an act of love for Rob –forgiveness of his mistake, and rejection of Dennis's ridicule. This episode demonstrates how dismissiveness and responsiveness do not neatly map onto bullying and love. It also exemplifies the way mathematics can be a refuge for people who are bullied, which is a reality both of us have seen in our school teaching experiences.

The last transcript is further into their conversation. The group has not moved forward very much mathematically. Caleb was still the one taking leadership but was beginning to realize that his mathematical ideas might not work.

C78	Caleb	[*Takes the cube*] because in the middle here, it's covered because one, two, one, two,...	SS/DA-R
C79	Dennis	Oh, so only the ones in the middle.	SS-R
C80	Caleb	That's what he was saying, we have trouble.	SS/SP-R
C81	Dennis	I am an idiot.	SS-D
C82	Caleb	We all are.	SS-R
C83	Tim	[*speaking to Dennis*] Compared to Rob, you're a genius.	SS-D
C84	Rob	[*Not speaking, he stands up dramatically*]	
C85	Dennis	I'm really not guys. I am bad at math.	SS-R

In turn 78, Caleb took the cube as if they belonged to him and explained his idea to the others as if it was the only possible answer (SS). Because his communication included explanation he was harkening to the mathematics discourse as authority as well (DA). He continued to close the space for the others' contributions –even if it seems as if Dennis have shared some possible ideas earlier (turn 79 and 80). Dennis was responsive to Caleb's explanation, and continued with the sense that there is only one correct path (SS). Caleb responded to this and referred back to a statement by Tim, which showed Caleb's realization that something was amiss in his thinking. The form of his communication –"we have trouble"– suggests that everyone agrees, but it also acknowledges that there may be more ways of seeing the situation (SP). Dennis's realization that he was being an "idiot" was a judgment of himself, and thus ignored the others. After this, the positioning within the group went awry, with more ridicule of Rob (turn C83). The space for Rob to speak or otherwise contribute is closed, almost locked.

While the group members express their feelings of stupidity, there is again especially strong ridicule of Rob (turn C83). We see that the mathematical task along with the traditions of mathematics classroom storylines, positions the group in this way –feeling stupid, ranking their levels of stupidity, and thus positioning one person especially poorly. We ask if there is any love here. Perhaps there is a sense of care among Caleb, Tim, and Dennis as they form solidarity around the exclusion of Rob. Perhaps Caleb's recognition that the whole group was on common ground as they could not come to terms with the problem (turn C82) was a way of expressing camaraderie in recognition of the feelings of stupidity. Nevertheless, the one-right-answer tradition of school mathematics seems to have laid the ground for feelings of ineptitude and for ranking, which positions one or some as the lowest of the low.

DISCUSSION

We can say from the very short excerpts analysed above, that the way communication happens in mathematics classrooms can open or close space for the other, and that this impacts students' experiences of love, bullying, and separateness.

It is important to emphasize again that all of the incidents described above come from the same class. Whatever the teacher and school culture (including other students) did before the day of these mathematical interactions facilitated interactions of love, interactions of bullying and solitude. We argue that what happens in a classroom cannot be laid at the feet of the teacher or school administration alone. There are larger discourses of school and of school mathematics that strongly influence the students' perceptions of what they could do and what they should do in mathematics class. And these discourses also significantly direct and constrain the teacher and other school personnel.

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BE THE BEST VERSION OF YOUR SELF! OECD'S PROMISES OF WELFARE THROUGH SCHOOL MATHEMATICS

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This work aims at tracing, from a Foucaultian perspective, the taken-for-granted truths, the promises, in OECD´s discourse on a state of welfare. These promises are built under the assumption that mathematics skills are needed to achieve citizens full potential. OECD's reading of 2012 PISA's outcomes reveals how discourses of numeracy proficiency are entangled and displayed for the making of a specific type of productive citizen for society. Also, this work problematizes OCDE's expressed 'desired citizen', by building in the double gestures of hopes and fears produced by the intention of include "all" student and of equity.

INTRODUCTION

There exist some circulating discourses about what is important for citizens 'to know' and 'be able to do.' OCDE has presented within the several documents released an approach which "reflects the fact that modern economies reward individuals not for what they know, but for what they can do with what they know" (OECD, 2014, p. 24). Mathematics has been taken, by OCDE, not as a knowledge students should acquired, but as a necessary skill –proficiency– for personal fulfilment, for future employment, and also, for a full participation in society (OECD, 2014).

> With mathematics as its primary focus, the PISA 2012 assessment measured 15-year-olds' capacity to reason mathematically and use mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena, and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens. Literacy in mathematics defined this way is not an attribute that an individual has or does not have; rather, it is a skill that can be acquired and used, to a greater or lesser extent, throughout a lifetime (OECD, 2014, p. 17).

Several studies in the field of mathematics education have already problematized the assumption of "mathematics for all" (see, e.g., Diaz, 2013; Valero, 2013). Nowadays, there is a new 'truth' added to the one of "mathematics for all". The taken-for-granted truth that has been currently (re)produced is the need for "inclusive and equitable quality education and promote lifelong learning opportunities for all" (OECD, 2016a, p. 13), as part of the sustainable development goals for education. This result ought to be achieved by a series of indicators that "spell out what countries **need** to deliver by 2030" (Op. cit., p. 13, emphasis added), with the aim of promoting social progress. Within these discourses, numeracy –as the ability to "access, use, interpret and communicate mathematical information and ideas in order to engage in and manage the mathematical demands of a range of situations in adult life" (OECD, 2016a, p. 38) –becomes a key skill to achieve social progress and, also, to ensure welfare.

This work aims at tracing the naturalized truths or the promises (re) produced by OECD's discourse toward a state of welfare and social progress. These promises are built under the assumption that the 'proper' acquisition of mathematics skills –numeracy– is needed for citizens to achieve their full potential, making them able to excel and, therefore, to have better lives. OECD's reading of 2012 PISA's outcomes takes an important role, given that it reveals how discourses about numeracy proficiency have been entangled and displayed for the making of a productive citizen for society. Finally, this work, by portraying the desired subject, problematizes the undesired citizen through 'abjection', as the double gestures of hopes and fears produced by the intention of including 'all' students and of equity.

OECD'S PROMISES OF WELFARE

Over the years, OECD has been positioning itself as the main global network for the (re)production of policy discourses toward economical progress. Tröhler, Meyer, Labaree, and Hutt (2014) recognize OECD as a central node for local and peripheral policy ideas to be expanded on and amplified. In this regard, "[p]olicies that might have a hard time becoming accepted in local contexts seem that much more irresistible when offered as uncontested consensus of the world's leading democracies" (Op. cit., p. 2). And, therefore, many countries take into consideration what OECD states as guiding for the development of their national agendas. After the last economic crisis, OECD published several indicators and strategies for countries to improve and invest in their educational programs, targeting the making of a particular type of citizen.

The claim "be the best version of yourself" is a motto that portrays the many discourses circulating about social progress and the necessary skills for citizens to achieve better lives and for countries to achieve a greater economical progress.

Be the best version of yourself

> Equipping citizens with the skills necessary to achieve their full potential, participate in an increasingly interconnected global economy, and

> ultimately convert better jobs into better lives is a central preoccupation of policy makers around the world –poor skills severely limit people's access to better-paying and more rewarding jobs. The ongoing economic crisis has only increased the urgency of investing in the acquisition and development of citizens' skills –investing in structural reforms to boost productivity, such as education and skills development, is key to future growth. More and more countries are looking beyond their own borders for evidence of the most successful and efficient policies and practices –in a global economy, success is no longer measured against national standards alone, but against the best-performing and most rapidly improving education systems. PISA 2012 results show wide differences between countries in mathematics performance –all countries and economies have excellent students, but few have enabled all students to excel. PISA is not only an accurate indicator of students' abilities to participate fully in society after compulsory school, but also a powerful tool that countries and economies can use to fine-tune their education policies –every country has room for improvement, even the top performers (OECD, 2014, pp. 3-4, bolds removed).
>
> OECD. Better policies for better lives

The above quotation is a summarized part of OECD's *PISA 2012 Results: What Students Know and Can Do.* In this publication, OECD states a concern regarding the economic crisis and it poses the urgency of having well-equipped-citizens. In this short text, many naturalized truths are entangled to (re)produce a discourse in which education is believed as the key to future growth of nations. For example, that few countries have enabled all students to excel, and, so, even top performers have room for improvement. This discourse embodies the idea of an ever-growing economy for social progress and of lifelong learners to achieve that goal.

It has been said that, in an ever-growing economy, citizens should be equipped with the necessary skills to achieve a state of welfare, given that educational attainment is used as an alternative measuring for human capital (OECD, 2015). These necessary skills –such as literacy and numeracy– should enable citizens to reach their full potential. A full potential not only regarding students performance in PISA or in national standardized assessments, but also 'high - attainment - individuals' "generally have better health, are more socially engaged, have higher employment rates and have higher relative earnings" (OECD, 2015, p. 30). This statement means that it is taken as possible to correlate higher levels of educational attainment and positive individual and social outcomes, resulting in a state of: 'the higher educated, the better health, the better job, and therefore, the better life'.

According with OECD's readings on PISA's outcomes, higher educational attainment is due to literacy and numeracy skills. However, it has also been stated by OECD that "[c]ompared to literacy skills, numeracy skills have a more significant impact on employment outcomes" (Op. cit., p. 168). Henceforth, the odds of being employed increases directly proportional to the level of numeracy proficiency (see chart A9.2 in OECD, 2015). It is also stated by OECD that numeracy proficiency has an impact even on employees' hourly earnings. In this regard, the 'equipped citizen' not only will aspire to having a healthier life, but, depending on the level of proficiency in mathematics, will also aspire to having better opportunities of being employed and of earning more income than a 'not–so–well equipped–citizen'.

> Across the OECD, the average return to below upper secondary education stands at approximately 2.5% (ranging from 0% for those in possession of the Level 0/1 numeracy proficiency to approximately 4% for those in possession of Level 4/5 proficiency, while at upper secondary, the range of earnings outcomes (compared to an individual with the lowest level of formally recognised qualifications and numeracy skills) stands at approximately 10% (upper secondary and Level 0/1 numeracy) to 18% (upper secondary and Level 4/5 numeracy). At tertiary level, the earnings outcomes (compared to the reference group) range from approximately 33% (tertiary and Level 0/1 numeracy) to 53% (tertiary and Level 4/5 numeracy). (Lane & Conlon, 2016, p. 21)

Educational attainment has also been correlated with lower morbidity from the most common diseases –heart condition, stroke hypertension, cholesterol, emphysema, diabetes, asthma attacks, ulcer– and correlated with life expectancy, in which life could be increased up to 5 years (Cutler and Lleras-Muney, 2006). And so, educational attainment rises as an important factor for well-being (OECD, 2016a), by boosting specific features of the desired productive citizen for society.

> Highly skilled people are also more likely to volunteer, see themselves as actors rather than as objects of political processes, and are more likely to trust others. Fairness, integrity and inclusiveness in public policy thus all hinge on the skills of citizens. (OECD, 2015, p. 3)

All of the above translates in a promise of 'well-equipped-citizens' that live longer, are socially active, healthier, volunteer, engage in political processes, trust others, are more likely to be employed and earn more, all because they reach their full potential thanks to their numeracy proficiency. After all, the first OECD commitment is 'better policies for better lives'. So, be the best version of yourself!

FROM THE HOPED TO THE FEARED

OECD (re) produces discourses emboding salvation narratives about who the desired citizen is and how the desired citizen should be. The image of the productive citizen is enunciated in every statement above. The 'well-equipped-citizen' OECD expresses is a lifelong learner that should be wiling to engage in a perpetual process of making choices and problem solving (see, e.g., Popkewitz, 2008b). Also, a high skilled citizen that is willing to participate fully in society. This portrays the image of the hoped, an entrepreneur, an “individual who is continually pursuing knowledge and innovation in a never ending chase for the future” (Popkewitz, 2008a, p. 310), all because of mathematics proficiency.

Within the making of the citizen, OECD plays the role of ‘homogenizing the heterogeneous’ (Tröhler, et al., 2014). In which, PISA has been shaping “the “accountability” agenda in ways that rival and even overshadow the influence of national policy makers” (Op. cit., p.5). Under this homogenizing role, numeracy proficiency has been granted with the feature of producing social equality (Diaz, 2013). All students having the same opportunities, access, and possibilities translates into “reform projects assum[ing] that social, economic, and educational inequalities can be minimized if all children have the opportunity to learn mathematics” (Op. cit., p. 36). But equity, in this sense, is an illusion (see, e.g., Bullock, 2012); there are many differences amongst students. Although, according to OECD, these differences become less visible if nations invest in the acquisition and development of citizens’ skills, enabling students to excel.

> When the “all children” is examined, there is no universal and undifferentiated “all” but a particular continuum of value that differentiates and divides. The “all children” implies a unity from which identities of difference are generated. As quickly as reforms state that the purpose is for “all children to learn”, however, the discourse shifts to the child who is different and divided from the space of “all children”. The different child is to be rescued and saved from his or her unliveable spaces. The space of the all children is the space of a difference and abjection that cases the Other into unliveable spaces. (Popkewitz, 2011, p. 42)

While certain discourses promote inclusion: “all students could become the well-equipped-citizen”, at the same time there are double gestures of what is desired and what is feared that produces processes of exclusion with those of inclusion. And so, these normalizing and regulating discourses on welfare become in “ways of reasoning about who and what is normal [and also] who and what is abnormal, and in need of social administration, intervention, and salvation” (Bloch, et al., 2003, p. 15). To talk about the “all” implies to talk about the “abjected”; the one who does not fit (Popkewitz, 2008a).

OECD describes the ‘abjected’ as the ‘low performer’. Lower performance in mathematics is attributed to an accumulation of students’ ‘disadvantages’ throughout their lives (OECD, 2016b) –economical, social, educational, gender disadvantages– making this ‘abnormal child’ in need of salvation to fit the “all”. In fact, OECD has even calculated the variables and the percentages of likelihood of being a low performer.

> Who is most likely to be a low performer in mathematics? On average across OECD countries, a socio-economically disadvantaged girl who lives in a single-parent family in a rural area, has an immigrant background, speaks a different language at home from the language of instruction, had not attended pre-primary school, had repeated a grade, and is enrolled in a vocational track has an 83% probability of being a low performer (OECD, 2016b, p. 13)

Above was stated that the social, economical, and educational differences of students apparently play no role in the acquisition of skills, and even that this ‘girl’ is recognized as a low performer, does her disadvantages make her a ‘fear child’? Does this girl pose any danger to the future? Well, it depends on the point of view. But, pushing a little deeper the analysis. Who will become a ‘threat’ for economic progress and the welfare state?

On a neoliberal mentality, discourses about progress and welfare shape and reshape citizens modes of living –ways of being and acting in the world– to be the expected product of a neoliberal and capitalist society. Marketing, consumerism and competition are some of the key elements of neoliberalism (Kaščák & Pupala, 2011), where “people are reconfigured as productive economic entrepreneurs of their own lives” (Davies & Bansel, 2007, p. 248). Neoliberal modes of governance aim at reconfigure a lifelong entrepreneur learner, given that it will be beneficial for the economic productivity of society (Rubenson 2008). Entrepreneurs are taken as the human capital necessary for personal and social and economic prosperity. In this sense, the feared is the one that does not become the productive citizen for economical and social progress, the one who does not consume, who does not engage in market labor, who does not participate fully in society, and who does not compete; the one who cannot fit the “all”. Imagine, for example, a student refusing to participate in PISA.

WELFARE AND SCHOOL MATHEMATICS

> The technology of schooling was not invented ab initio, nor was it implanted through the monotonous implementation of a hegemonic ‘will to govern’: the technology of schooling –like that of social insurance, child welfare, criminal justice and much more– is hybrid, heterogeneous, traversed by a variety of programmatic aspirations and professional obligations, a complex and mobile resultant to the relations amongst persons, things and forces. (Rose, 1999, p. 54)

The promises of welfare emerged in the late 20th century, as ‘new patterns of governing’ (Bloch, Holmlund, Moqvist & Popkewitz, 2003), as a “way of securing or “policing” the well-being of citizens and populations through the “cultural reasoning system” that orders the individuality of the welfare person” (p. 6). Within the promise of welfare, school mathematics is taken as necessary to achieve numeracy for the pursuit of individual happiness and human progress (Popkewitz, 2013).

There is a promise to shape students to fit in certain category for the development of nations, the better economy, and their own welfare. OECD's narratives of what a productive citizen should be have effects of power in the shaping of students' subjectivities –in which "the self is constructed or modified by himself" (Foucault, 1993, p. 204). OECD discourses normalize and regulate whom the productive citizen is, how the productive citizen should be and should act. To be a productive citizen means students should engage in practices to conduct their own conduct to achieve the 'well-equipped state', a will of fitting in the "all" and becoming a lifelong learner. But it also means to recognize in school mathematics an opportunity to reach welfare... All in the name of economic growth!

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SOCIAL NATURE OF MATHEMATICAL REASONING: PROBLEM SOLVING STRATEGIES OF MIDDLE GRADERS' WITH DIVERSE OUT-OF-SCHOOL EXPERIENCE

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Middle graders coming from low-income, disadvantaged families in developing world context often have varied exposure to work-contexts, diverse handling of goods and rich cultural resources as well as funds of knowledge in the community. Such exposure and experience create affordances for them to gather mathematical knowledge and build a connection to their school mathematics learning. This paper explores middle graders' mathematical reasoning while solving routine arithmetic tasks that indicate their social nature of mathematical knowledge. It argues that learners' problem solving strategies derive from cultural resources and work practices and call for tracing curricular implications. Data is drawn from a study done in one of Mumbai's large low-income settlements with huge economic output.

TOWARDS SOCIAL ASPECT OF MATHEMATICS KNOWLEDGE

Late sixties and early seventies witnessed a spurt in research focusing on alternate ways of learning outside of school, which drew on cultural anthropology to analyse various contexts that created the ground for learning in general. Cultural contexts of thinking and learning soon became part of the main focus of the mathematico-anthropological studies. For example, Michael Cole and John Gay's study in Liberia in western Africa in the sixties (Gay & Cole, 1967) looked at Kpelle children's learning of mathematics embedded in their cultural practices. Around the same time, Claudia Zaslavsky's study in Nigeria and East Africa focused on numeracy learning and use of patterns and shapes as part of the culture and work practices (Zaslavsky, 1973). Zaslavsky's work on “sociomathematics of Africa” as she called it, revolved mainly around the “applications of mathematics in the lives of African people” (p. 7). This was also the time when Sylvia Scribner explored literacy and numeracy development among Vai people in Liberia in their everyday practices (Cole & Scribner, 1974; Scribner & Cole, 1978). These are just a few names from those beginning days when the need to understand the social aspects of mathematical knowledge was felt. Around the same time came a shift in focus of the studies in cognitive psychology from individual psychological to socio-cultural aspects of one's cognitive development. There was also a shift in research from using Piagetian developmental psychology framework to Vygotskian socio-cultural psychology and social learning theory. It was the time when research in mathematics education research (henceforth MER) began to use tools from cultural anthropology and drew on other socio-cultural tools looking for alternate ways of development of mathematical thinking and reasoning in individuals and this new trend bore a parallel to the shift towards cross-cultural studies that was already underway in developmental psychology. It was the time when studies in MER increasingly started looking at one's cultural resources as well as at work practices as possible locations of mathematics learning and development of mathematical cognition. These studies provided evidence to the growing belief and claim that school is not the only site of mathematics learning but there exist other alternative sites as well. These alternate sites are interesting for educational researchers to look at, for they entail diverse everyday contexts and situations that create opportunities for children to acquire mathematical knowledge.

This paper discusses middles graders' reasoning towards solving arithmetical tasks which I claim emerges from their mathematical understanding in out-of-school contexts such as work-contexts, everyday shopping and socio-cultural practices. In particular, this paper explores a few instances of the nature of mathematics that remains embedded in the everyday world and the ways in which such knowledge emerges or is gathered. This paper tries to understand the social construction of mathematics and its pedagogical significance for classroom learning. I first describe the underlying theoretical underpinning of what is known as *Funds of knowledge* followed by a description of the study and the learners, analysis of the problem-tasks and solution strategies, and emergent insights from the discussion.

COMMUNITY BASED KNOWLEDGE PRACTICE

A few studies in the past have focused on how the notion of *Funds of knowledge* has been used as a framework to examine the potential resource available to the community in the form of embedded mathematical practices in the work-contexts. There are not many studies in MER other than those by Civil (1995), de Abreu (2008), Moll, Amanti, Neff and Gonzalez (1992) and a few others (viz., Andrews *et al.*, 2005; Velez-Ibanez & Greenberg, 2005) who have used this framework to explore the resources available in the community that can potentially support school math learning of the children. Although there are studies in India and elsewhere (for example, Rampal, Ramanujam & Saraswati, 1998) that looked at community knowledge resource and its implications for various numeracy and literacy initiatives, there are not many studies that have looked at middle graders' varied exposure to work-contexts, diverse handling of goods and availability of rich cultural resources and *funds of knowledge* in the community in the literature on out-of-school mathematics. We have argued elsewhere that such opportunities available to the children create affordances for them to gather mathematical knowledge and to build a connection to their school mathematics learning (Bose & Subramaniam, 2013). This paper discusses middle graders' mathematical problem solving strategies as they are shaped in their work-contexts embedded in economically active everyday settings, and community's funds of knowledge that they have access to.

It is widely seen that children in low-income conglomerations are often bound in social relationships and work practices from an early age and the broad features of their learning develop at their home as well as in their surroundings. Households and their surroundings contain resources of knowledge and cultural insights that anthropologists have termed as *funds of knowledge* (Gonzalez, Moll & Amanti, 2005; Moll, Amanti, Neff & Gonzalez, 1992; Velez-Ibanez & Greenberg, 2005). The perspective of "funds of knowledge" brings to mathematics education research insights that are related to, but different from the perspectives embedded in the studies of "culture and mathematics". In contrast to restrictive and sometimes reified notions of "culture", "funds of knowledge" emphasise the hybridity of cultures and the notion of "practice" as "what people do and what they say about what they do" (Gonzalez, 2005, p. 40). This perspective opens up possibilities of teachers drawing on such funds of knowledge and relating it to the classroom work (Moll *et. al*, 1992).

Funds of knowledge (FoK) are acknowledged to be broad and diverse. They are embedded in networks of relationships that are often thick and multi-stranded, in the sense that one may be related to the same person in multiple ways, and that one may interact with the same person for different kinds of knowledge. In other words, FoK points to the diversity of contexts and settings from which knowledge is acquired. FoK are also connected and reciprocal. When they are not readily available within households, they are then drawn from outside of household from the networks in the community. The concept thus emphasises social inter-dependence. Further, from the funds of knowledge perspective, children in households are active participants, not passive by-standers. This paper looks at "funds of knowledge" as a resource pool that emerges from people's life experiences and is available to the members of the group which could be households, communities or neighbourhoods. In a situation where people frequently change jobs and look for better wages and possibilities, members of the household need to possess a wide range of complex knowledge and skills to cope and adapt with the changing circumstances and work contexts. Such a knowledge base becomes necessary to avoid reliance and dependence on experts or specialists, particularly in jobs that require maintenance of machines and equipments.

Socio-cultural studies in mathematics and science education have argued that cultural resources and *funds of knowledge* (Gonzalez, Andrade, Civil & Moll, 2001) of people from non dominant and underprivileged backgrounds are often not leveraged (Barton & Tan, 2009) in school teaching and learning practices. Neither is their knowledge from everyday life experience valorised (Abreu, 2008) and built upon in the classrooms nor is their identity acknowledged. Access to such school education that is seen as meaningful and relevant by the underprivileged communities and connected to their life settings has remained elusive. What is offered in schools at present is a structured educational package detached from most students' everyday life experiences, yet accepted as “legitimate” knowledge since it acts as the “gate-keeper” (Skovsmose, 2005) to different kinds of opportunities and future social well-being. The legitimacy and necessity of the “formal” school mathematics renders all other forms of mathematical knowledge not only insignificant but also ineffective. This “package” of formal school mathematics either repels or attracts people depending largely on their socio-economic status. In this backdrop, it is widely accepted that hierarchical social structure (for example, caste and class division in the Indian society) has bearings on academic achievements including mathematics learning (Kantha, 2009; Weiner, Burra & Bajpai, 2006).

LOCATION, COMMUNITY AND THE LEARNERS

This paper draws on from a larger study that explored the nature and extent of everyday mathematical knowledge possessed by middle graders from a financially disadvantaged, low-income but economically active community based in central Mumbai. The objective of the study, done in three overlapping parts, was to unpack and document the connections between students' mathematical knowledge, work practices and identity formation, and inquire into the implications of these connections for school learning (Bose, 2015). This paper draws on the first part of the study which looked into learners' arithmetic knowledge - numeracy knowledge, use of arithmetical operations and solution strategies of textbook-type word problems. What emerges from the study is the hybrid nature of middle graders' mathematical knowledge building on both the domains - school and outside. But more importantly, it is the access to diversity of goods and artefact knowledge which afford development of mathematical ideas and endeavours.

All the learners came from a densely populated, culturally diverse low-income settlement where economic production contributes to almost one-third of entire Mumbai's gross domestic product (GDP), according to some estimates (Sharma, 2000).This economically active low-income neighbourhood is spread over a 2 square kilometre area beside locations that fetch some of the highest property values (real estate) in the world (Campana, 2013). The population of the settlement is estimated to be around one million which indicates high population density of the locality (Campana, 2013). As a characteristic feature, the settlement has a vibrant economy in the form of micro and small enterprises dispersed among households, which include manufacturing, trade and service units with high economic output. People living in the settlement are mostly immigrants from different states of India who come to Mumbai in search of livelihood. They are financially poor and most of them are unskilled labourers. The entire community is multi-religious and multilingual with a strong social network and prevalence of funds of knowledge to which children living in the neighbourhood have access to right from an early age. The settlement is thus a co-location of workplace and home for most of its residents.

The learners came from sixth grade of two government-run schools with English and Urdu as mediums of instruction respectively. Their age varied between 10 and 12 years and most of them were engaged in income generating work practices after school hours wherein they either assisted their family elders or worked independently. There were 31 randomly selected sample learners and 30 of them took the interviews about their arithmetical knowledge.

INTERACTION WITH THE LEARNERS

I, as the principal researcher visited the schools and the community as a non-participant observer almost regularly for close to two years and a half. Such visits helped in building a rapport with the learners, teachers, school authorities as well as the community elders. My visits often included lesson observations, informal discussions with the learners and the teachers and visit to the settlement workplaces, homes and neighbourhood. First phase of the study from where data for this paper is drawn was ethnographic in nature involving eclectic exploration of the learners' life-world and opportunities available to them to gather everyday mathematical knowledge. Individual interviews of learners about their arithmetical knowledge were conducted which had items on “number knowledge”, “currency knowledge”, “count-on strategies (number enumeration)”, “computation using arithmetical operations” and “proportional reasoning”. For every item learners were asked whether they were sure about their answers and whether or not they wanted to make any changes. The changes made were recorded. Learners were asked to explain the solution procedures.

THE PROBLEM TASKS AND SOLUTIONS

This paper discusses learners' solution strategies for two items: a) finding the price of 25 *burfi* when 20 *burfi* cost 42 rupees, b) 981 divided 9. 14 out of 30 learners found the prices of 10 and 5 *burfi* by halving 42 and 21. Some learners however tried to employ the unitary method (to find the price of one *burfi* first and then raising it to 25) at the outset but upon getting stuck or when the calculations became complex, turned to alternative convenient methods. Only 1 out of 30 students could correctly complete the task using unitary method. This task showed that students were able to switch between their out-of-school and school mathematical knowledge which supports our contention about the hybridised form of mathematical knowledge drawing elements from both the domains. As justification for using the halving method, a few learners described that what was required was to find the price of 5 *burfi* and since 5 is a *paav* (quarter) of 20, they halved the number twice [*do baar aadha karenge*] - and arrived at 21 after halving first and 11 after halving again. Learners who arrived at 11 as a quarter price of Rs 42 concluded that the price of 25 *burfi* would be Rs 53 and not Rs 52.50. When asked to explain their solution strategy, each of those learners (it was an individual interview) described that although half of 21 is "ten and a half" but since the shopkeepers do not return *aath anna* [eight annas which is equivalent to 50 paise or half a rupee], the price would thus be 53 rupees.

It was noted that some learners brought in reality perspective from their everyday experience of rounding off which is a common practice in economic transaction and trade these days. Learners justified that sellers and shopkeepers often do not return change as balance amount rather round-off to next rupee. This is part of learners' *funds of knowledge* derived from their everyday context and interaction in the community that is reflecting from their solution strategies. A few learners also used strategies like build-on or “workable guesses”. Many of them used “closed” numbers, “convenient” numbers and different “units” from daily usage.

Most learners drew on the old currency units based on a base-16 system that is no longer in practice but has remained as part of the social language of the community. In the old base-16 system, 16 *annas* made a rupee which was equivalent to 64 paise then, against 100 paise now (Subramaniam & Bose, 2012). Hence, *aath anna* or eight *annas* made half a rupee which is equivalent to fifty paise now (100 Paise = 1 Rupee). Till recently, when the 50 paise coins were in circulation, they were often referred to as *aath aana* although the base-16 system made way for the decimal system a few decades ago. *Aath anna* also represents "fifty percent" and refers to half of a whole in the local social language register. It is also a part of the local trade language widely followed in the community and children living in the neighbourhood pick up such social language as part of the community's *funds of knowledge* and learn about the inter-conversions between the old and the new currency systems.

There were a handful of students who would come to me with problems from the mathematics and science textbook or to learn topics like long division method, fractions and operations on them. Such interactions indicated that many learners (fifth graders then) knew exactly where they lacked in arithmetical proficiency following the formal algorithms and wanted to get them addressed.

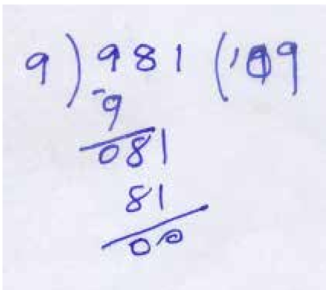

Figure: 1

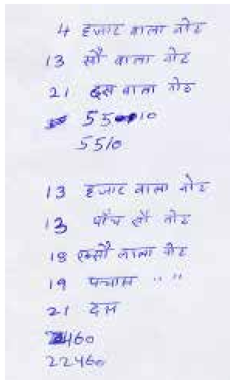

Figure: 2

Many learners had developed number sense building on their currency knowledge. Numbers for them were amounts of money and arithmetic operations signified “summing up”, “getting more”, “giving away” or “distribution” and so on (Bose & Subramaniam, 2011). For example, when asked to divide 981 by 9, Abdul (pseudonym) a fifth grader of Urdu school (at the time of interview in 2010) looked at the problem as “equally distributing” Rs 981 among 9 children. This was after he had arrived at "19" while doing the calculation on a worksheet following the long division method learnt in school (shown in Fig. 1 above). He himself noticed the discrepancy in the obtained result and pointed out to me that "19" cannot be the correct answer. He argued that although he had followed the method that his teacher had taught in the class (*jaisa teacher ne sikhaya hai*) but "19" cannot be correct since if we were to distribute 981 rupees among 9 of us, "each one would get at least hundred-hundred sir" (*har ek ko kam se kam sau-sau to milega na sir*). He mentally calculated and divided Rs 900 among 9 children and arrived at Rs 100 for each of them and then divided the remaining Rs 81 among 9 children for each to get Rs 9. Hence, each child gets Rs 100 plus Rs 9, i.e. Rs 109. Abdul then hesitatingly put a “0” between “1” and “9” in the worksheet probably because he had “more faith” in the oral procedure than school taught algorithms. Abdul had metacognitive awareness about where he was in the middle of the calculation and gauged the possible range of the answer. Leveraging his everyday mathematical knowledge he could clearly see the absurdity in "19" as an answer to 981 ÷ 9. Abdul's justification and mathematical reasoning drew on the social language and his social identity as a worker who distributes wages to junior co-workers. His *funds of knowledge* and work context experience helped him formulate and verify his solution strategy which subsequently informed and refined his school mathematical knowledge. Unfortunately, he believed that school solution can be different from those obtained outside. He said "*yeh* (indicating "19") *school mein sahi hai aur yeh* (pointing at "109") *bahar sahi hai*" (This is correct in school ("19") and this is correct outside ("109").

Abdul like other students in the same grade had number sense built on his currency knowledge. During the above interview, he worked as a learner (novice) in a garment making workshop after school hours. Interactions with him indicated that his interest in school studies brought him back to studies after a two-year gap during which period financial condition of his family had forced him to work than attending school. Discussions with him earlier had shown that he could add currency-values sometimes involving 5 digit numbers purely mentally. For example, when asked how much money would be represented by, 4 thousand rupee notes, 13 hundred rupee notes, and 21 ten rupee notes (see *Fig. 2, previous page*), Abdul correctly replied, “five thousand five hundred ten rupees” but initially wrote the sum as 550010 and subsequently corrected it to write 5510. When asked to add 13 thousand rupee notes with 13 five-hundred rupee notes, 18 one-hundred rupees notes, 19 fifty rupees notes and 21 ten rupees notes, Abdul had the accurate answer as, “*twenty two thousand four hundred sixty”*. Numeracy for Abdul was derived from the social aspect of his mathematical understanding while number-literacy (formal representation) was not.

Everyday work contexts create affordances for making decisions in relation to work and for optimising resources and earnings. Such optimisation processes often entail quick mental calculation and estimation skills. Learners engaged in garment recycling work (collection of small garment pieces in large quantities and selling) earmark workshops where they are most likely to get a handsome quantity and know how quickly they can fix a deal before other groups can drop in. One also has to keep in mind the weight that can be carried easily and which can fetch a good amount. The control over and extent of decision making, need for optimisation, knowledge of backward and forward linkages are strongly related to the sense of ownership that participants had about their work.

Such linkages are often derived from community's funds of knowledge. Similarly learners engaged in mobile repairing task reported the need for optimising costs and quoted price, time required for carrying out repair work, use of different kinds of parts based on the customers' paying capacity and estimating the profit margins. All these work practices involve different arithmetical skills such as computation, maintaining accounts, sorting, estimation apart from decision making and optimisation to varying extent. In addition, there are tasks that require specialised skills, viz., block printing work (dyeing) or *zari* stitching or tailoring work that requires training through a series of stages. These out-of-school work contexts entail different levels of interaction with arithmetical problem and their solution procedures.

SOCIAL NATURE OF MATHEMATICAL KNOWLEDGE

Economically active low-income settlements dotted with micro enterprises are usually rich in the occurrence of work-contexts involving dealing with quantities of different kinds, the use of multiple units, problems involving proportions and a cluster of related mathematical concepts such as fractions, proportions, multiplicative reasoning, division and measurement. Some of the strategies used by children to solve proportion problems appear to arise spontaneously in the context of everyday mathematics (Nunes and Bryant, 1996). From the interviews it also became clear that many learners were comfortable in using social language such as binary fractions that are part of everyday discourse like half *(aadha),* quarter (*paav*) and half-quarter *(aadha-paav, i.e.,* one-eighth) or three-fourths (pauna), one-eighth (adha paav) and one-sixteenth (paav-paav), one-and-a-quarter (sawa), one-and-a-half (dedh), and two-and-a- half (adhai). Some of these number-words are stand-alone units - for example, unlike in English, the corresponding word for three-fourths or three-quarters is *pauna* which is not three (times) *paav* in additive literal sense but it means three *paav*. This is because the semantic meaning of the word *pauna* is *paav-una* which stands for "a *paav* less than" that is to say, "a *paav* less than a whole" which is equivalent to three-fourths or three-quarters. It is however interesting that fractions other than these (non-binary and decimal fractions) were difficult to comprehend for most learners and poorly developed despite these being present in the school curriculum. Their everyday experience does not include most fractions dealt with in school (for example, decimal fractions) or any kind of visual support for arbitrary equal partitions. In their experiential world there is not much insistence on precision, or fair division. It was noted that on many occasions the social language entails numeracy words and mathematical terminologies that are not part of the formal pedagogy processes or mathematics textbooks.

Many learners were unsure about how to represent these fractions symbolically. A seventh grader in response to a question about how to express *pauna* (three-quarters) in decimal representation came up with alternative representations in binary system and not in decimal, namely, *teen paav* (three quarters), *aadha aur paav* (half and a quarter), *ek se paav kam* (quarter less than a whole) and also *pachhattar* (seventy five). The latter representation indicates his access to social language on percentages.

Access to community's funds of knowledge shapes the social nature of mathematical reasoning. Most workplace activities have distinct features as compared to the mathematics used in formal school contexts. School mathematics is largely textbook driven which as Freudenthal says, comes as a packaged “ready- made mathematics” (1971, p. 431). But, mathematics that is embedded in socio-cultural and economic activities are mostly routine and fragmented in nature, some of the work-contexts entail on-the-spot decision making and optimisation that goes beyond “ready-made mathematics” prescribed in schools and comes closer to “mathematics as an activity”. Aided with such real life experience of dealing with mathematics in their work-contexts, some students possessed rich potential resources for gaining deeper understanding in the course of formal learning than what they currently acquire.

SO WHAT?

The above examples underline my claims that the whole gamut of everyday experiences including diversity of cultural and work practices shape students’ everyday mathematical knowledge and has structural difference with school mathematics (Bose, 2015). However, the inter-penetration between everyday and school mathematics indicates that learning in one domain has relevance for the other which remains to be unpacked. From the standpoint of socio-economic influence of math learning, analysis of such hybridised embeddings of one domain knowledge onto the other has remained an area that calls for systematic exploration.

Out-of-school mathematics bears the functional aspect of mathematical knowledge that is available to all and not hidden (Subramaniam, 2010). This calls for leveraging learners' social nature of mathematical reasoning which can possibly pave way for developing skills and interests in learning mathematics. Pedagogy of mathematics needs to draw on "social reasoning" for deeper conceptual understanding. At present, there remains a disconnect between the syntactic as well as semantic differences in the language used in everyday contexts and the language used during classroom-teaching. For example, the arithmetic tasks in the interviews showed flexible competence of the students in the contextual problem tasks, and better levels of competence in handling currency or in doing calculations when currency was given as a cue. These tasks also revealed students' propensity towards using their own situation-specific competencies and often these out-of-school learnt strategies were amalgamated with elements of school mathematics, for example, the use of school learnt algorithms and multiplication tables. School mathematics offers "ready-made" mathematics and learning of its application is stressed upon, while social nature of mathematical reasoning remains disconnected and even unacknowledged which otherwise has a potential to increase meaningfulness. It is therefore self-defeating for an education system to merely aim to produce the trappings of social class, while depriving learners of knowledge that has power because it illuminates aspects of life and brings in meaning and hence long-lasting understanding.

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SOCIAL INQUIRY WITH MATHEMATICS IN TWO HIGH SCHOOL CLASSROOMS

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What do the inquiries afforded by socially relevant mathematics applications look like? This paper discusses the nature of the inquiries in two high school mathematics classes. A critical theoretical perspective on the mathematics curriculum informs the framework used to examine these inquiries.

INTRODUCTION

Recommendations for reforming the high school mathematics curriculum in the United States of America (USA) emphasize the importance of incorporating mathematics applications involving societal issues in the curriculum so that students may gain a better understanding of these issues and deepen their knowledge of mathematics (CCSSM, 2010; NCTM 2000). What does the inquiry afforded by these socially relevant mathematics applications look like in high school classrooms? Do they embody ideas about curriculum advocated by critical mathematics education scholars? The desire to answer these questions led to a study of two high school mathematics classes in which students used mathematics to explore a wide range of societal issues.

THEORETICAL PERSPECTIVE

Chassapis (1997) argues that mathematical constructs acquire meanings for students beyond their mathematical meaning when applied to real-world problems. Similar mathematics constructs are used to determine the growth over time of a business' profits or nuclear waste, for example. The constructs share a common mathematical meaning in both applications. However, these mathematics applications implicitly emphasize different ways of thinking about human activities and support different values. A mathematics curriculum that is dominated by applications of the first type sends the message that increasing profits is a valued human activity and that real-world applications of mathematics are about figuring out ways to support capitalist markets.

The mathematics curriculum experienced by many students consists mainly of pure mathematics problems (Apple, 1992; Ernest, 1991). To the extent that the curriculum in the USA incorporates applications, they are largely inauthentic, chosen for the purpose of teaching a mathematics concept or skill rather than to obtain a better understanding of a real-world situation (Forman and Steen, 2000), they do not address social justice issues because classicism racism, sexism (and other "isms") remain "taboo topics" in mathematics classrooms (Gutstein, 2003), or they "reify hegemony, the exploitation of people, and a marked disregard for the environment" as Bright (2016, p. 1) found in her examination of mathematics textbooks. As a result, the mathematics curriculum has distorted the knowledge students learn in schools about themselves, their communities, and society and what they learn about mathematics as a tool for social critique (Apple, 1992; Frankenstein, 1995).

Furthermore, applications in the mathematics curriculum have promoted mathematics as an "ideology of certainty" (Borba & Skovsmose, 1997). A belief in the applicability of mathematics to a broad range of situations is not problematic in and of itself. The problem arises when mathematics is uncritically viewed as a tool for representing and making sense of virtually any situation and that the use of mathematics vouches for the reliability of any results obtained. This may impede a serious discussion of mathematics applications by limiting the types of questions that are pursued. The ideology of certainty is supported by traditional classroom experiences where mathematics problems have one correct answer and the task becomes finding it, or where students deal exclusively with situations that spare students' exposure to the kinds of challenges encountered in the design and implementation of real-world applications.

While the mathematics curriculum has largely restricted social inquiry, critical mathematics education scholars argue that it can foster robust social critique. Empirical studies of mathematics applications involving concrete instances of discrimination and exploitation (or privileging), based on class, race, gender, and other social group identifiers support scholars' claims that they enhance students' social awareness (Gutstein, 2003; Tate, 1995; Turner, 2003). By surfacing contradictions between sociopolitical ideals and lived experiences, these mathematical investigations led to changes in students' perceptions of social life that are consonant with an emerging critical social awareness. Additionally, a positive change occurs in most students' perceptions of the utility of mathematics when they have engaged in social inquiry with mathematics in the classroom (Brantlinger, 2007; Frankenstein, 1995; Gutstein, 2003; Tate, 1995; Turner, 2003).

Critical scholars also argue that the mathematics curriculum must engage students in a critique of mathematics in its applications, if it is to foster the development of views of mathematics that are critically-oriented. For Skovsmose (1994) the principal "tasks" of a critique of mathematics applications are uncovering assumptions, monitoring processes involved in the design and implementation of applications, and evaluating the effects of using mathematics to solve a real world problem. In conceptualizing a critique of mathematics applications, Christiansen (1996) distinguishes between technically-oriented and critically-oriented reflections. The former are concerned with such matters as whether the application's calculations address the right problem (often narrowly defined) and have been performed correctly, the reasonableness of methods in view of what was to be mathematized, and the reliability of results obtained. In contrast, critically-oriented reflections address the broader consequences of using mathematics to address a problem by uniting social, political, and ethical concerns with technical considerations. These more critically-oriented reflections are frequently the kind of reflections that are silenced, marginalized, or supplanted by technical concerns when mathematical applications are discussed, both in schools and outside of schools.

METHODS

The research sites for this study were a Mathematics Modeling class and a Statistics class in two public high schools in the Midwestern USA. These courses are electives taken mostly by students in their last two years of high school and were selected because their curricula incorporated several socially relevant mathematics applications. Mathematics topics addressed were mainly from the advanced algebra and statistics curricula. The study's teacher participants created the mathematics applications that the researcher observed. Both teachers stated that they are committed to increasing the use of socially relevant mathematics applications in their mathematics courses, even though neither teacher was formally schooled in critical theories of education.

Admission to both schools is selective and highly competitive. Student selection is based on grades and test scores on nationally normed tests. Race is also a factor as both schools are committed to increasing the diversity of their student body. The Mathematics Modeling class was very diverse, racially and ethnically: 24% White, 33% Black, 23% Hispanic, 7% Asian/Pacific Rim, and no Native Americans. According to their teacher, 13% of the students in the class self-identified as bi-racial; 63% of the students were male and 37% female. Socioeconomically, more students came from working-class or poor families than middle-class families (the class included a few students from upper middle-class families). With few exceptions, the students in this class did not have a track record of high achievement and interest in previous high school mathematics courses. The Statistics class was very diverse ethnically and culturally but not very diverse racially, reflecting the demographics of the area in which the school is located: 74% White, 21% Asian/Pacific Rim, 5% Black, and 0% Hispanic; 51% of the students were male and 49% female. While most students were from upper middle class, well-educated families, about a quarter of the students' families lived in struggling small towns or rural areas. The Statistics class included students who had a track record of high achievement and interest in previous high school mathematics courses as well as students who were not among the high-achieving mathematics students at the school and those for whom mathematics generally did not hold much interest.

Data were gathered over a six month period through classroom observations, interviews of teacher and student participants (93% of students in the Modeling class, 82% of students in the Statistics class) and review of student work and curricular materials. Data collection focused on discourse (written and oral) about inquiry assumptions, methods, and conclusions; they are the elements of the analytic framework discussed later in this section. Interviews were semi-structured, guided by a series of open-ended questions about the topics of interest to this study. At the same time, interviews allowed for the pursuit of questions or topics raised by participants or the researcher during the interview. For each mathematics application, teachers were interviewed individually, and a sample of students was interviewed individually or in groups to two to four students. A sample of student written work was collected for each application. These data sources –classroom observations, interviews and documents, were triangulated to produce and enhance the credibility of study findings. Seventeen socially relevant mathematics applications were observed, nine in the Mathematics Modeling class and eight in the Statistics class. They are identified by issue in Table 1.

Table 1. Class Observations and Student Interviews by School and Application

Application	Days observed	Students in groups	Students interviewed	Number of interviews
MATH MODELING CLASS				
Death penalty	1	5	5	3
Affirmative action	3	5	12	4
Distribution of income	1	5	5	2
Social security	1	5	5	2
Nuclear waste	2	5	10	4
Toxic dumps	3	5	10	4
Voter's guide	4	8	11	5
Better transportation	14	12	23	13
Global warming	2	4	10	5

STATISTICS CLASS				
Fall projects	1	N/A	10	5
Air pollution	4	4	5	5
Fairness #1 (various issues)	1	3	8	6
Spring projects (various issues)	5	4	4	2
Death penalty	2	7	21	15
Fairness #2 (various issues)	2	6	18	14
Scarce resources	2	N/A	5	4
Equity in school funding	2	N/A	8	4

The analytic framework created for this study, referred to as the framework for social inquiry with mathematics, draws heavily on Skovsmose's conceptualization of tasks of reflection on mathematics applications and Julie's conceptualization of domains involved in mathematical modeling –the extramathematical reality, the consensus-generated reality, and the intra-mathematical.

The framework sets up a relationship between inquiry contexts, or domains, and objects of inquiry, or components, as they are respectively called in this study. Inquiry is conceptualized as a series of acts that are articulated in terms of what they do with respect to these objects in different contexts. The framework contains three overarching components: (1) assumptions, (2) methods, and (3) conclusions. Each component exists in three domains: (A) the mathematical domain, (B) the sociopolitical domain, and (C) the mathematical/sociopolitical domain. It may be graphically depicted by a 3 X 3 matrix whose cells represent combinations of inquiry components and domains. The cells of the matrix are populated with the actual contents of inquiry, which is to say, inquiry acts related to different components in different domains. The matrix is shown in Figure 1. It contains a sample inquiry act in each cell from one of the applications observed –the capital punishment (death penalty) application in the Statistics class.

Figure 1: Framework with Example Acts of Inquiry

		Domains of Inquiry		
		Mathematical	Mathematical-Sociopolitical Interface	Sociopolitical
Components of Inquiry	Assumptions	(Cell 1) Defining the χ^2 statistic	(Cell 4) Defining statistical hypotheses (null and alternative) to test assertions about the fairness of the death penalty	(Cell 7) Identifying the race of the victim and race of the defendant as relevant factors in an investigation of bias in the death penalty
	Methods	(Cell 2) Calculating χ^2 using observed and expected values	(Cell 5) Determining how expected values for a fair distribution should be computed	(Cell 8) Posing questions about the legal process by which the death penalty is imposed
	Conclusions	(Cell 3) Rejecting the null hypothesis as a result of hypothesis testing	(Cell 6) Interpreting the results of hypothesis testing in context	(Cell 9) Concluding that the death penalty should be abolished or reformed

Figure notes:

(1) Acts in cells 1-3 are non-contextualized (pure) mathematics. (2) Acts in cells 7-9 are non-mathematical. (3) Acts in cells 4-6 are contextualized mathematics (mathematized sociopolitical). (4) The component "Assumptions" encompasses assumptions, values, and interests. "Methods," encompasses methods, processes, and procedures. "Conclusions," encompasses conclusions, effects, and implications.

Socially relevant mathematics applications map mathematics onto a problem situation involving a societal issue. Thus, the resulting inquiry necessarily contains both mathematical and sociopolitical content. The framework's domains reflect these differences in content. Inquiry acts in the mathematical domain address the inquiry's purely mathematical assumptions, methods and conclusions and are necessarily technically-oriented. The mathematical/sociopolitical domain is the interface between the mathematical and sociopolitical domains. Inquiry acts in this domain involve mathematized sociopolitical assumptions, procedures and conclusions, or alternately contextualized mathematical assumptions, procedures, and conclusions that may be technically or critically-oriented. Inquiry in the sociopolitical domain concerns non-mathematical, or "extra-mathematical" matters (Niss, 1996). Inquiry acts address strictly sociopolitical assumptions, methods, and conclusions. When social actions and policies are viewed through a critical lens -the social and political interests of critical theory - they are evaluated with an eye on how they help perpetuate or challenge existing inequitable social arrangements.

Analysis of each inquiry began with a deductive approach –the use of a priori categories derived from the study's analytic framework. It focused on the distribution of inquiry data across cells of the matrix, noting where they were concentrated and which cells were sparsely or densely populated. The identification of such patterns delineated the scope and foci of the inquiry. The examination of matrices across inquiries focused on identifying inquiries with similar patterns. These inquiries were grouped together. Each group was further analyzed using a modified form of Spradley's (1980) domain analysis to distill features shared by inquiries or unique to an inquiry, leading to insights about the functions and topics of inquiry.

RESULTS

Each mathematics application in this study afforded inquiry in all three domains of the study's framework for social inquiry with mathematics –that is, part of the inquiry was purely mathematical in nature (inquiry in the mathematical domain), part of the inquiry was purely sociopolitical (inquiry in the sociopolitical domain), and part of the inquiry involved the transformation of sociopolitical content into mathematical content or vice versa (inquiry in the mathematical/sociopolitical domain). In the following discussion, some key findings about the nature of the sociopolitical inquiry (inquiry in the sociopolitical domain) at research sites will be presented followed by key findings about the mathematical inquiry (inquiry in the mathematical and the mathematical/sociopolitical domains).

At one end of the spectrum were mathematics applications (3 out of 17, 17.6 %) that afforded minimal inquiry in the sociopolitical domain. The inquiry was typically limited to the identification of a social problem to investigate with mathematics and a few elements of the problem situation to mathematize. At the other end of the spectrum were mathematics applications involving considerable inquiry in the sociopolitical domain (14 out of 17, 82.5%). Inquiry ventured well beyond what was necessary for the class' mathematical investigation of the problem. There were recurring topics of conversation in the sociopolitical domain across applications and research settings. In the main, conversations involved: (1) identifying a social problem, including related social justice or fairness issues, (2) describing social practices and mechanisms –how they work and how they're supposed to work, and factors that influence them, (3) discussing plausible explanations for findings of the mathematical investigation related to these practices or mechanisms– including but not limited to questions about their fairness, and (4) proposing possible remedies for social problems. It is important to note that an application did not usually address all of these topics, although all topics were addressed by the applications collectively. Furthermore, data analysis does not support a relationship between the kind of social issue addressed in the application and the topics addressed in sociopolitical inquiry. That is, all applications could potentially address all of these topics, even if they did not.

Data analysis revealed many inquiry acts in the mathematical and mathematical/sociopolitical domains related to assumptions, methods and conclusions. While many acts were routine in nature, this discussion will focus on inquiry acts that involved student reflection. Students reflected when an aspect of the inquiry gave them reason to pause, that is, it raised a question, or concern, or presented a challenge. Students also reflected when they looked back at their work, examining assumptions, methods or conclusions to see if what they had done was reasonable and made sense. All applications afforded mathematical inquiry that contained acts of reflection (some applications more than others). Reflections largely occurred in the mathematical/sociopolitical domain. The following examples illustrate topics of reflection from the death penalty inquiry.

When presented with numerical data on executions, students asked how racial categories are constructed, seeking to reveal sociopolitical assumptions behind the data. During interviews, some students argued that race is "more of a socially constructed idea" (Dinesh) or an arbitrary classification system reflecting the interests of those in power (Adam and Matthew). Subsequent to the class' use of chi square hypothesis testing to investigate a variety of fairness issues, including racial bias in the death penalty, their teacher asked them to reflect in writing on the value of this method for investigating "fairness" issues. Here is an excerpt from a typical reflection: "Chi square is useful in looking at fairness because it tells you how far away an actual situation is from what you expect it to be [if the situation is fair]....The greater the chi-square value, the farther away the actual values are from your expected values and the less fair the situation is" (Lauren). The following excerpts from interviews were representative of conclusions reached by the class about the death penalty inquiry: "The justice system isn't quite just.... Racism still really isn't dead in this country, and it's still good to be rich and White and influential"

(Ming). "I found it interesting that the true unfairness [is] that murderers of White people are far more likely to be executed than murderers of Black victims. It seems a way, even if unconscious, for the judges and juries to vent and yet hide their racial prejudices" (Christopher).

Many applications (10 out of 17, 58.8%) problematized mathematics' use in the sense that they afforded mathematical inquiry involving ambiguity or complexity. These applications were productive for inquiry acts involving reflection on mathematics. Data analysis revealed common topics of reflection concerning assumptions, methods, and conclusions across mathematical inquiries. They are incorporated in the following questions: (1) What mathematical tools or methods do I use to investigate this problem? (2) What data do I need? (3) How do I represent the factors or ideas in this problem situation mathematically? (4) What conclusions can I draw? (5) How sound are the inferences I have drawn? (6) What are the social implications of the results of the mathematical inquiry? (7) Is mathematics relevant in this situation? It is important to note that an application did not usually afford inquiry that supported reflection on all of these topics, although all topics were addressed by the applications collectively.

Inquiries involving the quantification of fairness were productive for many of these types of reflections. Mathematical inquiries about social justice issues often, but not always, involved ambiguity in depicting fairness. Fairness is a sociopolitical construct. These mathematical inquiries raised the question: How should fairness be measured? For example, in an inquiry about the fairness of capital punishment, the principal challenge involved operationalizing fairness, an inquiry assumption, in relation to death penalty executions. Determining the fairness of executions begged the question: fair compared to what? Students struggled with figuring out which population's racial distribution the racial distribution of executed individuals should match as they contemplated what a hypothetically fair distribution should look like. This was a matter for reflection as Ming stated during an interview:

If you want to look at a particular type of distribution, like by ethnicities on death row… then you have to find another distribution to compare it to. And that was one of the main things I had to think about, because otherwise, if you come up with a distribution, that really doesn't mean anything by itself. You had to come up with something that you would expect it to look like, if things were fair.

During some inquiries (4/17, 23.5%), students openly questioned the appropriateness of mathematizations to address issues with obvious social, political and ethical dimensions. Here is an example of a reflection that arose during the affirmative action application. The goal of the inquiry was to assess the validity of claims of discrimination in hiring practices advanced by female applicants for faculty positions at a university. The class had been analyzing numerical data to determine whether a university's hiring practices were fair, as the university claimed, when one student remarked: "Wouldn't it be more fair, I'm not invalidating the problem, but wouldn't it be more fair if they just didn't look at...[gender]... or whatever and just hired the best qualified people? I think that universities do that: hire the best qualified employees" (Julia). This student seemed to imply that the fairness of a university's practices could not be determined on the basis of the statistics, that the quality of the applicants was paramount. Her comment represented a "teachable moment" that led to an unplanned yet substantive and interesting class conversation about the use of mathematics to evaluate a variety of social justice claims.

DISCUSSION AND CONCLUSIONS

Critical mathematics education scholars assert that due to the dominant ideology of egalitarianism, equality is often assumed in our society. They argue that the use of mathematics to scrutinize this assumption promotes critical social awareness. Several applications in this study afforded students the opportunity to test the validity of this assumption in the lives of historically marginalized social groups. Students used a variety of mathematical tools to examine the distribution of social goods and opportunities. They analyzed numerical data to determine whether particular social policies and practices had differential effects on social groups. In many cases, the evidence discovered during their investigations supported claims of discrimination. In examining social practices and public policies that have historically fostered inequality, these inquiries went against the grain of the canonical secondary mathematics curriculum in the USA. It is worth noting that during interviews, all students indicated that these were the only mathematics courses where they encountered applications involving social justice issues.

In the case of many (but not all) applications, inquiry ventured beyond the mathematical investigations of these issues. In fact, each of these applications became a gateway for a fairly substantive conversation about the respective issue. Students discussed plausible explanations for findings, methods for alleviating or eliminating the problem investigated with mathematics, ways of reforming aspects of social systems, and the role of government and individuals in reform efforts. In this way, these socially relevant applications promoted social awareness beyond that which was obtained from the class' mathematical investigation of the issue.

Other applications, however, afforded no inquiry about social issues beyond what was needed for the class' mathematical investigation. These applications did not engage students in a discussion of the social implications of the data or sociopolitical assumptions underlying the data. In these ways, applications constrained what students could learn about these issues. A way that all applications in this study constrained students' social awareness was by not connecting an inquiry about a particular social issue or injustice to a broader inquiry about the ways in which various social systems (e.g. economic, political) perpetuate injustice.

Critical mathematics education scholars argue that mathematics must be problematized in its application to social issues to combat the dominant, uncritical view of mathematics in society. A few mathematical inquiries in this study involved a fairly routine application of mathematical methods, even as they used real data and some fairly complex (multi-step) procedures. There was no ambiguity involved, so there was little need for interpretation and no need for deliberation. These applications lent themselves to evaluation in simple and rather straightforward ways, supporting the dominant ideology about mathematics.

However, most inquiries involved authentic problems that could be described as open-ended, ill-structured, or messy. They required that students interpret reality and make assumptions about it, determining what was relevant about the problem situation from the standpoint of the application, deciding how this information could be represented as data and what mathematical procedures and tools should be used to analyze the data. Assumptions, methods, and conclusions were problematized. Operationalizing fairness mathematically proved to be problematical in many cases, the outcome of a method fraught with ambiguities. What is fair mathematically? It was not clear in several inquiries. Thus fairness could not be "objectively" depicted with the mathematical tools; rather, students had to make assumptions or choices about how it would be mathematized, be it a fair distribution of income or a fair distribution of executions. The transformation of the sociopolitical construct of fairness into its mathematical representation required collective reflection. The ambiguity promoted a view of mathematics as subjective, contesting the presumed objectivity of mathematics (as objectivity is typically conceived) in its applications.

Several inquiries underscored the need to take into account all relevant real-world considerations of the problem situation investigated and the constraints of available tools. This caused tension between technical and ethical concerns: what was technically feasible (e.g. chi square hypothesis testing required severely limiting the number of factors that could be taken into account) and what was right (e.g. what would make for a thorough investigation). Inquiries also contested the certainty of mathematics by having students conduct various statistical hypothesis tests and interpret the statistics and results obtained. Teachers pressed students for a proper interpretation of the results, which required stating confidence or significance levels, respectively for statistics and hypothesis tests. These statements embodied the recognition that the certainty of conclusions is compromised by limits of confidence. In addition, teachers required that students be clear about what mathematics could or could not allow one to conclude about the problem.

The ambiguity, complexity and uncertainty of mathematics' use to investigate social issues posed authentic challenges for students. Thus, collectively, the applications in this study provided opportunities for students to reflect more deeply on specific mathematical tools and their uses. That said, inquiries in this study largely promoted critique from the standpoint of the discipline, what this study calls technically oriented inquiry reflections on mathematics.

Finally, from a critical perspective, applications are strongly influenced by the broader social and political contexts in which they are developed and used. On a few occasions, applications afforded inquiry that addressed this larger context: the interests and values of the individuals who would use mathematics to promote an agenda to influence mathematics applications. However, the inquiry did not tie these individuals to social groups and ideas about how injustice operates.

FUTURE DIRECTIONS FOR RESEARCH AND PRACTICE

This paper provides an account of social inquiry with mathematics in two classrooms using a framework that may prove useful to researchers interested in examining the nature of social inquiry in mathematics classrooms. Its domains and components of inquiry were useful analytic categories for capturing the diversity of inquiries, identifying commonalities and differences in their features, with an eye on how they support and constrain the development of social awareness and knowledge of mathematics as a tool for social critique. Further testing and refinement of this analytic framework is needed. An avenue for further research is a conceptualization of the interplay of the framework components and inclusion of the pedagogical influences on the facilitation of social inquiry in classrooms.

This study also has implications for educational practice. Mathematics teachers need considerable support in the development and selection of curricular materials. Although teachers in this study developed their own materials, both noted that they would have done more with socially relevant applications in their classes, but that time and their own knowledge constraints precluded this. Less time was spent in class discussing a social issue, when a teacher felt pressured to cover more mathematics topics, or when the teacher's knowledge of the societal problem was not sufficiently deep.

One way to support more robust, critically oriented social inquiry in high school classrooms is to incorporate more interdisciplinary projects across the high school curriculum. This requires changing school structures and teaching practices and providing professional development to teachers as they implement these projects. Using one discipline such as mathematics to study virtually any societal issue is an inherently weak approach, as many student participants in my study pointed out during interviews. Furthermore, expecting high school mathematics teachers to consistently engage their students in substantive critically-oriented social inquiry during mathematics class seems to be unrealistic, given that their primary responsibility currently is to support student learning of mathematics.

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ASSEMBLING MATHLIFE CHRONOTOPES
'STREET MATHEMATICS' AS HYBRID OF EPISTEMIC/ONTIC KNOWLEDGE DISCOURSES URBAN CIRCULATION IN TEACHER EDUCATION

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Could mathematics teacher education courses be part of assemblages that grasp and circulate affective, sensorial, mnemonic and political temporalities going beyond a mechanistic reincarnation of thinking that deprives mathematics from the drama of life? By means of the project 'street mathematics', a hybrid of assembling mathlife chronotopes, the present paper attempts to explore the above question and its political significance for student-teachers in a teacher education program at times of crisis. It is argued, that through specific urban interventions in the cityscape student-teachers can experience such assemblages as events of epistemic/ontic knowledge discourses circulation through the public space of teacher education institutions and/or the streets in the city.

MATHLIFE CHRONOTOPES IN TEACHER EDUCATION

Bergson (1896/2004) at the turn of the 20th century critiqued time as a strict rationalist view of geometric space that reduces life into an ordered 'clock time' experience and fails to encompass the duration of inner life, irregularity, acceleration or deceleration. In mathematics teacher education such a limited sense of life is, currently, exemplified through institutional, national and global demands to conform with certain requirements for the production of efficient, flexible and competent neoliberal subjects through curricular and assessment practices. However, discourses of the ideal mathematical subject as a rational reasoned problem solver obeying the ruling stratum of 'proper mathematical activity' as quick methods of deep understanding, is being constantly subverted by subcultures such as our youth including student teachers, adolescents and children in the early years, or unschooled, marginalised and diverse groups, or disabled, racial and gendered bodies that tend to think, talk and behave otherwise in asymmetrical, paradox or unorthodox tropes.

In contrast to a rationalist time-space of conceiving time and life, Bakhtin (1981) suggests the notion of 'chronotope' to express the inseparable intersections of space and time -taking time as the fourth dimension of a space conceived as a material whole. Based on examples from literature and the arts, Bakhtin insisted that time takes on flesh in becoming aesthetically visible and, equally, space becomes alert to time movement as plot and history. Narrative genres involve chronotopes (e.g. encounter, road, castle, parlours and salon, threshold and crisis) that relate characters with classes of identities, value systems and morals, make linkages with social, cultural and historical contexts and play a key role in the production of meaning and sense. In similar terms, the time-space through which a mathematical activity or practice is being narrated and reconfigured provides the habitus or the life-world where mathematical experience is connected to its sociopolitical field –creating, thus, mathlife chronotopes. Such cultural housing of maths is not neutral and can vary from word problems, to thematic contexts, project work, playful outdoor activity, indigenous mathematics, dramatizations or even fragments from cinema, poetry, literature, choreography, painting or photography. All these consist mathlife chronotopes where children, adults, materials, maths, life are assembled together in stories that reveal the affordances, pleasures and desires, but also, the symptoms and disorders of mathematics education hegemonic hierarchies.

Deleuze (1984/2006) employs the notion of chronotope to analyse thinking as a tacit temporal/spatial order where the subject encounters the means and obstacles to thought into a 'geographic' boundary that works as the milieu of situated meaning-making. Resorting on Bakhtin's question of '*what is a novel*' and answer that '*novel is never given*' but always forms, transforms and grows within specific chronotopes, Deleuze, in turn, contemplating on '*what is thinking for the philosopher*' locates its chronotope to a 'scream'. For Deleuze, the philosopher needs to attend primarily the biopower of 'scream' that forces the posing of a problem and not to any particular 'method' of resolving a problem or creating a concept. Whilst, the 'scream' cannot determine the outcome or provide the solution, it is this very embodied social act that makes the terrain of struggle visible and supports a determined desire, not a joy, to act on affections experienced by body or mind, superstition or reason. For Deleuze, such basic chronotopes to thought can extent into cartographies of; a) integrated spatio-temporal frameworks including order and mapping (i.e. temporal order inscribed in a map and a map evolving in temporal process), b) generic activities (actual and potential), generic roles and characters located into spatiotemporalities and c) boundaries and crossings. MathLife chronotopes as vignettes of mathematical practice can voice the possibility for thought in mathematics education. Such chronotopes are not always in harmony of the senses, but may come in discord as they 'scream' out the symptoms of the practice such as specific cases of repression, epistemic violence or disobedience, racial or gendered exclusions, language use limits, or body work boundaries that orient us to search the disorders in the field (Straehler-Pohl et al, 2016). Could the 'scream', or at other times the 'laugh', the 'smile' and 'cry', enforce the researcher's will to encounter a cartography around a complex trap amongst hegemonic discourses of mathematics as disciplinary knowledge, mathematics as school subject, mathematics as everyday resource, historical and cultural product, or a formatting power for social orders?

The above question is particularly pertinent in the context of mathematics teacher education where the 'what' of mathematics needs to be, on the one hand, connected with the whom, why, where and when of children, activists, teachers, practitioners and people in the community, and, on the other hand, disconnected from a threatening rhetoric that assumes mathematics as key for ensuring national or world security, progress and development. Guttierez (2013), along with others, claims for political knowledge in mathematics teacher education programs as a urgent need in a neoliberal society where education becomes part of a consuming market of qualifications, competences and skills. How could mathematics teacher education support student-teachers and teachers not only for critical citizenship, but also for agency to navigate, resist, disrupt, trouble or subvert such essentialist discourses? How could our teacher education courses invest more into not only representing but also performing creative critical and aesthetically challenging mathematical interventions in ways that address the invisible or voiceless in our diverse worlds and destabilise the 'myths' around mathematics? In other words, how could we encourage our student-teachers and us to discern the 'scream' and the 'smile' in mathlife chronotopes?

Along these lines, the project of 'street mathematics' focuses on the centrality of questions of significance in sociopolitical life and mathematical creation, exploring the ways in which language-use and body-work in discursive practices of diverse mathematical experiences register the conflicts amongst social groups as they seek to meet and connect in public at the urban space.

STREET MATHEMATICS AS HYBRID

Although the term 'hybrid' steams from biology, the last three decades has been extensively used, and critiqued, in postcolonial, cultural and feminist studies, as well as, in sociology, history and anthropology of science. In postcolonial studies, Bhabha (1994) made an influential argument that the border or boundary region between two spatial domains is a new region of overlap that produces hybridity. This region, often called 'the third space', contains an unpredictable and changing combination of attributes of each of the spaces that contribute towards producing something new but, yet, related to the old –called the hybrid. His area of concern was colonization politics, in which some native people find themselves caught in between their own culture(s) and the newly imposed culture(s) of the colonizers. This hybrid lives in-between, and despite/because its contradictions, conflicts and power politics, the hybrid is, always, where the polyphony of languages, cultures, discourses and identities exists.

The notion of hybrid in science studies has been discussed by Bruno Latour and Donna Haraway who argue how science is, ultimately, a factory of hybrids and maintain that a hybrid is the result of any process of association amongst species, methods or ideas. Haraway (1991) extends the notion of hybrid to cyborg, a metaphor borrowed from science fiction, in order to break binary distinctions amongst nature, science and technology or humans and non-humans, clearing, thus, the way for acknowledging diversity and difference. In human computer interface design studies, Lucy Suchman (2002) argues in favour for hybridity as a crucial aspect for both users and software professionals so that to maintain and foster the presence of multiple voices in constructing new knowledge and technology products. 'Hybrid' spaces where knowledge becomes re-circulated seem essential for facilitating co-construction, re-negotiation and re-configuration of concepts, ideas, meanings and alliances. The boundary-crossing or the becoming-hybrid as mutual learning in-between different standpoints, epistemologies and ontologies gain ground and appeal to theorists, designers and researchers who work in complex fields.

Bakhtin (1929/1981) also discusses hybridity as a way to capture the complexity of language(s) and discourse(s) and views hybridisation as fundamentally inherent in every discursive practice or language-act as heteroglosia, polyphony and dialogicality. Reading Bakhtin, Sholat and Stam (1994) describe hybridity as '*an unending, unfinalisable process*' which is '*...dynamic, mobile, less an achieved synthesis or prescribed formula than an unstable constellation of discourses*' and argue how hybridity becomes the unmarked case of social life foregrounding life's dynamism, contingency and uncertainty. Experiencing uncertainty in (mathematics) education is related to people positioning themselves in different global landscapes, networks or social worlds (e.g. ideological, cultural, technological) having to deal with contradictions, ambiguities and contrasting interests (see Skovsmose, 2005 about a discussion of uncertainty in mathematics education). Bakhtin, in a series of texts, negates language or discourse as essentialist or abstract systems but turns to emphasize their intersubjective consciousness and social nature. Any social interaction, including mathematical activity, carries within it diverse social biographies involving give-and-take of multiple utterances, languages, discourses, identities as situated interactive multi-voiced hybrids of knowledge as both epistemological and ontological experiences (Chronaki, 2009). The concept of hybrid might well remain open for critique, but it is relevant for this study as it encourages to consider the epistemic/ontic dimensions of knowledge discourses circulation with people in the urban scape.

As explained elsewhere, the project 'street mathematics', rooted in the spatial metaphors of 'street', 'body' and 'move', evolves as a hybrid where vignettes of mathlife chronotopes can be re/presented, re/located and re/assembled. Such vignettes are depicted through audio-visual or text media and derive from long term ethnographic research (e.g. in situ observing and interviewing), and selections from artworks (e.g. paintings, literature, poetry) or pop culture (e.g. movies, graffiti, literary texts) in partial and impure connections to the cultural life of the city of Volos (Chronaki, 2015). Based on this hybrid space tapestry specific urban interventions have been curated, stories concerning our ways of valuing and relating with knowledge, knowing, and so-called knowers or 'specialists' (vs non-specialists) have been revisited allowing mathematical subjectivities to be reconfigured. As such, particular taken for granted discursive constructions of 'truth', values and valorisations concerning mathematical knowledge sharing, identifying and learning have been re/circulated, re/presented and, even, disrupted. The present paper, based on previous work, aims to discuss the conceptual frame of this work and its potential for teacher education.

KNOWLEDGE DISCOURSES URBAN CIRCULATION

A range of discourses exemplifying our relation(s) to/with mathematics, power, race, gender, desire, pleasure, society, identities, values, body-work, objects of creation or making through interview vignettes with local artists, craftsmen or scientists, or selected scripts in movies, literature, poetry, or work-pieces in arts, crafts, off-hand constructions, choreographies consist the material fragments of 'street mathematics'. Each of these offer opportunities for curating potential urban interventions in the form of installations and/or performances in connection with the social space of the locality. For example, specific scenarios have been organized in routes where one can walk into a virtual and/or physical route in the city (i.e. the cinemas route, the literature room, the dance-studio route etc.) that invites embodied interactions with specific clips from movies (e.g. Agora, Pi or Proof) or artwork connected with scripts form poetry, literature or vignettes from choreographies[1,2]. These can then be experienced next to interview vignettes with local artists, craftspeople, scientists, youngsters or lay people, and along with children's activity inside and outside classrooms. As we move within the city either through the screen or the physical context we become the carriers of such knowledge fragments, and, we,

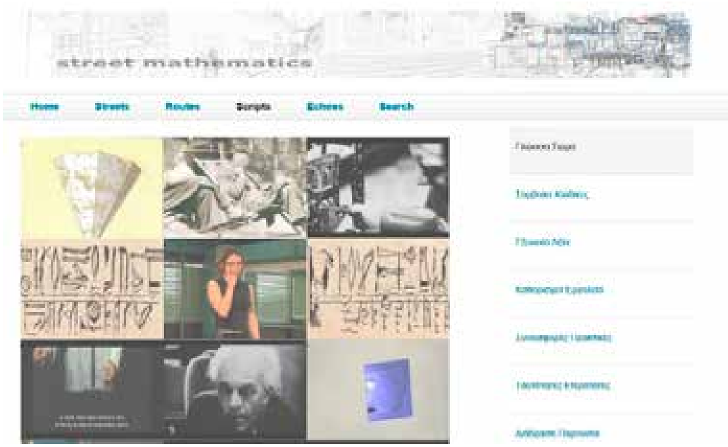

Figure 1: Aspect of the 'street mathematics' webpage

1. These fragments have been digitized and become re-assembled with certain layering technologies, such as; a) a cloud typology related directly to the dynamic data base, b) a virtual reality mapping of imaginary routes in the city of Volos, and c) locative aware media based detours, performances and narratives in the urban tapestry of the city. Users can walk into the streets of the city, through a multiplicity of embodied performances by means of their full 'body', their hands, eyes, feet, senses and their extended 'body' with the help of mice, touch screens, QR codes, locative games or the virtual reality model (Chronaki et al, 2011, 2014).

2. Specifically, the 'street mathematics' incorporates four distinct spaces, namely a) a dynamic data base where the project material as knowledge fragments are being archived, stored, retrieved, moved and relocated in other spaces such as the virtual reality model of the city, b) the physical space of the city of Volos from where much of the data originate, as they have been collected via in-situ observation and interviewing, have been reformatted into small episodes, scenarios and narratives, and now can return back to the city, c) the virtual reality model of the city Volos which becomes a canvas for artwork installations exemplifying specific spatial imageries in routes, landmarks and places, and d) the website of the project where the three previous spaces become presented, connected and accessed by the end users as they navigate the site. Moreover, the website interface provides ways for a meshwork amongst virtual and physical experiences within the city of Volos through specific sub-areas, namely; 'streets', 'routes', 'scripts' and 'echoes' (http://streetmathematics.ece.uth.gr/portal & Figure 1). It is within these sub-areas that the actual first-person experiences of end-users as navigators, players, learners, teachers, educators, simply derives, or even larkers, takes place (Chronaki, 2015).

ultimately, are not part of merely the embodied act of navigating, walking, strolling, or wandering in the streets of the city, but, in fact, we ourselves perform knowledge circulation. It is this bonding amongst the hybrid urban space of ‘street mathematics’ and the metaphors of ‘street’, ‘body’ and ‘move’ that, potentially, expand imagination, foster creativity and urge us for a revolutionary vision to ‘see’, ‘touch’ and ‘experience’ knowledge in other ways –as part of our life-worlds.

Some questions persist: How do we relate with our student-teachers in the locality and how do we reclaim the energy of everyday life that space and people hold for us? How do we deal with academic and everyday knowledge in the chaotic conditions of our worlds and how could we invent ways in which knowledge circulation does not work towards remaking ‘colonial science’, but, instead, paves subtle disruptions of colonial dispositions, binary meanings and poisonous affects? How do we relate with the presence and/or absence of mathematics and the multiple genres of narrating mathematical activity as we stroll and wander in the cityscape? Could the social space of the city enable us to open up relations with diverse mathematical practices? How does the urban environment become challenged and troubled by mathematical activity? How do such experiences affect mathematical subjectivities?

Contemporary life in the urban scape of the city of Volos is strongly affected by the economic debt crisis with serious implications on the closing down and fast desertion of a number of spaces in the public domain such as shops, bookstores, cultural centers, galleries and cinemas. An example is the Lido cinema located in one of central commercial streets (see Figure 2). Effects of the debt crisis are also discerned with our graduate and undergraduate students who, despite coping they can be in despair. On the one hand, unemployment rates have reached the highest level making their potential of finding a job at Volos, or even their possibilities of hoping for a better future, extremely slim –exemplifying the low exchange value of qualifications. On the other hand, lecturing methods assuming a direct knowledge transfer, especially now, at times of crisis, cannot be tolerated. Under conditions of wide social uncertainty, monopolies of academic imperialism cannot work with young people. Knowledge not as a ‘matter of fact’ but as a matter of concern and as a matter of care is of urgent significance today (Latour, 2005). Taking this into account, how knowledge as a mathematical practice of every day or scholar activity can be experienced in embodied performances that trigger them, and us, to question, resist and disrupt taken for granted ‘truths’.

The ‘street mathematics’ project-in-progress provides ways towards addressing some of these questions as it becomes more and more utilized by student-teachers, children, adolescents, educationists, designers, urban planners, performers, artists and architects. However, the aim is not really to provide answers, but, in a more modest way, to try address these questions in ontic terms, focusing more on mathlife chronotopes. Specifically, the collected scripts –as fragile life-world knowledge fragments in the form of inscribed accounts, visual images or text- concern human action in direct relation to a multiplicity of concepts, practices and activities that, conventionally, is named 'mathematics'. Reassembling them into the spatiality of streets and routes but also into the discursive configurations of scripts and echoes (see Figure 1) can, perhaps, work towards, not so much answering a question, but into reformulating its significance and scope by means of a bodymind urban materialisation and in ways that 'a question' is further stated as a matter of concern and care for the people involved.

Figure 2: The Lido Cinema at Kartali Street in the city of Volos. /
Figure 3: A hybrid of Lido Cinema

Interaction with mathematics concerns multiple narratives on how mathematics is being creatively or/and violently employed in varied practices (e.g. art, choreographies, constructions, crafts, games, gambling, economy, commerce, history, personal storytelling etc.), but also the relations people form with mathematics or through mathematics as part of specific economic, social, cultural or aesthetic practices. These narratives consist parts of stories with people who aren't necessarily mathematicians or formally educated. They consist scripts from varied sources such as films, literary texts, poems, art, constructions, or events having to do with everyday life, work, and leisure. Questions that enforce the scenario creation along with our student-teachers are; *'what might mathematics be for us or for others?', 'how mathematics appears or disappears in people's life?', 'in what chronotopes?'*

One such scenario has evolved around the experience of working with the simultaneous 'presence' and 'absence' of artist Giorgio de Chirico and his artworks form the, named after him, cultural gallery in Volos –the town of his birth (see Chronaki et al, this volume). The gallery is located just next to the University Library building at the parade of Metamorphosis Street. De Chirico's internationally renown artwork remains allegorical in how geometry is being employed to reveal 'emptiness' in engineer's passion for geometrizing the urban space. What might be the significance of Giorgio de Chirico's artwork today for us? How could we relate to his/our allegories of geometry use, but also how do we identify with the artist's 'scream' in his/our urge to discord with rationalist geometric space? How could we explore these by moving into critical urban (mathematical) interventions with our student-teachers and the locals in the street where de Chirico gallery is placed?

Another scenario of urban (mathematical) intervention, which is underway at the moment, focuses on reconfiguring the turbulent life of Hypatia, the Alexandrian mathematician through making relations with contemporary women scientists who live and work in the city of Volos. Scripts related to Hypatia's life, selected from the cinematic and literary sources, become present and accessible just outside the Lido Cinema (Figures 2 and 3) awaiting to make virtual and physical contact with local passers (Chronaki et al, 2011, 2014). What might be the thoughts, affects and affective experience created by such an associology, in Bruno Latour's words, for mathematics and science, gender and mobility? And, what else can this association may signify when it takes place outside of the closed Lido cinema?

The birth and growth of scientific facts as knowledge discourses circulation has been explored not only in the context of laboratory studies (Knorr-Cetina, 1999. Latour & Woolgar, 1979/1986) but also in remote communities where science becomes either 'discovered' by western gaze or imported with western interests (see research studies in ethnomathematics, indigenous and first nation people, specifically D'Ambrosio, 1985). Particular issues have grasped research attention including the effects, processes and ethics of re/distributing cognitive activity, knowledge within social networks, between people and via inscriptions for much of the work of science. Of most importance is the critique of the vision of science as travelling from the 'metropolis' to the 'periphery' as 'colonial science'. Certain knowledge mobility practices that have been organised towards crossing cultural borders were opposed as colonializing acts. In addition, the idea of modern science moving towards a racial or cultural mingling has been also problematized by post-colonial theorists who wanted to emphasize that knowledge circulation is not just an issue of 'translating', 'interpreting' or 'moving' from the west to the east or to the south. Rather, scientific knowledge remains in a constant shaping and reshaping of ideas and, according to Winterbottom (2011), '*a continual dialectic exchange of information and techniques between Europeans and Asians throughout the colonial period*', but as she explains '[...] *unequal power relations often meant that this shared knowledge benefited the colonizer more than the colonized*' (p.268). However, this discussion has mostly left mathematical knowledge practices untouched, as if mathematics, in sharp contrast to science, remains a neutral domain of practice.

Scientific knowledge has been addressed by Bruno Latour as 'immutable mobile' in his work *Science in Action*. With mobility he refers to the transportation of knowledge in networks of interest whilst with immutability to its capacity of retaining key features whilst moving. Immutable mobiles are effects of costly technoscientific infrastructures (material, discursive, technological) and their study reveals power-control hierarchies in society. Whilst Latour (2005) stresses knowledge as immutable mobile, others have pointed to the fact that knowledge/objects transform as they become transported to other cultural contexts or networks of interest. Referring to previous analysis of the usage, development and transformation by locals of a bush pump in Zimbabwe in this mode, Law and Mol (2001) argued about the hybrid topology of technoscientific knowledge/objects. Although, such knowledge discourses circulation discussion refers mainly to science and not to mathematics, I would argue here that it is important to explore further its significance for mathematical activity and artefacts exchange and mobility. The 'street mathematics' project develops into such a direction of a hybrid space topology by deliberatively reassembling mathlife chronotopes. Hybridity becomes possible through the liminal re/constructions of virtual and physical border-regions where distinct spatial experiences of knowledge exchange co-exist and blur. In this, enacting with the metaphors of 'street', body' and 'move' an epistemic/ontic knowledge circulation is being orchestrated amongst spaces, locals and ideas creating new imaginaries for our student-teachers, our students and ourselves.

KNOWLEDGE AS EPISTEMIC/ONTIC

As has been outlined in a number of related ethnographies, 'street mathematics' was inbred and ingenerated within the minds of children, adolescent and adult street sellers, in Brazilian cityscapes where a great number of people from the nearby rural areas had emigrated for a better life. The survival of those people was depended on their unskilled manual labour, their competences to establish small businesses, their tolerance and ability to deal with a periodical and unsure income and their solidarity in self-organised communities. The debt crisis of the 80s in Latin America made them face complex problems of unstable currency increases due to huge inflation rates that affected commercial transactions in both high and low commerce. Quoting Geoffrey Saxe (1989) '*Brazilian children address mathematical problems when they use currency in their everyday lives in such activities as purchasing a grocery item at a store. Because of the inflated currency, these activities give rise to the need to represent large numerical values and –to a limited extent- arithmetical calculations involving large values. Eventually, some urban children take up 'street professions' such as candy selling, and in the context of this practice, mathematical problems of everyday life increase in complexity. Candy sellers must produce frequent computations involving multiple bills as well as compute and compare pricing ratios*' (p.1424). Children along with adults, despite being unschooled, were encountered into these complex socio political contexts as competent problem solvers without the resource of schooling, without using pencil and paper, with no resorting on algorithms and formal tools but through collective, mental, haptic, and oral strategies (Nunes et al, 1986). In retrospect, one might note how their desire to live a better life or simply to survive embodied them with courage for taking risks, staying together and overcoming obstacles. It must be emphasized that in doing so, children, involved their minds by, primarily, involved their bodies in specific places.

Helen Verran (2004) through her ethnographic work with bilingual Yoruba children in mathematics classrooms in Nigeria discusses 'ontic' as a way to move beyond a Kantian notion of knowledge as being mainly epistemic. Following A.N. Whitehead, her perspective on 'ontic' denotes how agents, always, contribute with embodied participation in collective action. She explains that the ways Yoruba children participate in problem solving is not through quantifying concrete items into abstract number entities as in 'western' logic, but through quantifying matter in relation to their bodies.

Their generalizing attempts proceed with bodily gestures and acts of speaking referring to bodily qualities like thingness or volume. Ontics can be seen as a bodymind coupling that does not aspire completeness, certainty or singularity, but, instead, seeks connectivity and accepts vagueness. As such, ontics is a major part of knowing and becomes a politics of rendering our life commitments visible through embodied performance.

Diverting from a pure epistemological standpoint brings forward also the question of 'who, when, why is constructing knowledge' and urge us to explore knowledge in direct relation to questions of geo-body-political significance (Mignolo, 2009). The body metaphor is already utilized by information technology designers in web interfaces for configuring the screen in relation to how representations and spectators connect to each-other as they navigate the chaotic space of the internet (White, 2006) or locative based media in relation to embodied performances, body presence, sensual and sensing experiences or self/other interactions. For the purposes of our research in the 'street mathematics' project, the notion of body, as the ontic axis, becomes an important issue in coming to terms with knowledge. Primarily, a resort to the body metaphor signifies the urge to move beyond body-mind dichotomies where knowledge is solely located in the individual mind and the body is conceived as a closed container. Secondly, the body signifies the ontic substance of knowledge promoting the importance of encountering embodied performativity. Lastly, the bodymind bond signifies how we best respond to chaos, as well as to inconsistency, uncertainty, complexity and vagueness of life itself and life-worlds. At the same time, it unveils how times of crisis affect our relation, access and abilities to deal with knowledge discourses circulation amongst poor and affluent, haves and have-nots, subalterns and bourgeoisie in urban cityscapes. In all, the resort to body encourages us to think embodied interaction in relation to our flesh, senses and sensibilities but also to how and why our body can or cannot relate with the social space outside the body, but still connected with it, as a matter that matters to our lives and our living environments.

Today, 30 years later, Greece (and Europe) is under an equally severe economic debt crisis, faced by Latin America in the 80s, that affects seriously urban life and challenge modern ways of dealing with knowledge, knowing, knowers and formal learning. The current debt crisis is deeply immersed into a contemporary culture and social life described in cultural flows or networked society terms. Cultural diversity along with ongoing access to open technologies tend to blur and/or reinforce variously borders and boundaries amongst illiterate and literate, poor and affluent, subalterns and bourgeois, south and west, primitive and privileged. Today, we witness the increased complex presence and impact of social media and, in consequence, a rapid medicalization of knowledge, as well as, the growing potentials for hybridizing cityscapes due to locative aware technologies.

Charitos et al (2013) observe how such systems have paved new revolutionary encounters such as the Arab Spring, the Occupy movement, or the anti-austerity demonstrations in southern Europe which, albeit being ephemeral, are '*...both embodied and mediated, and influence community dynamics, giving rise to networks around common interests and collectives of affect'* (p.xv). It is important to delve more into their virtual potentialities and to explore how they might be set to work towards mobilising mathematics education discourses and subjectivities into more sociopolitical routes.

AS A WAY OF CONCLUSION

Lefebvre (1991) invites us to open up the container of any fixed images of a city, a house, or a street and urges us to consider them as '*a complex of mobilities, a nexus of in and out conduits*' (p.92). The urban landscape possesses the elements of a potentially fluid geography that is being reproduced by ongoing movements or '*streams of energy which run in and out of it by every imaginable route*' (p. 93). Our research, as part of a contemporary 'street mathematics' hybrid is geared towards reconfiguring the synergy amongst content, people, space and technologies production -thus opening up the mathematical activity container. Saying this, urban (mathematical) interventions with our student teachers and others aim not only to unravel the mathematical activity but also to re-think our relationship with knowledge and to re-designate its social value and power. In this way, it urges our imagination to experiment with knowledge as epistemic/ontic. Such urban interventions can potentially mobilize multiple cultural presentations of mathematical ideas and make them move, drift and wander with/in the streets of the city.

However, the act of wandering in a city is not neutral. It is a cartography of spatial embodied relations with/in the metropolis where both the 'power of the city' and the 'city of power' need to be encountered as manifolds of social, racial and gendered inequalities. The urban interventions that the hybrid of 'street mathematics' can generate have the potential for exposing diversities and making subtle disruptions of dominant mathematical subjectivities. As such, assembling mathlife chronotopes becomes a place-based laboratory of temporary and deliberate heterogeneous arrangements of sensorial material and immaterial elements. Such assemblage can work, under certain conditions, to further key features of sociopolitical thought in mathematics teacher education. It can hold together an experimental mingling of life-worlds and mathematical creations of affective intensity, especially important for reaching student-teachers. As such, it can serve to unfold bio-political thinking through denoting where is the 'scream', the 'cry', the 'laugh' or the 'smile'.

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HOW CONFIDENCE RELATES TO MATHEMATICS ACHIEVEMENT: A NEW FRAMEWORK

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While there is increasing acceptance within mathematics education that confidence plays a crucial role in students' mathematics achievement, not much is known about the nature of this relationship and students' confidence profiles. This paper uses the Trends In Mathematics and Science Study (TIMSS) Student Survey data to establish relationships between students' level of proficiency and how they describe themselves in terms of confidence doing mathematics. Based on a set of eighth questions from the TIMSS survey a theoretical model influenced by the Johari Window was developed. In this paper, we discuss the development of this model and its implications to instruction.

INTRODUCTION

Historically, eighth grade students in the U.S. who participated in algebra were successful white male math students from upper socioeconomic status (Flores, 2007; Oakes, Ormseth, Bell, & Camp, 1990, Sells, 1973). Recognizing the disproportionate number of minorities and females in Algebra 1, researchers and educational policy makers supported the movement to offer "algebra for everyone" (NCTM, 1992). The "algebra for all" movement began as an initiative designed to increase equity and diversity amongst the population of students taking Algebra 1 by providing an opportunity for students from all backgrounds (Moses & Cobb, 2001; Richardson, Ball, & Moses, 2009). The challenge, however, is that not all eighth grade students are well prepared for this course leading to persisting achievement gap. The reported achievement gap necessitates an understanding of the underlying factors rather than engaging in stereotyping of certain student groups (Gutiérrez & Dixon-Román, 2011; Martin, 2012).

Recent research focusing on the affective domains suggests that under-preparedness (prior ability) may not be the only reason and that psychological constructs may also be related to student achievement in mathematics (Reyes, 1984; Reyes & Stanic, 1988). One of such affective constructs is students' mathematical confidence which is believed to serve as a mediator between their motivation and achievement levels. Morony, Kleitman, Lee& Stankov (2013) in an investigation of the structure and cross-cultural (in)variance of mathematical self-beliefs between Confucian and selected European countries reported that confidence is the "single most important predictor of math accuracy" (p.1). Despite issues related to affected domains such as confidence being seen as vital in explaining students' achievement in mathematics, Op 'T Eynde, deCorte and Verschaffel (2002) noted that "research on this topic has not yet resulted in a comprehensive model of, or theory…" where confidence was taken as part of four characteristics of beliefs (cited in Burton, 2004; p. 358). Morony, et al., (2013) also called for further research that highlights the extent to which students' calibrations of their confidence both reflect and enhance their learning. The model presented in this paper responds to these calls for further research on students' confidence and its relationship to their achievement levels. The current study is not experimental, nor is it attempting to identify causal relationships in mathematics achievements. Instead, it is a quantitative study that examined the relationship between students' mathematics confidence and mathematics achievement which was followed by the development of a model, *CCL Confidence/ Achievement Window* that is related to the Johari Window (Luft, 1969). The following research questions guided this study: 1) What is the relationship between mathematics confidence and mathematics achievement? 2) Do higher levels of confidence towards mathematics lead to higher levels of mathematics achievement? 3) What is significant about the relationship between confidence and achievement?

LITERATURE REVIEW

Confidence "consists of an individual's perception of self with respect to achievement in school" (Reyes 1984, p. 559) and is related to an individual's self-concept. Pajares and Miller (1994) define self-efficacy as "a concept-specific assessment of competence to perform a specific task" (p. 194). Based on these definitions, self-concept (domain specific) and self-efficacy (item specific) differ at the level of the construct.

Consistently, both researchers use the term confidence in their definitions. Reyes states,

> Confidence in learning mathematics is a particular component of self-concept that is specific to mathematics…Confidence in learning mathematics, or self-concept specific to mathematics, has to do with how sure a person is of being able to learn new topics in mathematics…. (pp. 560- 561).

Pajares and Miller (1994) report, "It is clear that beliefs regarding confidence are part of an individual's self-concept…confidence in learning mathematics [is the] conceptual forerunner to math self-efficacy" (p. 194). Despite other constructs such as self-concept and self-efficacy being used interchangeably in the literature, there is an increasing use of the term *confidence* as a bridge between these two constructs. The importance of confidence in explaining students' motivation and success levels has led to a number of studies being carried out to determine how well students are able to calibrate their confidence (e.g. Atherton, 2015; Foster, 2016; Hong, Hwang, Tai, & Chen, 2014).

Calibration is used to generally refer to how well a student's confidence and competence match (Foster, 2016). Empirical studies reveal that there are three ways in which students calibrate themselves namely: under calibration (high achievement versus low confidence), over-calibration (low achievement versus high confidence), and high achievement versus low confidence being well-calibrated (good match between achievement and confidence) (Foster, 2016). A consistent finding from the calibration studies is that students tend to be poorly calibrated leading to the confidence-achievement paradox (e.g. Atherton, 2015; Foster, 2016). For example, Atherton (2015) reported that females were more likely to be less confident compared to males. A way of understanding students' confidence level profile, therefore, is useful in determining students who know what they know and those who project over-confidence relative to their actual achievement level. Such a profile, we argue, will help teachers implement measures that can help students better calibrate their mathematical confidence and know when to ask for help instead of being overly confident and obtaining a wrong answer (Fischhoff, Slovic, & Lichtenstein, 1977; Foster, 2016).

METHODOLOGY

Ladson-Billings (1997) suggests that even when a study is focusing on a specific group, in this case eighth grade students, it is important "to situate it in the larger context of mathematics teaching and learning..." (p. 698). For that reason, we begin our analysis using the U.S. TIMSS data to situate our study in the larger context of the U.S. We also explored the findings from the TIMSS data analysis by focusing on journal entries collected from a sample of African American students (although this is not included in this paper). We chose to focus on a sample of African American students because data concerning the mathematics achievement of African American students indicates a prominent achievement gap across various levels and concerns about instructional quality (Hallett & Venegas, 2011; Horn, 2008; Ladson-Billings, 2006; Palardy, 2015). Hallett and Venegas (2011) in a study of college-bound high school students concluded that despite increased access to Advanced Placement courses for students enrolled in low-income urban high schools, students in the study did not have a sense of being adequately prepared for college. Subsequently, research is warranted given the disparity among student groups.

Data

The International Association for the Evaluation of Educational Achievement developed the Trends in International Mathematics and Science Study (TIMSS) to measure the mathematics and science skills of fourth and eighth grade students (Williams, Ferraro, Roey, Brenwal, Kastberg, Jocelyn, Smith, & Stearns, 2009). The United States was one of the 58 participating countries during the TIMSS 2007. The United States data is available for public use from the National Center for Education Statistics' (NCES) *Trends in International Mathematics Science Study (TIMSS) 2007 U.S. public use data file* (2009).

Sample

The TIMSS 2007 data was collected on a national probability sample of students nearing the end of their fourth and eighth year of school (Williams, Ferraro, Roey, Brenwal, Kastberg, Jocelyn, Smith, & Stearns, 2009). This study's focus is on the sample of eighth grade students. There are 7,377 eighth grade students sampled from 239 schools across the United States. Of the 7,377 students, 3,656 are males and 3,721 are females. The sampling design of the TIMSS data led to a racially diverse sample which is relatively proportional to the population of 8th grade students. Of the 7,282 students with valid data, 3,873 are White, 949 are Black, 1,787 are Hispanic, 243 are Asian, 90 are Native American, 58 are Pacific Islanders, and 282 are Multiracial (2 or more races). Finally, the sample included students with varying levels of socioeconomic status (measured with Mother's education level).

Student Questionnaire

The TIMSS 2007 assessment framework documents the importance of contextual factors on students' learning; therefore, all participating schools administer questionnaires at the school, teacher, and student levels (Mullis, Martin, Ruddock, O'Sullivan, Arora, & Erberber, 2005). The IEA conducts extensive analyses to verify the reliability and validity of all instruments used during TIMSS administration, and publishes the information in the TIMSS 2007 Technical Report and User Guide (Olson, Martin, & Mullis, 2008). We chose to focus only on data collected from the student questionnaire. The student questionnaires collect information on students' home background and attitudes towards mathematics and science (Olson, Martin, & Mullis, 2008). This study examines the relationship between confidence and math achievement; consequently, only the items of interest are further discussed.

Independent Variables

Eight items from the student questionnaire are used in this study. We theorized that the following items measure the latent construct of mathematics confidence: *I usually do well in math* (BS4MAWEL), *I would like to take more math* (BS4MAMOR), *Math is more difficult for me* (BS4MACLM), I enjoy learning math (BS4MAENJ), *Math is not one of my strengths* (BS4MASTR), *I learn things quickly in math* (BS4MAQKY), *Math is boring* (BS4MABOR), and *I like math* (BS4MALIK; see Table X). All of the eight items are measured using a four- point Likert scale, where “Strongly Agree” = 1, “Agree” = 2, “Disagree” = 3, and “Strongly Disagree” = 4. However, we decided to recode the variables in reverse order, so that larger values suggest more confidence and interpretation is straightforward. The independent variable is consistent, scaled -1 to 1 standardized with a mean of 0 and standard deviation of 1.

Dependent Variables

The dependent variable for this study is mathematics achievement. The TIMSS 2007 data provides five plausible values of mathematics achievement for each student: BSMMAT01-BSMMAT05. All analyses use the five plausible values as the measure of students’ mathematics achievement. The use of the plausible values ensures accuracy of students’ achievement (Williams, Ferraro, Roey, Brenwal, Kastberg, Jocelyn, Smith, & Stearns, 2009).

Criterion Profile Method

Profile analysis is a statistical procedure used to identify profiles patterns based on membership of a small subsample. Davison and Davenport (2002) proposed the Criterion Profile Method (CPM) as an alternative to existing profile analysis procedures. An advantage of CPM is that the profiles are related to an external criterion; therefore, the validity of the procedure is greater than other profile analysis procedures (Culpepper, 2008; Culpepper, Davenport, & Davison, 2008; Davison & Davenport, 2002). The CPM has been used to study out of school activities that are related to reading achievement (Culpepper, 2008) and the relationships between race, academic achievement, and college acceptance (Culpepper, Davenport, & Davison, 2008). CPM is a regression based procedure; consequently, the observed profiles are related to an external criterion (Dependent Variable). For this reason, the current study utilizes the CPM model to test the relationship between confidence and math achievement. Even though profile analysis is a statistical procedure frequently used in psychology, there is not a significant amount of literature that employs the CPM. More details of this model and analyses are available in a forthcoming paper.

The Johari Window

The Johari Window, developed by Joseph Luft and Harry Ingram (1969), is a four-pane window that symbolizes information known to ourselves and information known to others. This interpersonal communication tool has been used in diverse venues like psychology and leadership to describe the degree and depth of knowledge individuals have of themselves that is the same or different from the information others have of the individual. According to Wade and Hammick (1999), one purpose of the Johari Window is to "help people understand themselves, and to acknowledge barriers and defences (sic) that may be erecting subconsciously...[and can be used to help] develop more insight and self awareness (sic)" (p. 10). This, therefore forms the basis for the connection between the Johari Window and the CCL Confidence Window.

	Known to Self	Not Known to Self
Known to Others	I Area of Free Activity (*Public*)	II *Blind* Area
Not Known to Others	III Avoided or *Hidden* Area	IV Area of *Unknown* Activity

Figure 1: The Johari Window

In the diagram (see Figure 1), quadrant one (in the upper left-hand corner) describes what is known to self and others, the *Public* area. Quadrant two, upper right-hand corner, is known as the *Blind* spot; that is, information known to others but unknown to self. The *Hidden* area is in the lower left-hand corner and describes what is known to self but unknown to others. Quadrant four, lower right-hand pane, represents an area that is *Unknown* to self and others.

We combined the concept of the four-panes of the Johari Window with the Cartesian Coordinate Plane (x- and y-axes) from mathematics to reorganized the information into quadrants with values greater than or equal to zero or less than or equal to zero (see Figure 2).

	High Achievement		
	II (c, A) Low Confidence High Achievement *(Blind)*	**I (C, A)** High Confidence High Achievement *(Public)*	
Low Confidence			High Confidence
	III (c, a) Low Confidence Low Achievement *(Unknown)*	**IV (C, a)** High Confidence Low Achievement *(Hidden)*	
	Low Achievement		

Figure 2: The CCL Confidence Achievement Window (The Johari Window labels parenthetically)

Graphing dependent and independent variables on this plane disaggregates ordered pairs into four regions similar to but placed differently from the Johari Window. Our two characteristics, confidence as a result of the TIMSS confidence items and mathematics achievement as a result of the TIMSS test, are assigned values between -1 and 1. Thus, we redefined the quadrants as positive confidence, higher achievement scores in the upper right-hand corner (+, +); lower confidence and higher achievement scores in the upper left-hand corner (-, +); lower confidence and lower achievement in the lower left-hand corner (-, -); and low achievement despite high confidence in the lower right-hand quadrant (+, -). While the newly defined window does not match the corresponding regions of the Johari Window, the characteristics of the premises of the quadrants' similarities remain; and our new window closely mimics the mathematical meaning of the Cartesian coordinate system. The notation that we have selected to represent each quadrant uses an upper case A for positive achievement, an upper case C for positive confidence, a lower case a for low achievement and a lower case c for low confidence. Thus, the notation "(c, A)" represents the second quadrant because of negative confidence level and high achievement.

The characteristics of the Public area align to the new high confidence-high achievement quadrant (C, A); here the student and others have confidence in the student's ability to perform mathematically. The student's confidence is intrinsic and the high achievement is externally positive as well. In other words, both the individual and those around the student have evidence of success in mathematics. The Hidden area (unknown to others but known to self) is similar to the new quadrant where student's confidence is high but achievement is low (C, a). The student has confidence in their mathematical performance but has not demonstrated their ability to others. At this point, it is unclear where the root of the student's confidence lies. Negative confidence and high achievement (c, A) is analogous to the Blind area. Here, students lack confidence in their mathematical ability, yet their performance on the math assessment is inversely high. The characteristics of this quadrant are also difficult to explain so early in the research project. The Unknown area is parallel to the student and the public lacking confidence in the student's ability to perform mathematically (c,a). The student does not believe s/he can do math and others are not able to assume the student is able to perform positively based on past performances.

RESULTS

The complex sampling design employed by the TIMSS 2007 data requires the use of software which can properly estimate the standard error of parameter estimates (Williams, Ferraro, Roey, Brenwal, Kastberg, Jocelyn, Smith, & Stearns, 2009). Therefore, the analyses for this paper were generated using SAS® software, version 9, and AM version 0.06.03 (American Institute for Research & Cohen, 2005). In addition, all variables are standardized to have a mean of zero and standard deviation of one. This was necessary because CPM requires that all variables are on the same scale (Culpepper, 2008; Davison & Davenport, 2002).

Students' responses on the eight items of interest are used in the analyses. These responses only include those from the TIMSS 2007 sample of students. There is great variation in most of the responses on the items of the student questionnaire. The only item that does not have a "balanced" distribution of responses is "I usually do well in math". Even though only five percent of respondents disagreed a lot with this item, it was retained for further analyses.

Next, a correlation analysis using the eight items of interest reveals that 26 of the 28 bivariate correlations are above 0.3. This suggests that these items may share some common variance. As a result, an exploratory factor analysis procedure using principal axis factoring was performed to test the dimensionality of the eight items used for the mathematics confidence scale. The results indicate that the items are measuring a single latent construct. The factor analysis revealed that the dimension has an Eigenvalue of 3.98 and accounts for 94% of the extracted variance. Finally, a reliability analysis revealed that the items compose a highly reliable scale ($\alpha = .815$). Based on the description of the items, we decided that the latent construct measured mathematics confidence. Next, we used the items from the mathematics confidence scale and students' plausible values on the TIMSS mathematics assessment to test the relationship of mathematics confidence and mathematics achievement.

A standard multiple regression was performed between the TIMSS math assessment and the items from the Confidence scale. The Multiple Regression (MR) model was found to be significant, F (8, 68) = 175.06, $p < .001$; and accounted for 22.5% of the variation in the TIMSS mathematics assessment score ($R^2 = .225$). The results from the MR model suggest that the item "I would like to take more math" (BS4MAMOR) does not significantly add to the model; however, we decided to keep it in the model because of its strong correlation with the scale. Overall, the MR model suggests that the items on the confidence scale are related to the TIMSS mathematics assessment, and that they may aid in the prediction of students' mathematics ability.

Next, a Profile Analysis was performed using the Criterion Profile Methodology (CPM) model (Davison & Davenport, 2002). The CPM model was significant, F (2, 74) = 709.00, $p < .001$; and accounted for 22.5% of the variation in the TIMSS math assessment scores ($R^2 = .225$). The squared multiple correlations of the MR and CPM are exactly the same, which suggests that the variance from the confidence scale is partitioned into the level and pattern statistics.

Accordingly, two models were fit to the data, using the level and pattern as the sole predictors for each model. This allows for the researchers to test the amount of variation accounted for by the level ($\bar{X}_p$) and pattern (Cov_p) statistic individually (Davison & Davenport, 2002). The results indicate that both the model with level as the sole predictor, F (1, 75) = 121.16, $p < .001$, and the model with pattern as the sole predictor, F (1, 75) = 1401.08, $p < .001$, are statistically significant. Additionally, the results indicate that the pattern ($R^2 = .225$) accounts for more variation than the level ($R^2 = .027$). This suggests that there is a stronger relationship between the pattern of a student's confidence profile and mathematics achievement than that of the level of a student's confidence profile and mathematics achievement. That is, the results from the CPM analyses suggest that a highly confident student may not have a high mathematics achievement scores. Likewise, the results of the analyses suggest that students doing well in math may not want to take mathematics or enjoy mathematics. Instead, it is important that students feel that they are doing well and enjoy math class. These findings have the potential to greatly add to mathematics education research; therefore, the next step in the analysis is to answer a very important, yet simple question: WHY?

DISCUSSION

Participation in mathematics tends to be distorted with a one size fits all policy for all students which assumes that all students need the same fixing without a deep understanding of the underlying issues surrounding achievement levels (Meaney., Trinick., & Fairhall, 2013). Foster (2016) asserts that:

> …a person's *knowing whether they know* is extremely important, as it determines whether additional support (from other people, a computer or a reference source) is required. For example, it is much better to know that you do not know the value of −2+7, although you think that it might be −5, than it is to be sure that it is −5 when in fact it is 5. (p. 272; emphasis in original).

The *CCL Confidence/Achievement Window* reported in this paper builds on that of Foster (2016) by classifying the various ways in which students can calibrate themselves. The use of the Johari Window, we argue, makes it possible for teachers to be aware of the unique characteristics students thereby serving as an important step away from treating all students the same way. By identifying the size of panes like Hidden or Blind, individuals are able to increase the effectiveness of their interactions by shrinking these areas in order to open up other panes like Public. Likewise, identifying the confidence and achievement patterns of mathematics students, perhaps we as educators will be able to prescribe interventions specific to students with varying levels of achievement.

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WORKING CLASS, INTELLIGENTSIA AND THE "SPIRIT OF GENERALIZATION"

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Algebra constitutes here a background against which thinking and activity of the left are examined. The search for whereabouts of "the spirit of generalization" among the working class whose absence was observed by Marx in 1870 leads us to algebra as, on one hand, a generalization of arithmetic, and on the other, as in the metaphor: "dialectics - the algebra of revolution". These two metaphors together open a new point of view with the help of which we can throw some light upon the confusion and, ultimately helplessness of the left in the contemporary world urgently needing a revolutionary boost, in our opinion. The work here approaches the critical math education methodology (Scovsmose and Borba,2004) in reviewing the state of affairs (CS) and, through the explorative reasoning, leads to the imagined situation (IS) of the design of the program for Organic Intellectuals of the Working Class.

THE STATE OF AFFAIRS

I was struck by the Marx's words of 1870:

> *The English have all the material requisites necessary for a social revolution. What they lack is the spirit of generalization and revolutionary ardour.* (Marx,1870)

As a professor of mathematics in a community college in the South Bronx, that is in the working-class institution of higher learning, I know something about the absence of the "spirit of generalization" among students of arithmetic and algebra and, I have been breaking my head as a mathematics teacher-researcher how to re-kindle it. Generalization has been one of the two principles on which I designed the integrated course Arithmetic/Algebra for the working class remedial students at my community college recently.

What exactly is the spirit of generalization? What is generalization? What does it mean that algebra, the same algebra Alexander Herzen used in the metaphorical phrase: "Dialectics – the Algebra of Revolution" is generalization of arithmetic? One could also ask, what would be the Arithmetic of Revolution? Or, pushing the analogy farther, we could ask, what is the dialectics generalization of?

Some of it's simple. It means that (1) the algebraic symbol of the variable X, the essential concept of Algebra, includes in itself (or means) all real numbers at once, that is all numbers we have on the real number line, and (2) that XxY or X+Y, or X÷Y symbolize (or mean) multiplication, addition or division of any two real numbers. Thus, writing the symbol X means writing infinite number of reals in one stroke! Writing X+Y means writing infinite number of binary additions in one stroke. Quite an economy of thought. In terms of dialectics, it's the movement from Many→One, from particular to general. Its need at the present stage of development of the revolutionary movement is tremendous. Let's look at the examples:

Harvey (2010) argues in his address to the World Social Forum that "the central problem is that in *aggregate* there is no resolute and sufficiently *unified anti-capitalist movement* that can adequately challenge the reproduction of the capitalist class and the perpetuation of its power on the world stage (Harvey, 2010)."

The "central problem" hides a generalization. Generalization here is necessary in the process of transition between the "aggregate" that is the total collection of social movements in the world together with their ideological positions to the "unified anti-capitalist movement". To make that mental, and therefore organizational fundamental transition within the practice of the social movements, its participants should discern or formulate the "common denominator" of different struggles, separate it from their particular conditions, focus their attention *on the commonness together with the network of relationships that surrounds or incorporates* it and formulate a new, general theoretical/practical point of view based in contemporary praxis. It will lead to the new view about how to "wrench out the cancerous tumor of class relations" (Harvey, 2012) from the society. Maybe if within each struggle, we identify how exactly the particular movement, contributes to "wrenching out the class relations", we will see where we are, what aspects of the struggle need to be strategically and tactically emphasized.

Crowther and Lucio-Villegas (2012) mention that "Educators working in neighbourhoods have *to connect analysis of social change* and awareness of the wider context, but at the same time begin *with people in the communities* "where they are" (pp.59). The process of connecting analysis of social change with "people in the communities where they are" is the process of generalization, and, its reciprocal, particularization.

Occupy Wall Street (OWS) movement showing the injustice of the ratio 1:99 in terms of the national wealth distribution. The slogan "we are 99%" engaged the process of proportional thinking –the very first step in the transition between arithmetic and algebra; the very first step in generalization. While un-doubtfully that first step had imprinted itself very deeply in American imagination, it has been no more than that –it could not produce the full-fledged generalization, because the education of participants has not emphasized the process of generalization in their school and college preparation. While generalization is a natural process of thinking, its full development requires systematic learning environment focused on the process. However, the steady focus of educational policy (Hacker, 2012) is to eliminate such learning environment by eliminating elementary algebra from college requirements. Participating members of the General Assembly at the Zuccotti park were profoundly democratic but could not move beyond the listing of the relevant issue and their particular causes, later categorized into working groups themes.

The theoretical importance of the spirit of generalization can be easily grasped paying attention to another interesting quote of Marx, who, wanting to bring his dialectics closer to the natural science, offered the following analogy: ***"****The Sun is the object for a plant* –an indispensable object to it confirming its life– just as *the plant is object for the Sun,* being an expression of the life awakening power of the Sun…"(Ollman,2003). A similar relationship, indicated *italics*, exists between a variable (the Sun) and a number (the plant). *The variable is an object for a number* because the variable helps the number transcend its specificity. *The number is an object for the variable***,** because the number helps the variable to become specific. Algebraic processes of generalization and particularization are essential for grasping Marxian dialectics in a nutshell. In similar vein the dialectical relationship between a number and a variable carries a name in mathematics education, the "<u>process-object duality</u>", whose mastery is the criterion for understanding and mastery of algebra itself. Therefore an expression of the type 2x + 1 can be understood as the object, an algebraic expression with its own algebraic operations, and it can also be understood as a process during which the meaning of the expression 2x + 1 is attained by substituting a number instead of the variable x to calculate its numerical value with the help of arithmetic operations. Thus the mathematical object 2x + 1 can also be understood as an abstraction from the pattern of the series of arithmetic expressions: 2*3 + 1, 2*4 + 1, 2*(-5)+1, etc. Therefore the "process/object duality" is similar to Marx's type of abstraction, which joins the process with the result "…for Marx, as for Hegel before him, "abstraction functions as a noun [the result of abstraction] and as a verb [the process of abstracting]."(Ollman, 2003)

How critical is the presence of the spirit of generalization for the praxis of the actual struggle has been brought to strong relief by the Polish Revolution of Solidarity of 1980/1981, where that spirit was clearly missing from the rank-and-file Solidarity members of the rising. The fascinating chapter Dynamics of the Working Class: Consciousness of Staniszkis (1984) written almost simultaneously with the event itself informs that: "The workers could only keep referring to their isolated, concrete experiences (and thus gain only local concessions), or else they could remain silent. It was precisely this silence when they realized that Gierek's diagnosis was incompatible with their own but could not express it made the workers realize that differences in linguistic competence indicate not only dissimilarity but also a hierarchy" [both political and conceptual] *(p.120)*[1]. It is precisely that constant reference to concrete isolated experiences without the ability to draw and express more general conclusions applicable to wider environment than their own factory that tells us about the absence of the spirit of generalization. Here the absence of the spirit of generalization was the explicit cause for defeats in the Solidarity struggles.

Mastering that hierarchy means to develop the two-way linguistic bridge between workers' language and that of intelligentsia (together of course with the underlying conceptual frameworks). Properly designed present education system has no major problems in facilitating this process. Instead, however, Polish dissident intellectuals "*monopolized nearly all expressive roles in Solidarity movement*" (p.121) eliminating working class from participating in the decisions concerning future Poland.

Staniszkis' description sounds very much like a description of the situation in our arithmetic/algebra classes in the Bronx with missing spirit of generalization where students are locked into concrete arithmetical procedural thinking The algebraic rules that is the syntax of algebra as generalization of arithmetic operations do not fare well with students because they are not based on the developmental process from those operations –that is they do not arise out of understanding their arithmetic roots or manifestations. Instead they are handed down as rules not connected with student arithmetic experience. Because of this "banking" (Freire, 1971) pedagogical approach students get bogged down in concreteness of arithmetic thinking, do not see algebraic relationships and, therefore, have tremendous difficulties with algebra and its spirit of generalization.

How come Polish workers in communist Poland of 1980, thirty six years after the communist takeover, had a problem with generalization, and at the same time, as a class, they reached the high level of their consciousness manifested in the demand of the independent union, followed by the vision of new Poland with the factory workers' councils as one of its foundations?

The answer is quite complex and it reaches the very beginnings of the left movement and starting, possibly, at the Marx's note of 1870 whose part 2 states

1. She is referring here to moments of direct negotiations of workers with the 1st Communist party secretary at the time, E.Gierek.

> Only the General Council can provide them with this, and thus accelerate a truly revolutionary movement here and, in consequence, *everywhere*. The only way of bringing about this change is to do what the General Council of the International Association is doing. As the General Council, we are able to initiate measures (for example, the founding of the Land and Labor) *which later, after their execution, appear to the public as spontaneous movements of the English working class.*

It seems that Marx here is advocating manipulation using working class rather than facilitation of the spirit of generalization within working class. Similar attitudes became institutionalized first by Kautsky (1901), a leader of German Socialdemocracy at the turn of 19 and 20 centuries who asserts:

> "Modern socialist consciousness can arise only on the basis of profound scientific knowledge. Indeed modern economic is as much a condition of socialist production as, say modern technology, and the proletariat can create neither one nor the other, no matter how much it may desire to do so. The vehicles of science are not the proletariat, but the bourgeois intelligentsia. Thus the socialist is something introduced into the proletarian struggle from without and not something that arose from it spontaneously."

And soon later by Lenin in *What's to be done?* where he follows Kautsky line of thought condemning the intellectual possibilities of proletariat and reinforcing disbelief in proletariat's own strength and power:

> "This consciousness can be brought to them [to workers] from without. The history of all countries shows that the working class, exclusively by its own effort, is able to develop only trade union consciousness."

The definiteness of this dismissal of workers as intellectuals is surprisingly like the contemporaneous dismissal of teachers of mathematics as independent creators of "research" knowledge within the mathematics education profession. A colleague, Whittmann (1999), who wants to convince the profession to undertake the research investigation of "teaching units" that is a sequence of classes devoted to the development of a concept asserts that till recently, the design of teaching units was considered a *mediocre* task normally done by teachers and textbook authors. He asks, why would anyone anxious for academic respectability *stoop to designing teaching and put him-or herself on one level with teachers* (p.94) While he acknowledges that "many of the best units were published in teacher journals", at the same time he asserts that "by no means it [the design] can be left to the teachers, though teachers can certainly make important contribution within the framework of design provided by experts". We see here the process of disenfranchisement of teachers from their own didactic tools with the simultaneous characterization of their work as below standard, resulting of course in the familiar divide between the theory and practice, exactly the same divide created by the Marx, Kautsky, and Lenin in their dismissal of workers as intellectuals in favour of “bourgeois intelligentsia”.

The book Polish Workers (Malachowski, 1981) appeared more or less the same time as Staniszkis analysis, and it represents the knowledge about working class seen from the perspective of sociological research conducted during the communist experiment. He informs that young workers in Poland (15-29 years of age) constituted 51% of the young Polish generation. Among those, approximately 49% had no more education than the grammar school (p.97), and around 45% of those that went beyond the grammar school (~1% of the total of young workers) were the students of vocational schools which in the word of the author “is the type of education that doesn’t open, in practice, opportunity for further studies nor it does it create the general knowledge needed to function in the contemporary world. Instead, it gives them the knowledge necessary to become skilled worker (craft knowledgeable).”

That approach of *bringing consciousness from without* has survived till the times of Solidarity in the form of Solidarity “experts”. Their recent publications show the same technique of organizing consent through intellectual manipulation (Bugaj, Obywatel, 2014):”Us experts! My God, we were really manipulating them! *In the best of faiths*, but nonetheless, we were, doubtlessly, manipulating them.” On one hand, it was clear during the opposition time that “without the workers’ support, the elimination of the system or even its reform was impossible.” On the other hand, some of us were not very happy that “we are dealing here with people who don’t read books and do not go to the theater.” As a result, he asserts that “although obviously workers played an essential role in Solidarity, they did not become, apart from few exceptions, leaders nor even regular participants of political activities” (Bugaj, 2014). Polish workers of Solidarity had enough of revolutionary ardour. What they missed solely, was the spirit of generalization!

Thus, workers’ youth in Poland of the seventies was severely underrepresented at the institutions of higher learning; constituting most of young generation, at the end of sixties/beginning of seventies daughters and sons of workers’ origin constituted only 4% of student population at the Warsaw University (240 out of 6000 students at the time; anecdotical evidence). Clearly, the development of the spirit of generalization through public education was not in the interest of the communist rules in post-war Poland.

Nor it is in the interest of American academia. This situation in Poland of the seventies was not very different from our contemporary conditions of working class education in US. Andrew Hacker (NYT, July 22, 2012), a political scientist retiree from CUNY informs us that "Of course, people should learn basic numerical skills: decimals, ratios and estimating, sharpened by a good grounding in arithmetic". But algebra, the generalization of arithmetic is not necessary for workers, since "a definitive analysis by the Georgetown Center on Education and the Workforce forecasts that in the decade ahead a mere 5 percent of entry-level workers will need to be proficient in algebra or above". And then he adds "It's not hard to understand why Caltech and M.I.T. want everyone to be proficient in mathematics" – demonstrating explicitly the hierarchy of knowledge, where the working class of South Bronx occupies the lowest level in that hierarchy characterized by knowledge of rules and skills separated from its subject-natural integral development into a coherent schema of thinking. Aren't arguments of Hacker in 2012 surprisingly similar to the Malachowski's reasons, 30 years earlier in communist Poland?!

We see that both, "left" and "right" intelligentsia, proclaim and reinforce, for different reasons, the intellectual limitations of the working class anchored in the absence of the spirit of generalization, disempowering it at the same time. Considering that first public schools appeared in England around 1874, we see that, while education has made in general a tremendous progress, the degree of absence of spirit of generalization among working class has not changed within last 150 years. Just to make sure how general is the phenomenon of Achievement Gap between the professionals and working class (called in OECD terminology, elementary professions) we can note its evidence-PISA (2012) report. It shows the difference in achievements of children from different social grouping in every participating country confirming 150 years long absence of the spirit of generalization among children of working people.

ORGANIC INTELLECTUAL OF THE WORKING CLASS

It was Antonio Gramsci who for the first time in the context revolutionary movements, understood and recognized the issue that one can't separate *homo sapiens* from *homo faber,* the man - the thinker from the man – the maker, the worker from the intellectual. He recognized that "In any physical work, even the most degraded and mechanical, there exists a minimum of technical qualifications, a minimum of creative intellectual creativity" p (8*).* On the basis of this recognition he formulates, in his Prison Notebooks the fundamental new entity that of the organic intellectual of the working class, which is the first, most probably, attempt at the creative synthesis of the worker and the thinker. However, he does not formulate it completely hence interpretation conflict persists. Thus Hoare and Smith, (1971), the editors of the Quaderni, assert "that, ideally, the proletariat should be able to generate its own intellectuals *within* the class and who *remain* intellectuals of their class" (p. 9), while New York City's rank and file left intellectual Stanley Aronowitz, (2015) in his chapter on Gramsci defines it as *The organic intellectual* [of the working class] *is one whose work is the expression of the worldview of the proletariat.* Aronowitz supports this view by the later statement: "In this context Gramsci's famous phrase "organic intellectuals" refers *not primarily* to those who have sprung *from the ranks of the workers...".*

Gramsci circles around that issue but, possibly because of the absence of fundamental psychological and scientific discoveries which came only in the middle to second half of the 20th century, can't get to the core of it. On one hand, he very strongly asserts the presence of an irreducible creative intellectual component in any physical work (above, p. 8), however, already on the page 9 he says, "Each man finally, outside his professional activity, carries on some form of an intellectual activity, that is he is a philosopher, an artists, a man of taste...has a conscious line of moral conduct", what of course is true but it has nothing to do with primary professional activity of being a worker. Why does Gramsci point our attention to the worker's activity outside of his or her professional, having reminded us that any human labour, that is also worker's professional activity has an irreducible creative component? There is a fundamental divide between the two which he doesn't know how to bridge it.

At present, we understand that the process of transition between *homo faber* and *homo sapiens* takes place via metacognition, the concept formulated by Ann Brown (1987) in the second half of the 20th century, which clarifies the development of intellect through and from praxis. It points to the necessity for an individual to distance himself or herself from participation in the work, while at the same time to keep that role in the mental focus. This then enables the intellectual reflection as *homo sapiens* upon the practice of *homo faber* what is necessary to initiate the development of critical thinking rooted in practice –the fundamental need for the organic intellectuals of the working class. This critical metacognitive process partaking in generalization and abstraction involve both the development of intellectual awareness of oneself and of one's role in the process.

In other words, the process of generalization is developmentally different for a person of worker's origin, than for a person from "traditional" intellectual family. It has been recognized in mathematics education profession (Esmonds, 2009) that the pedagogical approach to the students from so called, "underprivileged" or "underserved" population needs to be anchored at their basis experiential, spontaneous base. On the other hand, learning of students from the “traditional” intelligentsia starts at the opposite level of “scientific” concepts within worker/intelligentsia ZPD, which as Gramsci elaborates in his essay On Education, “are breathed-in” during their childhood: “In a whole series of families, especially in the intellectual strata, the children find in their family life a preparation, a prolongation and a completion of school life; “they breath in” , as the expression goes, a whole quantity of notions and attitudes which facilitate the educational process properly speaking. They already know and develop their knowledge of literary language, i.e. the means of expression and knowledge, which is technically superior to means possessed by the average member of the school population between the ages of 6 and 12”. This difference in the process of generalization between children from middle class/intelligentsia family and children from worker’s family is strongly supported by the critical approach literature starting with Bernstein (1973) who observed that “more middle class mothers emphasize the transmission of general principles whereas more working class mothers tend to stress aspects of a specific situation.”(p. 66) Janet Holland (1981) investigated impact of this difference in the classroom setting and found that middle class student were classifying an unstructured collection of different foods into food groups based on their intrinsic characteristics, while working class students would classify them through local classification systems, such as “what would be offered for Sunday lunch”. Theoretically, this difference can be understood as the difference between complexes and concepts proposed by Vygotsky (1986), showing the lower level of generalization by working class students. There is a work to be done by the teacher in helping those students to develop the missing qualities of generalization; the middle class/intelligentsia students do not require such a development.

Thus the only way for a worker to grow into organic intellectual of the working class is to generalize with the help of metacognitive awareness from his/hers particular and concrete practice to the workers as a class. For me this means, to investigate the truth of every political moment independently of the past ideologies, and past truths, to find its internal dynamics and from that to derive the revolutionary strategy expressing both the interests and wisdom of the working class. It is thus fundamental for the organic intellectual of the working class to come from within the class itself because only then he or she has the possibility to creatively integrate the derived revolutionary strategy with the intrinsic wisdom of that class. And it is our, mathematics educators’ responsibility (as best acquainted with the role of generalization and its spirit) to facilitate that synthesis.

HOW TO INCORPORATE SPIRIT OF GENERALIZATION INTO THE EDUCATION OF THE ORGANIC INTELLECTUALS OF THE WORKING CLASS?

The sketch of the proposed program is based upon the curricular concept of "vertical integration" increasingly practiced in professional schools. It is supported by (1) the theory of adult learners' interest in meaningful learning (Kaufman & Mann 2010) –learners are willing to invest time learning a topic only after they understand the topic's relevance, and (2) the theory of organization of knowledge, which is most effective when the organization of that knowledge matches the way knowledge is to be used (Ambrose et al. 2010). Consequently we propose a network of linked project-based courses (PBL) spanning subjects like economy, sociology, language and arithmetic/algebra integrated with respect to the processes of generalization and particularization in each of the disciplines. Thus, embedding the algebra generalization within the social sciences serves as particularization while the course of algebra itself may serve as the generalization of the modes of generalization in the disciplines. The program has a leading Seminar whose purpose is to develop and to integrate different modes of generalization in disciplines with the guidance of arithmetic/algebra mode of generalization. The details will be discussed at the conference.

CONCLUSIONS

We have presented here the search for the whereabouts of the "spirit of generalization" throughout 150 years of the revolutionary struggles. We discovered persistent absence of concern for the development of spirit of generalization in public schooling, which led us to assess the relationship between progressive intellectuals and workers. We have identified some sources of this surprising absence of concern and proposed a program to develop spirit of generalization with the help of vertical integration of relevant disciplines along the model of generalization offered by arithmetic/algebra transition.

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ABLEISM IN MATHEMATICS EDUCATION: IDEOLOGY, RESISTANCE AND SOLIDARITY

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I attempt to sketch out a comprehensive view of how Ableism functions in Mathematics Education. Ableism is not simply the injustice faced by disabled people but is also the privileges enjoyed by non-disabled people. While Ableism constructs some students as having "learning disabilities", most students as "normal", it also constructs students as "gifted". Ableism functions as an Ideology where "inclusive education" serves as its "sublime object" that restricts possibilities for introspecting contradictions within mathematics education. As a mathematics teacher in a school for blind children I observed that oppressed people including school students are quite politically aware and conscious about the forms of oppression they live through.

INTRODUCTION

Ableism is often described as though being synonymous with the discrimination faced by people with disabilities. At MES8, I (D'Souza, 2015) had also spoken about Ableism as a form of oppression faced by people with disabilities. I had then advocated then advocated the definition of Ableism as presented by Campbell (2001):

> "A network of beliefs, processes and practices that produces a particular kind of self and body (the corporeal standard) that is projected as the perfect, species-typical and therefore essential and fully human. Disability then is cast as a diminished state of being human." (pp. 44)

Such a conception of Ableism is similar to the idea of deficiencialism coined by Renato (2015) in his thesis titled "Deficiencialism: an invention of deficiency by normality" in which he cites as an example of, "normal people defining abnormal people".

But there's more to Ableism that most discourses do not sufficiently convey. For example, although "deficiencialism" aptly connotes having less (than "normal"), it says little about what "having more" could entail. The concept, "Gifted" carries this connotation of "having more", but is unfortunately celebrated as an individual feat, even though giftedness (being blessed with more) can exist only in relation to deficiency (being cursed with less). So if we speak of giftedness as being inherent (not a socially constructed problem), we imply that so is deficiency (D'Souza 2016b).

Skovsmose (2016) discusses how “difficulties (that) arise from the relationship between Braille and mathematical symbols” (p. 3). However, we should relate these difficulties to our privileges, that arise from the relationship between dominant languages like English and mathematical symbols. “Mathematical symbols” does not mean visual-English symbols.

In relation to caste, Somwanshi (2015), argues how, that “caste is a structure that includes ‘everyone [...] (and) oppression can’t exist without someone getting undue privilege”. In a similar sense, Ableism is a structure that includes everyone, not just the disabled people. It is not just the oppression of disability, but it is also privilege. Finkelstein (1998) argues how “Human beings are by nature, frail animals.” and that unlike the natural world, “In the social world, however, experience in managing human frailty has provided us with an amazing cornucopia of interventions that make possible the susrvival of those possessing the greatest physical and mental deficits.”(pp. 29)

I would actually problematize the term “able-bodied” and replace it with “enabled bodied”, since “ability” among human beings is as social as disability!

In education and cognition, it is quite acceptable to state and rationalize the claim that blind children also “visualize”. Arcavi (2003) argues how “Vision is central to our biological and socio-cultural being.” and locates the experiences of even blind people as a visual experience by stating that “visualization may go far beyond the unimpaired (physiological) sense of vision.” Making vision central to learning mathematics, he states as though it is a good thing that, “the centrality of visualization in learning and doing mathematics seems to become widely acknowledged. Visualization is no longer related to the illustrative purposes only, but is also being recognized as a key component of reasoning (deeply engaging with the conceptual and not the merely perceptual), problem solving, and even proving.”

Thus is becomes rather evident that, similar to the idea of deficiencialism, sighted people define the visually imparied. But further, they (we) also describe the experiences of blind people through the perspective of sight.

Such narratives have severe implications especially for blind students learning mathematics. When the discourses surrounding mathematics define it as a visual activity, one the hand it constructs blind students as naturally and biologically less capable of pursuing mathematics, but on the other hand, it constructs (and privileges) sighted students capable of visual reasoning, as being more capable of doing mathematics.

In such discourses, although there isn’t a direct defining of deficiency by normality, there underlies an assumption of what constitutes “normal” thus implying, as a corollary, the definition of the deficient. The object of the definition is thus in the absence.

The dominance of visual representations of mathematical ideas should not be taken for granted as the norm, but rather as contradictory to the claim of mathematics as being about ideals and abstractions.

ABLEISM AND IDEOLOGY

Conservative approaches towards addressing social, political or educational crises often ignore systemic contradictions, and present a stereotypical narrative that locates the source of the problem on a group of people.

In mathematics education research too, we ignore contradictions within what we understand as mathematics education, and tend to blame, for example, “teachers who passively follow the traditional rote learning method”, “students with special needs in mainstream schools”, etc.

Lundin (2012) presents two characteristics of, what he calls as, the *standard critique* of mathematics education. The first characteristic involves, taking for granted what mathematics knowledge is, how it is formed, and how it can benefit its bearers. He further states how such knowledge could be actively discovered or constructed by learners through meaningful and realistic problems. And such knowledge is assumed to be beneficial for self and society. The second characteristic of the *standard critique* involves describing school mathematics as boring, mechanical and based on memorization of facts and algorithms.

The standard critiques generally imply that with better teaching methods, and with assistive tools and technologies, the problem will be solved and mathematics will be accessible for all. It thus calls for a reform in the same direction.

However, by ignoring the contradictions within mathematics education, the mere focusing on better teaching approaches is doomed to failure. Lundin thus concludes that such “a path is a dead end and we need to look for other ways forward.”

While presupposing a nature of mathematics from a visuonormative perspective, even (teaching-learning) experiments on visually challenged students become problematic. Inclusive mathematics education has often been not about real students. The understanding of mathematics and blindness by sighted educationists are rarely developed through authentic engagement with blind people. And even if it is, it is often to teach them mathematics or to test the feasibility of their technologies. Their learning is often derived from their own experiences in learning mathematics in a visual manner, enjoying the visual mathematics, but projecting it as accessible for all by attempting to fix the “symptoms” of mathematics by developing technologies and pedagogies for “the blind student”. But this too does not “fix” the problem.

Visuonormativity in mathematics, which serves to (in)validate what counts as mathematics operates to prevent mathematization even before the students come into the picture. And even when students are encouraged to construct their own mathematical ideas they are actually made to construct the acceptable mathematics. Mukhopadhyay and Roth (2012) point out that:

> Even though constructivist theory emphasizes the personal construction of knowledge, actual mathematics education practices generally aim at making students construct the "right", that is, the canonical practices of mathematics –not realizing that for many, this may mean symbolic violence to the forms of mathematical knowledge they are familiar with, and that the standard processes typical of mathematics education contribute to the reproduction of social inequities. (p. vii)

"The blind student" thus serves as an ideological figure created in an attempt to fix the contradictions in the ideology of curricular mathematics education that is on one hand abstract and idealist in discourse, but "visual" or rather, occularcentric in practice. Thus, every attempted solution to the problem if constructed through the lens of vision cannot solve these, as Lundin mentions, "malfunctions ...ultimately created by mathematics education research itself."

However, as mathematics is very much linked to schooling and education, such contradictions are easily dismissed through the narrative of "education for all" and "ICT for teaching mathematics to blind children". Building up from Žižek's (2008) ideology critique, Pais and Valero (2012) show "how "mathematical learning" has become the sublime object of the field's ideology and, as such, a stumbling block in the process of reflexivity" The exclusion of blind children legitimized through the process of failing (verb), are generally addressed through making better teaching methods. But this exclusion is in fact a "symptom" of the ideology. In the words of Pais and Valero (2012), we need to "posit them as a window into the entire contradiction of schooling" and curricular mathematics.

ABLEISM IN CONSCIENTIZATION?

Another aspect of Ableism is in addressing disability with the assumption that oppressed (disabled) people have a false consciousness about their lived reality and need the help of politically conscious sighted educationists to help them liberate themselves. For example, Freire (2005) speaks about the oppressed people in the following manner:

> Submerged in reality, the oppressed cannot perceive clearly the "order" which serves the interests of the oppressors whose image they have internalized.

While there is a lot of truth of the statement, it often translates into a call for a certain kind of intervention that locates the people as lacking a political consciousness. This is generally coupled with the researcher's taking mathematics knowledge for granted. With an already preconceived notion that the oppressed people have a false consciousness, we tend to overlook and disregard their political tools and political consciousness. And if it is necessary to further conscientize the people, it is necessary to begin from, and maybe even learn to use their political tools from whence a solidarity can develop. Although, to a large extent false-consciousness does operate among oppressed people, approaching oppressed people with the lens of false-consciousness, is problematic.

While we have critical mathematics, Freirean pedagogy and Giroux as political tools, the oppressed people too have a political theories of resistance and empowerment. While initially the oppressed may come across as having a false-consciousness, it is possible that they do so only in the presence of the privileged researcher, who is from the group of oppressors. For example, in my course of my teaching mathematics in the blind school, it was only after three years did a teacher from a marginalized background speak about being discriminated by some school authorities due to her caste. The fact that I was personally quite close to the school authorities (owing to my privileged location) definitely contributed to her not feeling like talking about caste to me. But when she did speak about caste, she would cite Ambedkar in her narrative about knowledge and empowerment. She too would say how she would not criticize the school authorities since doing that could threaten the space (which was achieved after a long struggle) where the children interact with each other. Some students too were rather politically conscious about their location. But would of course not be open about it in front of rich visitors who were potential donors.

In the context of mathematics education, for example, in MES8 I had cited examples of one of my blind students raising the question, “If mathematics is all in the head then why is there an emphasis on the paper and pencil?” Another incident I had cited was of when I asked my students what was their most difficult topic in mathematics to which they replied, “Steps”. They could solve mathematical problems, but had to show all the in-between steps on paper.

I had presented these and other incidents as examples of the injustice faced by students. However, it was wrong on my part to locate these episodes within the framework of only injustice. The students were in fact asserting themselves as mathematics doers who were conscious of having their form of mathematics invalidated were aware that their right to self-determination in mathematics was being denied.

I share more recent incidents. I began a discussion on numbers with a group of 10 (visually challenged) students (ages 11-19), seated in a circle on the floor. Through the course of discussion the concept of odd and even numbers came up. So I asked how to characterise numbers as odd or even. In the beginning, a number of students agreed that those numbers which could be evenly divided by 2 should be categorized as *even* (eg. 2, 4, 6,...) while those that could not, as *odd* (eg. 1, 3, 5,...). When asked about the number zero, all seemed to arrive at a consensus that zero is both odd as well as even. The justification was that zero leaves no remainder when divided by 2, hence it is even; however, we cannot divide zero by two since we have nothing to divide. Hence, zero is odd. During further discussion, one student named Faizan raised his discomfort with including zero as an odd number. He argued, that numbers have the property that "odd + odd = even"; "even + even = even"; "odd + even = odd" and "even + odd = odd", for all natural numbers. Based on these properties he defined odd and even numbers. During a further course of our discussion, the number -4 turned up. On asking whether -4 is an odd or even number, Faizan stated that "before deciding that, we need to know where did these numbers like -1, -2 come from? i.e. there has to be a reason. For example, when we found numbers that could be divided by two, we called them even and those that could not be divided, as odd." As we would continue the session, he interrupted, saying that "when we visited the mall, the lift had numbers -1 and -2 to indicate the upper and lower basement."

The discussion that followed surrounded the need to conceptualize negative numbers. In between, Faizan interrupted stating that the idea of having negative numbers is very old, while malls with basements are comparatively new. He later on hypothesized that maybe during the Harappan civilization building structures which had some sort of basements could have given rise to the concept of negative numbers. The discussions continued with other hypotheses and examples that led them to conclude that it makes more sense to categorize -2, -4, ... as even numbers so that it fits into a continuous pattern of alternating even and odd numbers whether read backwards or forwards. I wrote about the above incident in a publication D'Souza (2016a).

But a few days later the students would again choose to dismiss zero as an even number, since zero had a property different from other even numbers - "If you kept dividing an even number by 2, you'd reach an odd number. This does not work if zero is categorized as even." This contradicted my claim of zero being just another even number. (D'Souza, 2016b) This incident needs to be located within a context. The NCERT[1] Class VI Mathematics textbook introduces the concepts of odd and even numbers in the following manner:

1. National Council of Educational Research and Training is a governmental body that provides consultation the the Indian Government in academic matters related to school education.

> "Even and odd numbers
> Do you observe any pattern in the numbers 2, 4, 6, 8, 10, 12, 14, ...? You will find that each of them is a multiple of 2.
> These are called even numbers. The rest of the numbers 1, 3, 5, 7, 9, 11, ... are called odd numbers.
> ...
> Fill in the blanks:
> The smallest even number is ________."

Mathematical ideas are presented as facts. Even though questions are posed, the "correct answer" is decided by the book. The need for talking about standard, current, accepted definitions never comes out. This essential fact that definitions are modified by people depending upon their need for mathematical ideas, is hidden. Such a form of introducing mathematical ideas may lead a researcher to hypothesize (and consequently find evidences) that such a textbook would lead students to passively accept the content as fixed and unquestionable, which are to be memorized and applied, and which justifiably decides their fate. And this would be true in a significant number of cases. But despite the fact that mathematical concepts are presented in such a canonical manner, the students would, to some extent, freely redefine concepts. And in doing so would resist the ideologies.

FINAL WORDS

My argument is not to mean that our role as sighted privileged people is to do nothing. I think that our role is to acknowledge how we are privileged by structural oppression and that the oppressed too have their means of resistance. As mathematics educationists we should look at teachers, students as activists and comrades rather than mere receivers of political consciousness, in a joint struggle against structural oppression. They are activists. Their activism just needs to be nurtured "and that the educator must [sic] himself be educated".

I am currently exploring whether Ideology is an appropriate framework to analyse Mathematics education from the perspective of Ableism, and what could that entail. Ableism which needs to be distinguished from Disability requires a deeper theoretical understanding from a perspective of both philosophy and social justice. Without addressing the philosophical aspect, our social justice approach may just be futile. But the resistance will continue with or without us. And we need to be open to the possibilities that the oppressed are in fact resisting us as well, and quite justifiably so.

If mathematics education does not lead to a direction towards liberation then we need rethink why we teach mathematics in the first place. We need to ask whether we use mathematics to arrive at a more just society? Or are we using the narrative of social justice to sell our mathematics?

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MACROINCLUSION AND MICROEXCLUSION IN MATHEMATICS EDUCATION

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Microexclusions take place through subtle and often covert practices, which can be devastating to the victims. Microexclusions tend to isolate an individual in a given environment, also in cases where this environment is considered inclusive. In this paper we want to elaborate the concept of microexclusions, presenting examples of how such practices may operate in the mathematics classroom. We develop the concepts of macroinclusion and microexclusion and discuss how they can be connected to an inclusive educational policy. We present examples that elucidate how macroexclusion can take place in mathematics education. Finally we relate the notion of microexclusion to the notion of microaggression.

INTRODUCTION

"*I became angry*", told a mother when her child, who suffered from dwarfish, was denied enrolment in a private school in the State of São Paulo, Brazil. In 2012, this was the title of a widely circulated news report. Furthermore, the mother said: "*I knew the school* [...]. *I talked with a member of the staff and explained my child's needs. I was told that I would not have problems due to my child's dwarfism, as they already had two students in wheelchairs at the school.*"

It is well known that dwarfism does not affect intellectual capacity, but it requires adaptations in the school environment as for instance with respect to tables, chairs, drinking fountains and toilets. However, while on the first day the child and the mother were on their way to school, they received a call saying that the registration of the boy could not be completed. The school claimed a lack of vacancies. This caused great upheaval in the child's family, as they felt discriminated against. In 2016, the school was judged by the Brazilian court to pay compensation of 20,000 *Reais* (approximately 5500 Euro) to the student's family (Belinni, 2016).

Processes of exclusion have been carried out by public and private schools, but since 2015 there has existed a Brazilian law referred to as the "Laws Concerning People with Disabilities", which defines as a crime if a school imposes extra fees for students with disabilities. Thus in Brazil "education for all" in an inclusive school is firmly secured legally. However, as illustrated by "*I became angry*", processes of exclusion continue to exist. In the following we will illustrate how such processes could take place also in cases where the students have been included in the educational system. We will address this possibility in terms of *microexclusions*, which we are going to relate to mathematics education.

MICROEXCLUSIONS

Microexclusion can be explicit and blunt as in the case of "*I became angry*". However, microexclusions can also take place through subtle and often covert practices, which also can be devastating to the victims. Microexclusions might tend to "isolate" an individual in a given environment, including in cases where this environment is considered to be inclusive.

When we talk about microexclusion, the prefix *micro* does not mean *minor*. Thus microexclusion could be brutal and severe. Instead *micro* refers to the context of the exclusion. While macroexclusion refers to exclusions that operate at a general socio-political level, microexclusion takes place at the level of individuals and groups. Western history has been accompanied by the most brutal forms of macroexclusions: think of the history of slavery and of racism. Black people have been excluded from the protection of human rights, but when slavery was abolished, macroexclusions took new forms. Black people were excluded from having votes and from access to further education. All such forms of macroexclusions may lead to a range of microexclusions. Thus, one finds intimate connections between macroexclusions and microexclusions.

As macroexclusions can lead to microexclusions, macroinclusions can lead to microinclusions. Thus the overall inclusion of black people in the education programme, including access to universities, could lead to a range of microinclusions, for instance with respect to job opportunities.

There is, however, an important phenomenon that we are going to explore in this paper, namely that *macroinclusions might lead to microexclusions*. In Figure 1, we show that macroinclusions can lead to microinclusions and that macroexclusions can lead to microexclusions, but also that macroinclusion might lead to microexclusions.

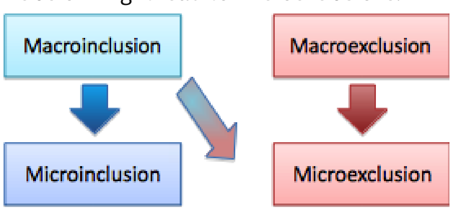

Figure 1: Macroinclusions might lead to microexclusions.

Inclusive education represents an overall effort to engage everybody in the same educational programmes: no difference should be made due to gender or race; no difference should be made due to political, social, religious or cultural preferences; and no difference should be made with reference to abilities. Thus the issue we are going to address is to what extent any such programme of macroinclusion could lead to microexclusions.

MACROINCLUSIONS AND MICROEXCLUSIONS IN A BRAZILIAN CONTEXT

In Brazil, macroinclusions were established through a policy supported by various political parties and social, cultural and educational movements advocating the rights of education for everybody in an inclusive school environment. Macroinclusions can be considered an expression of an educational approach that seeks to unite equality and difference as inseparable values. Such an approach aims at overcoming difficulties and limitations of the existing educational system by recognising the need for both structural and practical changes. Thus, it became crucial to consider the requirements of students with special needs in order to ensure their participation in various educational processes.

In Brazilian basic education, macroinclusions have been strengthened by the "National Education Plan", approved in 2014, which provides twenty goals to be achieved in a period of ten years. Among these, goal number four states that everybody between four to seventeen years old with special educational needs should have access to basic education as well as to specialised education. As a consequence, in the general educational system one now meets students with a variety of disabilities. It could be students with hearing or seeing impairments, if not deaf or blind students (see, for instance Moura,in progress).

Although inclusive education is legally ensured in Brazil, as by the "Law Concerning People with Disabilities", one finds many obstructions to such a general educational programme. Thus in most of the educational institutions one finds a lack of preparation to receive students with disabilities. Antunes (2014) emphasises that processes of exclusion that occur in the day-to-day school life, can originate from practices that have been claimed to be inclusive. The way the school is structured, physical in terms of architecture and pedagogically in terms of educational practices, may reinforce exclusion rather than inclusion of students.

Thus, even when a macroinclusion policy provides access for students, there are several factors within the school context that work in the opposite direction. We have presented this phenomenon with reference to the Brazilian context. However, we have to do with a general phenomenon: in very many different contexts macroinclusions can lead to microexclusions.

MICROEXCLUSIONS IN MATHEMATICS EDUCATION

Freire (2002) points out that being marginalised means experiencing a situation of oppression. He finds that exclusion does not mean that human beings are “out” and therefore should be placed “in”. Being excluded means that human beings become “deleted” through processes that dehumanise and marginalise the individual. According to Freire, marginalisation is not an option for those who suffer, but the result of a process of violence. We agree with Freire and conceive microexclusions as discriminatory acts that can occur within different environments, including the school. In this case, the students suffering microexclusions are those who, being in the school, are experiencing situations of oppression. Despite being included, they are continuously reminded that they are different and inferior. In the following we present different examples of such phenomena, and mainly refer to situations in the mathematics classroom.

Microexclusion through patterns of communication

Microexclusions could take implicit forms, both veiled and subtle. Such processes can be conducted consciously or unconsciously, also in an apparently inclusive environment in the mathematics classroom. Microexclusions could occur in an environment where some students are not “seen” or “heard” by the others.

The mathematics teachers, for instance, might have presented the groups in the classroom with a particular task, and each group might seem to work concentrating together. Within a group different students might propose ways of addressing the problem. However, it need not be everyone who is making suggestions, and some might be ignored. We are not dealing with any explicit exclusion, yet the pattern of communication might tend to isolate some students. Thus in mathematics education one can witness microexclusions taking place in the middle of a group work session.

In the mathematics classroom certain patterns of communication may open space for some students while for others it might be closed off. This could be to do with the format of the questions and with the expected pattern of answers. The teacher’s attention might be turned towards the students who provide the expected answer. (See, for instance Faustino, in progress).

Microexclusion by ignoring special needs

Teachers could practice microexclusions when they fail to recognise the particular characteristics of students with special educational needs when they organise activities in the classroom, for example, when the mathematics teacher bases his or her presentation on verbal communication. It could well be excellent presentation, but students with hearing loss (who in Brazil would be present in the common mathematics classroom due to the overall policy of inclusion) do not have the possibility to grasp what the teacher is saying. Verbal communication alone is not enough to ensure that students will come to learn mathematics.

Thus, if the teachers do not pay attention to other possibilities for interacting with the students, certain patterns of communication could generate microexclusions. The presence of the interpreter and the use of sign language can enhance communication between hearing-impaired and hearing, however in many situations the teachers leave to the interpreter to complete all the interaction necessary for teaching and learning, and only interact with the interpreter in order to respond to questions from the students. Some students may feel unappreciated because the teacher does not give them the opportunity for interaction. (See, for instance, Moura, in progress).

Microexclusions can also occur when the teacher expresses a surprise or admiration with respect to some skills demonstrated by the students with special needs. Thus a comment like "he is deaf, but still managed to understand much in mathematics" may appear to present an appreciation of the deaf students' performances, but simultaneously such a remark might include the message that the deaf student is inferior compared to hearing students (Moura, in progress).

Microexclusions may reflect features of the economic policy through which macroinclusions become implemented. Such exclusions could be the consequence when such implementations are not accompanied by the necessary financial and material support. Thus to include deaf students in the regular mathematics classroom would presupposes the presence of an interpreter. In the case of blind students it would presuppose the construction of particular materials; for instance, Marcelly (2015) has illustrated how it is possible to develop material for mathematics education that could be used by both seeing and bind students when working together. In all cases it would presuppose a development of new teaching practices and a new professionalism among teachers. Otherwise we might witness a severe process of exclusion taking place within an overall inclusive educational programme.

Microexclusion through performances

Microexclusions may take place when the mathematics teacher concentrates on students who perform well in mathematics and not paying the same attention to other students. This applies to all levels in the educational system. It is broadly documented, however, that at the university level students without the profile of being good at mathematics become the object of derisive comments. Such comments could refer to their questions during class, which could be labelled as being banal or as demonstrating a profound lack of mathematical understanding. Thus Silva (2016) shows that some students from engineering majors experience being excluded from the field. Repeatedly their questions are labelled as infantile. Thus formulations like "I have learned this when I was in secondary school" or "it is very easy, do you not see it?" were used by teachers and their classmates during calculus lessons. As a consequence these students "silenced" themselves in the classes.

Microexclusion through affirmative actions

In higher education, affirmative action provides an important kind of macroinclusive policy. Through such actions underrepresented groups get access to further education, which may change not only their individual lives, but also the prospect of the whole social group to which they belong. However, accompanying affirmative actions, microexclusions have taken place. Silva (2016) has pointed out a profound lack of concern within university departments with respect to the particularities of groups of students who now are getting access to universities. This lack of concerns may turn into microexclusions. Thus, in engineering and sciences subjects, Brazilian indigenous students who have got access to universities through affirmative actions have been labelled by teachers and other colleagues as "*coitados*". The Portuguese word "*coitados*" means "wretched".

A common idea is that nothing can be done in order to help the "*coitados*" to remain at the university: as soon as the first semester comes to an end, they would drop out due to failing their exams. From this perspective, pedagogical initiatives are not required in order to support the "*coitados*" in mathematical disciplines, for instance. However, according Silva (2016), in order to prevent the exclusion of these students, it becomes important that affirmative actions also address the educational requirements that an affirmative policy does presuppose.

Microexclusion through normalisation

Microexclusions become experienced by the students. However, it might not always be clear to the students what is the nature of these experiences. Students might feel put aside in the mathematics classroom and, as a consequence, they might lose interest in the topic.

Subtle forms of microexclusions might be acted out through processes of normalisation. In all contexts a conception of what is normal might be presents; however the very notion of normality can provide a departure for severe forms for microexclusions. In mathematics lessons it is common that teachers give more attention to students that ask the "normal" questions and solve the exercises in the "normal" way. (See, for instance, Muzinatti, in progress).

The conception of normality can operate in brutal ways when we consider students with special needs. Thus a conception of a blind student based on a general conception of a seeing student, can tend to bring about microexclusions (Marcone, 2015). Such a conception might emphasise the limitations instead of possibilities of a blind student, and can create a false impression that the blind student has a reduced capacity for mathematics abstraction. As a consequence, teaching strategies may become limited to the use of concrete materials. Although we know the potential of the use of such material in the teaching of mathematics, not creating opportunities for blind students to move beyond the concrete material may generate an exclusion from the broader landscape of mathematics.

FINAL REMARKS

Our discussion of microexclusion has found inspiration in research that emphasise the notion of *microaggression*. This notion has been developed with reference to verbal abuse and nonverbal practiced, more or less subtly applied, against individuals based on race, gender, ethnicity, social class, dialect or religion. Macroaggression might be done automatically or unconsciously by the offenders; still they might have a profound negative impact on the lives of abused (McCabe, 2009; Minikel-Lacocque, 2013; Silva & Powell, 2016; Solórzano, 1998; Sue et al., 2007).

Despite microexclusions show characteristics similar to macroaggressions –like subtlety and opaqueness– microexclusions have their own characteristics: they are often practiced in contexts considered inclusive, and they can lead to isolation, marginalisation and tend to dehumanise the "included" students. We have tried to develop the discussion of macroinclusion, macroexclusion, microinclusion and microexclusion with particular reference to a Brazilian context. However, we find that these notions might find applications with reference to many other contexts as well.

Microexclusions mean violence. Thus we do not see violence as just being of a physical nature; it can take many different forms. Certain discourses can be violent, for instance when they provide a stigmatisation of certain groups of students. Political structures can be violent by preventing certain groups of children and young people from access to adequate educations. Economic structures can be violent by keeping groups of people in poverty. Violent microexclusions can be acted out in the mathematics classroom. Also in such cases there could be many reasons for a person stating: "*I became angry.*"

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MULTIMATHEMACY: TIME TO RESET?

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This article criticizes the current mathematics curriculum and the cultural biased dropout as a result of it. We analyse the inequality of the school system and the cultural biased dropout based on philosophical and empirical evidence. At the philosophical level, evidence is shown by relying on the work of mathematician and philosopher A.N. Whitehead and contemporary theoretical insights on (mathematics) education. Empirical evidence for our argument is given by the analysis of the international PISA 2012 figures and empirical research at the national level of the Belgium/Brussels school system(s). Based on our own experience, our research and teaching practice, we will give some good practices to illustrate our concept of multimathemacy and to provide an idea of how to reset the curriculum.

INTRODUCTION: WHITEHEAD'S LEGACY

The mathematician-philosopher Alfred North Whitehead (1861-1947) was definitely not a Platonist. In his later works (Whitehead, 1925 and 1953) he became very explicit about his insights in philosophy of science. He insisted there that any theory (and hence any scientific representation) is only one particular perspective, one window on reality (esp. in Whitehead, 1925). Any further pretension has the scientist slip into a biased position. Put differently, even the claim that scientific knowledge is an approximation of reality is a religious statement that cannot be guaranteed by scientific research. The results we produce may work to some, or even to a large extent, but that is all we can say. Any claim of 'ontological' superiority is without ground.

In the remarkable bundle published after his death (Whitehead, 1953) the consequences of such a position for the way one can think and work with mathematical notions are elaborated on. In this case he does this for geometry. Whitehead shows that a whole list of notions (and not just one: the point) in geometry can be used as a 'primitive': it is perfectly legitimate to use the notion of volume as the primitive of geometry and construct 'point' as a highly complex derived concept, instead of the (usual or traditional) opposite. There is no ontological –or for that matter cultural– more 'true' or more 'natural' primitive. It is only a matter of perspective, not dictated in any way by 'reality' as the Platonists seem to imply.

Our aim of using Whitehead's proposal is only to invite educators to consider this point of view in the light of the tremendous dropout rates in mathematics education. In the light of the growing importance of mathematical skills in the globalising knowledge society, we invite all readers to think along these lines in order to develop an approach which recognizes the relevance of other (cultural, knowledge) perspectives in mathematics education. Our hunch is that such an approach may well be successful in the fight against massive dropout.

MULTIMATHEMACY

What Whitehead did was to look at any mathematical theory as just one particular perspective on the domain covered. In older terms we can translate this as follows: Whitehead understood that any scientific theory starts from a set of intuitions about reality. In more modern words one would say that scientists, as human beings and hence members of a particular cultural tradition, develop hypotheses and work with a frame of interpretation of data and meanings which are primarily laid down in a worldview (Apostel, 2002). In still other words, concepts, hypotheses and models do not emerge in any direct way from external reality, but are constructed, in more or less conscious chains of interaction between the religious, the cultural and/or the linguistic categories within which the researcher is raised, and the learned –more or less logically consistent– window on reality that is held as the valid or most likely scientific theory at any particular moment. When we adopt this view on the relationship between science, reality and sociocultural factors in order to characterize the status of a scientific theory at any moment, we get a better grip on what it may mean to look at it as a particular perspective on reality. Because of the nature of the debates in science (preferably with open criticism rather than dogma as the guiding principle) the dominant perspective of the moment may be the best we can imagine at that time, or the more encompassing one, or the more promising one (in terms of applications and such). But presenting it as the best approximation or even the ontologically warranted view of reality, is a rhetorical move that cannot be defended: it introduces an external argument which stands as infallible and hence escapes critical scrutiny. Traditionally –that is within the cultural tradition of the religions of the book– this was termed a dogma. Our point is not to make a simple, ideological plea for or against this type of reasoning, but rather to unearth such intuitive, deeply cultural (here religious) elements of thinking and worldview. We do this, because one of us (R.P.) has had deep contacts with a completely other cultural group (Native Americans) and attempted to understand how they conceive of learning, of spatial notions and so on. It appeared that there, and by extension in many other cultures (Pinxten, 2016), other intuitions and (slightly) other reasoning styles obtain, which are nevertheless viable: they allow people to cope with reality and survive, notwithstanding the manifest differences in perspective on reality. For example, in the verb-languages of this world (without a genuine noun category, that is) the intuition is that of a world of events or processes, rather than things or static phenomena. Now try to imagine set theory without 'objects' like sets and elements, but rather built up on changes and events. Or another example: the predominance of the logical operator of implication (and hence of related causal explanations) in western ways of conceiving good knowledge is not found in other traditions. Or the peculiar emphasis on a particular notion of proof in the past centuries of mathematical research, which proves not at all to be universal through time or space (Raju, 2007). Our suggestion is that such (deep) differences are mostly denied in the ways mathematics education is set up. The question we want to formulate then is: could the dropout rates be understood and hence mended against the intuitive and otherwise culturally framed background knowledge of the pupils who 'fail'?

EMPIRICAL EVIDENCE

International comparative research on mathematical literacy (OECD-PISA) shows cultural and language background biases. Time to reset our curriculum.

In the figures below (Figure 1 & 2) we will focus on two variables as analyzed by De Meyer, Warlop & Van Camp (2013). The figures from PISA 2012 show the following pattern. Figure 1 represents the PISA 2012 scores for mathematical literacy focussing on the difference between native pupils and pupils with a migration background (first-generation students). Figures are controlled for socio economic status (SES) variables. A first observation is the degree of inequality between natives and first-generation students for most countries –with a OECD mean of 34 points. Secondly, after control for SES variables, the gap decreases for all countries –with a OECD mean of 21 points. This means that SES can explain a small part of the observed differences. A considerable difference between natives and immigrants remains and needs to be explained by a 'cultural' background factor.

As demonstrated in Figure 2, the language at home also plays a major role in mathematics performance. The figures indicate the score of mathematical literacy and represents the differences between the achievements of native and non-native speaking pupils. Figures are controlled for socio economic status (SES) variables. At the bottom of the figure, the percentage of non-native speakers is indicated. The figures from PISA 2012 show the following pattern. A first observation is the gap between native and non-native speaking pupils –with a OECD mean of 43 points. A SES control was added to the report. After control for SES variables the gap decreases for most of the countries with a OECD mean of 23 points.

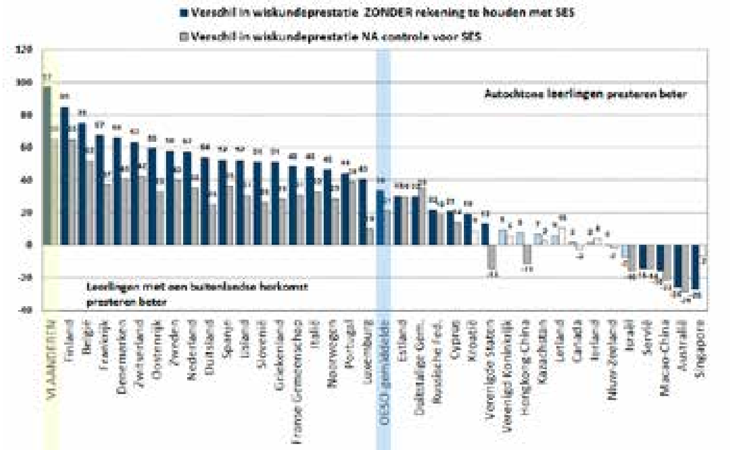

Figure 1: PISA-2012 Mathematical Literacy of 15–years old pupils: difference in score between native pupils and pupils with migration background (before and after control for Socio-economic status) (De Meyer, Warlop & Van Camp, 2013, p. 110).

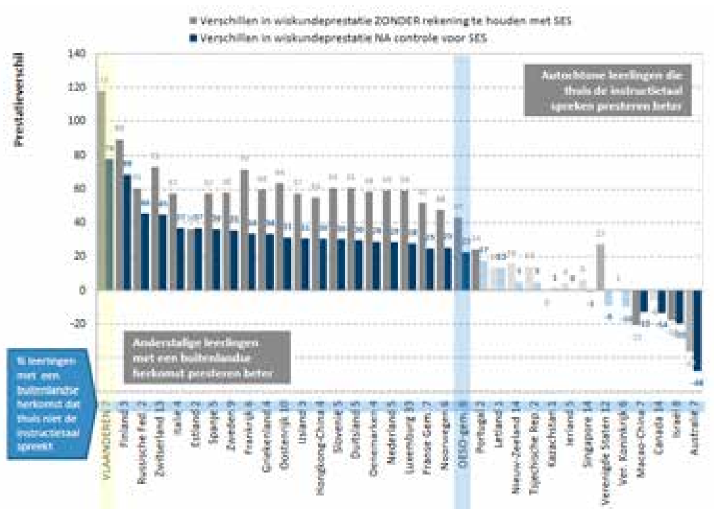

Figure 2: PISA-2012 Mathematical Literacy of 15–years old pupils: difference in score between native pupils and non-native pupils (before and after control for Socio-economic status). The figure after the country indicates the % of non-native pupils. (De Meyer, Warlop & Van Camp, 2013, p. 115).

This means that SES can explain part of the observed differences although a considerable difference between native and non-native speaking pupils persists.

Both variables, cultural background and language at home, indicate considerable difference between natives and immigrants and between natives and non-native pupils, although both variables have a considerable overlap. François (2005) studied the progression in Dutch-language secondary education in Brussels (see Figure 3). Brussels –as the officially bilingual capital of Belgium– has a separated school system with a Dutch-language and a French-language secondary education system depending on the Flemish and Walloon community. Both school systems attract pupils from the other language community (for many reasons) and they both attract pupils with migration background (although more in the French-language secondary schools). Looking at the outcome of progression in Dutch-language secondary schools one can observe considerable differences depending on language background.

Language background	% (15 year old) pupils who succeeded every year
(1) Dutch	88,2
(2) French	76,7
(3) Bilingual: Dutch and French	73,7
(4) Bilingual: Dutch and other language (not French)	73,5
(5) Bilingual: French and other language (not Dutch)	68,4
(6) Other language (not Dutch or French)	62,7

Figure 3: Progression in Dutch-language secondary education in Brussels (François, 2005).

With these figures one can explain that 'different language background' is not the decisive factor for school success and failure. French speaking pupils (who have a different language background in Dutch speaking schools but do have the same –non migration– cultural background) have a higher success ratio (76,7%) then pupils with a migration background (73,5%; 68,4% and 62,7%). The cultural background has more explanatory value. Time to 'culturally' reset our curriculum.

Our analysis of PISA 2012 figures and the conclusion that the educational system of the OECD countries reflects social inequality is confirmed by the recent UNICEF report on how inequality affects children in high-income countries (UNICEF, 2016). This inequality already starts at primary education and it increases in secondary and higher education (Groenez, Van den Brande & Nicaise, 2003; Hirtt, Nicaise, De Zutter, 2007). From this empirical evidence we have an additional argument to reset the curriculum, from the perspective of culture.

THE 'CULTURAL' FACTOR

Over the past decades it has become fashionable to blame all failures of integration or of lack of success in life on cultural differences, mainly with the failing groups. This is clearly a form of excessive 'culturalisation' in an era where identity has become the new buzzword (Pinxten, Verstraete & Longman, 2004). The main problem with this type of explanation of success and failure is that the notion of culture used here is vague and shifting. Sometimes it even has a status of container notion: it is used in such a vague way that everything and nothing can be understood by it. The criticism in anthropology on this notion of culture dates from over half a century (esp. Kroeber & Kluckhohn, 1952), when the concept of culture was starting to supplant that of civilization, which had been prominent before World War II. However, with the rapid growth of cultural (including religious) diversity in the past three decades because of worldwide urbanization (Castells & Martin, 2003), the old ideological distinctions in class and economic status faded to a large extent. Cultural identity grew stronger as a substitute principle to explain why a smooth and harmonious society did not emerge in the urbanized predicament we live in (with over 60% of the world population sharing urban contexts today).

In the work mentioned, our research group focused on culture and cultural identity concepts and tried to see how culture or cultural identity could become an analytic concept, and hence be useful for scientific research. Our proposal is to do away with the link between culture and group or community: stop speaking about culture X or Y, since any group is diverse and the linkage between culture and social entities denies that. This holds even more in the mixed societies of our presently highly urbanized world. When looking at what seems to be covered most of the time by descriptions of cultural anthropology is any and all process of meaning production. In the older discussion, the distinction between 'nature' and 'nurture' was held up for years. A refinement of this dichotomy refers to meaning production processes and their results. This is the stuff of what is called tradition, or 'ways' (as the Navajo would say) or what actually comes down to what is transferred over generations as characteristic of a family, a group or a human being in general; meaning and meaning production processes which are taught to children and to other newcomers. What we can determine though, is the diverse ways people learn and learn to learn. Moreover, social sciences have developed over the years at least three learning theories (behaviorism, genetic psychology and the socio-cultural theory). Hence, we possess workable analytical categories of learning, which can even be used in scientifically valid ways in experimental research to some extent (Pinxten, 2016). The so-called socio-cultural theory, elaborating on the Russian theory of learning developed by Lew Vygotsky in the decade 1920-1930, looks like the most potent of these three, since it has been proven useful in cross-cultural research (Cole, 1996).

When we extend this reasoning to mathematics education and its relative failure in the present-day world, we can open up an intriguing panorama of understanding and interesting avenues of educational opportunities. Indeed, when we agree that children are raised throughout the world in the particular meaning production processes and mental frames of their particular environment, it is important and sensible (i.e., possibly beneficial) to take into account and even actively study the contents and the learning strategies of the out-of-school knowledge and skills the child possesses and uses when first coming into contact with what we –western educators– call mathematics education. What intuitions, what preference for logical operators, what explicit concepts, what learning attitudes, what notions of relevance and sensibility, and so on do the children actually bring along and introduce in the context that mathematics education is? Disregarding this input will entail that the child will have to cope with all and everything in the mathematical curriculum and its learning styles it is running into. Our suggestion is that the blindness for these aspects of the 'home culture' (as it is often vaguely called) may well explain the remaining points of the gap between good and bad performers in the OECD reports cited above. The alienation the child feels when confronted with the usual mathematical classroom is manifold:

- linguistic differences: as shown in Figure 2 and 3, the language at home plays a major role in mathematics performance. In our research we noted that e.g., Turkish immigrant children had to try and cope with the singular/plural differences in Turkish and in Flemish in order to begin to understand what the problem amounted to in the mathematics textbook. Indeed, plural forms are rare and different in Turkish from most European languages (Huvenne, 1984),
- worldview: children come to mathematics classes with a variety of concepts and insights, but also practices, using and exploring mathematical operations and notions to a lesser or larger extent. Despite some criticism on the 'bridge' approach (Pais, 2011) we argue and consciously claim that good education should start from this out-of-school or folk-knowledge in

order to have the child help by making insightful steps towards those procedures and concepts of Academic Mathematics, which can be easily linked with the former. In Vygotsky's (1978) words education works within the 'zone of proximal development (ZPD) of the child. ZPD has been defined as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). Education works in a cognitive field which can be spotted at the fringe of the out-of-school worldview. Or in terms of Skovsmose's model: education has to be set up with roots in the Background Knowledge of the child, and looking for the Foreground Knowledge sphere, where the child will be able to connect easily and insightfully (Skovsmose, 2005),

- learning strategies: out-of-school practices occasionally makes use of a whole range of learning strategies and learning contexts, the so called informal learning procedures. In other words, different types of activities in the normal life of the child can show relevance for learning mathematical skills: not only counting, locating, measuring, designing, playing and explaining (as Bishop, 1988 shows), but also dancing, making music, storytelling, performing rituals, and so on can be mathematically salient activities (Pinxten, 2016).

Our suggestion is to study and identify all mathematically relevant notions and procedures in the out-of-school knowledge of the child and to develop from there, with the help of the children, pathways towards those elements of mathematical knowledge (or Academic Mathematics) which will fall at any particular moment within the 'zone of proximal development' of the child. Obviously, this pedagogical choice turns the current school strategies upside down: not the mathematician is the one who is prescribing what should be taught at school (top down, so to speak), but the trajectory for learning is decided upon in deep and constant negotiation between pupil and teacher (making it much more bottom up). Thus, the curriculum and the choice for particular notions and procedures at all times become specific for each child. It is this that we call the 'multimathemacy' approach. Obviously no assessment data on the results of such an approach are available yet, but we hope that –once this approach gets installed– they will be produced.

DISCUSSION: WHITEHEAD AGAIN

One thing we have empirical evidence about is that the current curriculum is not able to closing the gap between native pupils and immigrant pupils. The tremendous dropout rates in mathematics education are culturally biased, even when controlled for socio-economic status. This is what PISA figures –as illustrated above– can learn us about the massive dropout. On way of criticising the biased curriculum is to unravel the implicit philosophical perspective behind, the unspoken ontological claims and the ontologically warranted view of reality. In François (2007) we analysed the mathematics curriculum of Flanders secondary education on the basis of the philosophical dimension, both the explicit philosophical items and the implicit ones. We had to conclude that although some humanistic values are included, the mainstream of the implicit philosophy of mathematics is an absolutist one, "viewing mathematical truth as absolute and certain" (François, 2007, p. 13). With Whitehead we have philosophical evidence that any theory, any scientific representation and thus any mathematical representation is one particular perspective and one window on reality (Whitehead, 1925). It is one way of constructing a mathematics curriculum and one way of dealing with mathematical notions (Whitehead, 1953). Mathematics educators have to realize this philosophical evidence in order to rethink and to reconstruct the mathematics curriculum to become based in the experience of the pupils at stake. This brings us to a second way of criticizing the mathematics curriculum and again we will rely on the insights of Whitehead (1967). Whitehead's philosophy of education which he wrote during the second decade of the twentieth century is published as *The aims of education from 1929* (Whitehead, 1967). It is a collection of articles published as a critique to the educational system of England, the first one being delivered in the year 1912. In the preface Whitehead is very clear about the main idea of his educational philosophy which he states briefly thus:

> The students are alive, and the purpose of education is to stimulate and guide their self-development. It follows as a corollary from this premiss, that the teacher also should be alive with living thoughts. The whole book is a protest against dead knowledge, that is to say, against inert ideas. (Whitehead, 1967, p. v)

Whitehead's philosophy of education is connected to his process philosophy which can just as well be called philosophy of experience since the concept of process primarily refers to the process of becoming of experience (De Smet, 2014). The same way, Whitehead links the art of education to the stream of experience that forms our life. "There is only one subject-matter for education, and that is Life in all its manifestations" (Whitehead, 1967, p. 6-7). A mathematics curriculum has to be connected to the experience of the learners and the value of the ideas and theories has to be proved. With 'prove' Whitehead means 'to prove its worth' (Whitehead, 1967, p. 3). The learner should understand the application of ideas and theoretical concepts in the circumstances of the actual world, otherwise knowledge will become inert and deadly. Whitehead gives a great example how to deal with the teaching of Geometry, a great branch of general mathematical education one can find in every curriculum. The value of geometry and the experience of geometry in the real Life world is of particular importance. Plan-table surveying or special representations of real Life situations should lead pupils to a vivid apprehension of the immediate application of geometric ideas and concepts. Tony Brown (2011) introduced his 'Big Geometry' during conferences on mathematics education to let researchers and teachers experience what geometrical concepts are. By 'doing' Geometry, using our bodies, we can experience what abstract geometrical ideas are. Moreover, we can easily connect with the origin of abstract geometrical concepts. By doing 'Big Geometry' the highly abstract philosophy of Husserl's 'The origin of Geometry' (an appendix of Husserl's *Crisis*, 1962) becomes a vivid experience and a real connection with the evidence of the first geometer. With Pinxten, van Dooren & Soberon (1987) we have another example of 'doing' Geometry connected to the Life world of Navajo pupils by taking into account their cultural heritage and their experience. Pupils are engaged in the learning of mathematics based on the native concepts and the cognitive and cultural procedures.

With the analysis, the arguments based on empirical and philosophical evidence, and the practices of how to connect to the real life and to the informal learning procedures we call for a reset of the mathematics curriculum.

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RECRAFTING MANIPULATIVES: TOWARD A CRITICAL ANALYSIS OF GENDER AND MATHEMATICAL PRACTICE

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In this paper, we present a set of case studies of the mathematics that youth engaged when learning to knit and designing a simple rectangle. We present a range of experiences with knitting, and explore whether and how this manipulative can support and push students' proportional thinking. We consider the potential of this textile practice in the classroom, and whether the potential reach of these manipulatives make their high entry bar a worthwhile use of instructional time.

INTRODUCTION AND BACKGROUND

Despite the fact that there are generally no gender differences between girls' and boys' mathematics achievement (Hyde, Lindberg, Linn, Ellis & Williams, 2008), there continues to be a significant discrepancy in women's representation in STEM careers (Landivar, 2013). Studies of girls' and women's mathematical participation suggests that career choices are driven not by differences in ability between males and females, but rather by perceptions of the discipline of mathematics and the extent to which the cultures surrounding mathematics are welcoming to women (Alper, 1993). In contrast, women have a significant history in the production of textile crafts. These practices are, at their core, inherently mathematical, thus offering an existence proof of the kinds of mathematical activities with which women choose to participate across their lifespan. We focus on this unusual starting point to address the "STEM pipeline" by investigating not just the context of the craft, but also the culture of crafting that attracts and maintains women's participation.

Scholars have considered how the intellectual practices of STEM fields support the development of different identities and expectations about who is capable of being successful (Faulkner, 2007; Mercier, Barron, & O'Connor, 2006), and relatedly how workforce environments reify these identities, often leading to women abandoning these fields (Holleran, Whitehead, Schmader, & Mehl, 2010). Similar research on the development of mathematics and science identities in K-12 environments has also demonstrated that "school as usual" contributes to a perception that only certain types of people are good at math and science (Larnell, 2016). However, when schools reorganize instruction to include practices that include justification, argument, and inquiry, students' notions of "who can do" math and science can change (Langer-Osuna, 2015; Tan, Calabrese Barton, Kang, & O'Neill, 2013).

To date, much of the work on identities as they relate to mathematics and science has focused on the role of classroom practices that support different forms of participation, and, often, the role that teachers play in productively reorganizing these participation structures. Our work has taken a different starting place, to consider how the kinds of tools that are available in classrooms might organize and orient perceptions about what it means to do mathematics, with a particular focus on the kinds of manipulatives that we make available for everyday learning.

The belief that tangible-object manipulatives play an important role in children's learning stems back as far as the early 1800s. The use of physical objects to support learning in math education is particularly popular for young children who are developing representations of quantity. Manipulative use falls off dramatically after the early elementary years, due in part to skepticism about their efficacy for supporting higher-level ideas (Fennema, 1972, Friedman, 1978). Even with young children, the usefulness of manipulatives has been the subject of debate, primarily because manipulatives can be used in a variety of ways for a variety of purposes. Here our goal is not simply to propose new tangibles for students, but to explore a new context for students to experience mathematics through activity –a set of practices that embody and transform mathematics. In what follows, we present a set of case studies of the mathematics that youth engaged when learning to knit and designing a simple rectangle. Through these cases we present a range of students' reactions to the learning ecology of the summer knitting camp, and begin to illuminate salient contributing factors.

STUDY DESIGN AND METHODS

Background

This study was part of a larger project that that explores how mathematics is used in expert textile craft, and the kinds of mathematical thinking involved when creating textile designs. Here, we focus specifically on knitting, and consider the influence the materiality of the practice of knitting might have on mathematical reasoning. Prior research demonstrated that, for adult knitters, the explicit mathematics involved in authentic knitting practices were more salient in processes of design and reworking patterns, rather than in simply following directions (Gresalfi, Bell, Chapman, & Wisittanawat, 2016). As a consequence, the activities that we designed for novice knitters were all "design challenges," which involved specifying some kind of constraint, but still allowed youth to determine what they wanted to create.

KnitLab

Participants in the present study were eleven youth, all female-identified, ages 9-16, who self-selected to participate in a week-long "learn to knit" program called "KnitLab" offered over the summer at a public library. In previous iterations of KnitLab at this same library, researchers had intentionally recruited male participants in an effort both to be more inclusive and to avoid simply inverting and thus reifying existing binary structures; while in-person recruitment was largely productive in this regard, the mostly online/on-paper recruitment for the summer program saw a more stereotypical self-selection. The program was led by three researchers –two female, one male. Over the course of the week, participants were shown basic knitting stitches and invited to create a project of their own design based on a simple rectangle. This project offered a constrained problem space that still involved a considerable degree of aesthetic freedom and design choice. By asking participants to first create a swatch –a small sample of knitting that also served as a learning/practice space– and also to imagine the size they wanted their finished product to be, the design specifically targeted reasoning about rate and ratio. In order to create a rectangle of a particular size, a knitter will first calculate her gauge –a standard unit rate of stitches per inch– based on a swatch, and from there, determine the number of stitches necessary to make a piece of fabric of the target size.

Our facilitation aimed at reproducing some of the participation structures of more adult/expert knitting groups, including highlighting aesthetic choices and self-direction, developing community by encouraging collaboration with relative experts and among peers, and treating knitting mistakes as both inevitable and a place for interrogation and learning. Researchers addressed any questions from participants as they arose. For the most part, the calculation of gauge was facilitated in one-on-one encounters between participants and researchers, though there were two notable whole-group discussions that largely focused on these ideas.

In exploring student engagement, we considered the following questions:

- RQ1: What kinds of mathematical thinking are engaged by novice knitters in beginning and executing designs?
- RQ3: How do facilitator practices or the act of knitting support mathematical reasoning with knitting?
- RQ2: How do facilitator practices or the act of knitting undermine mathematical reasoning with knitting?

Data Collection and Analysis

All five days of the workshop were videotaped with standing cameras, set up to capture interaction and talk among participants. In addition, over four days a subset of focal students, selected with the goal of capturing the range of age and knitting fluency across participants, wore GoPro cameras on lanyards around their necks, which captured what was in front of them as they were knitting. These cameras thus captured a clear record of students' knitting and gesture, along with the conversations that they had with facilitators and other youth. All participants completed a basic questionnaire and a mid-week assessment of knitting-based proportional reasoning, and all but one were briefly interviewed on the final day about their impressions of math, knitting, and the potential overlap between the two disciplines.

For this study, three cases were selected for closer analysis; two cases were of individual students, and the final was of a pair of students. These cases demonstate a representative range in both age and craft experience. Case 1, Susie, was a 15-year old girl with experience knitting. She was proficient at the two main stitches in knitting –knit and purl– and had some experience with shaping using a simple increase and decrease –by far the most experience in the group. Susie's favourite subject in school was Composition. Case 2, Amy, was a 9-year old girl –the youngest of our participants– who had no prior knitting experience (though she did have experience with crochet), and was not confident in her math skills. Amy's favourite subject in school was spelling. Case 3, Tori and Stephanie, was a pair of 11-year old girls who often sat together while knitting. They did not know each other prior to the KnitLab workshop. Neither girl had prior experience knitting. Tori's favourite school subject was Science, Stephanie's favourites were Math and Science. This pair was considered as a single case because much of the mathematical reasoning they demonstrated emerged in conversation between the two.

Videos were initially coded for explicit evidence of mathematical reasoning in dialog, with chief focus on didactic episodes with the researchers, including the mid-week assessment. Next, videos from the GoPro cameras were coded for evidence that the reasoning from the initial didactic interaction was either repeated or adjusted, explicitly or otherwise (e.g. through material constraints). Finally, the highlighted episodes were analysed for evidence as to how the tools and emergent participation structures of the knitting group might be influencing mathematical participation.

FINDINGS

Overall, our findings suggest that knitting can be a useful resource for pushing on or supporting mathematical reasoning, but as with all manipulatives, whether or not that occurs is related to what students already know and understand, and their facility with the use of the manipulatives themselves. However, these general findings are more nuanced when studied in interaction, and thus we present three cases of reasoning with knitting below, with the goal of beginning to identify why differences in mathematical reasoning occurred. Finally, as we fit these observations back into the larger goal of the on-going project, we offer some preliminary conjectures about what the brief experiences of these youth may tell us about how knitting has traditionally persisted as a safe space for women despite the relative hostility of many other endeavours that involve mathematics.

Case 1: Susie

Susie's experience over the week presents a case of a student for whom the designed task did not require mathematical thinking, because of her familiarity with knitting and her comfort with mathematical calculations. As stated above, the challenge designed for this camp was fairly basic, and focused on one family of calculations –rate. It was clear that this challenge did not push Susie's mathematical thinking or knitting experience. Susie was the only participant in the group who had any knitting experience. She was also one of the older students, and was more familiar with the proportional reasoning involved in designing. Susie had enough fluency and speed with knitting that she was able to calculate gauge at several different opportunities, creating swatches in both garter and stockinet. The knitting design project thus afforded her many more opportunities to practice these skills than her less knitting-fluent peers. She also had enough fluency with the mathematics that she did "extra" calculations– she used her gauge to calculate how many stitches she would cast on in either direction before determining the orientation of her pillow (since only one side is cast on, generally a designer determines only this side in stitches, and the other in rows). She also remarked to a researcher that this might be useful to someone else who wanted to recreate her pillow with the nap of the fabric in the other direction. Thus, overall, Susie's case demonstrates ways proportional reasoning can be useful in creating knitting designs. However, from her videos and interviews, there was little evidence that the act of knitting actually contributed to or pushed back on her mathematical reasoning. It seemed that, while she had begun to see the utility of math in knitting, math and knitting remained separate spheres. For example, in the short interview on the final day, when prompted to imagine the overlap between the two spheres, Susie started describing gauge calculation, and then gave an example –calculating knitting speed– that an unspecified "somebody" had mentioned, that she did not take up at all. This is consistent with her patterns of behaviour during the design process, in which she moved away from her knitting to complete her calculations, returning to the materials only when her design work was finished.

In fact, Susie's facility with knitting allowed her to use knitting to circumvent some mathematical thinking. Whereas the curricular design expected students to first calculate unit rate from a larger piece and then use the unit rate to create the full pattern, Susie simply counted how many stitches she had in one inch of knitting –a simple way to find gauge that is often taught to beginning knitters. Thus, while she was proficient with the simple rate calculations, she was also able to circumvent some of the mathematical possibilities in the design problem space. While this did not interfere with her ability to design her pillow, during the assessment it became clear that she had not fully explored these other facets of gauge calculation.

> Interviewer: When we asked you to do the knitting problem on paper, did that seem like it was the same thing we'd been doing?
>
> Susie: No, it was different. Um, I don't know. It's just– it seemed a lot more complicated on paper to me. 'Cause I hadn't thought about the way that they'd written it exactly, I guess. [...] I just tried to think, um, I tried to think logically and hope for the best.

Susie's case demonstrates one way that the affordances of these manipulatives interact with the curricular design. Even gauge calculation is motivated differently for different projects. A simple design change, for example, might include a discussion of precision in measurement –counting the number of stitches in one inch is potentially less precise than measuring a larger swatch and calculating. The former will be "good enough" for a simple design like a small pillow, but the potential for error propagation in a larger project, or one with more specific dimensional requirements such as a sweater, might better motivate the more complex calculation. Susie's case thus demonstrates that facility with targeted math concepts can mean that knitting does not serve as a resource for thinking; likewise, facility with existing knitting practices can undermine the motivation for mathematical exploration.

Case 2: Amy

Amy's experience in KnitLab demonstrates the potential of knitting to serve as a manipulative to model thinking, with some disruptions. Amy was the youngest participant in our workshop, and demonstrated little confidence with basic mathematical calculations. She was enthusiastic about learning to knit, although she struggled with consistency in her knitting and purling. This challenge in knitting fluency influenced how she was able to reason mathematically about her knitting. For example, calculating gauge involves determining how many stitches will cover an inch. If each stitch is a different size, this calculation becomes less meaningful, particularly because the purpose of calculating gauge is to predict the eventual size of a piece. Thus, among other things, Amy demonstrates a case of when disfluency with a manipulative might interrupt or interfere with mathematical reasoning.

However, these challenges did not prevent Amy from engaging in mathematical reasoning, and indeed, there is evidence that ultimately her knitting became a resource to support her mathematical thinking. Initially, Amy saw math and knitting as completely separate spheres. When asked to calculate her gauge, Amy put her knitting down, picked up her pencil, and said: “Wait it’s a math problem, right? What, how do I write it, then? I don’t know how to write that.” When the researcher encouraged Amy to refocus on her knitting, however, it became a place where she could ground her mathematical thinking, rather than making “magical manipulations.” After looking at her swatch, holding a ruler against it, and counting stitches, Amy quickly noticed that the five stitches she measured in the first inch would stay consistent across the swatch, and was able to coordinate counting by fives with the inch markers on the ruler.

Ultimately, however, Amy’s disfluency with knitting limited the number and kind of encounters she could have with mathematical ideas, both because of how long it took her to knit, and how difficult it was for her to knit with consistency. Overall, this case demonstrates the potential of knitting for grounding mathematical reasoning, but also points to the high entry bar in knitting as a limitation for its usefulness.

Case 3: Tori and Stephanie

Tori and Stephanie’s experience in KnitLab presents a case of knitting serving as a both a means of modelling thinking and of pushing thinking, a true relationship between a manipulative pushing on thinking and thinking being explored through the model. Tori and Stephanie were two years older than Amy, and neither had any experience with knitting. They also did not know each other before the workshop, though they quickly ended up sitting together regularly, chatting and helping each other with their knitting. They are treated as a single case here because so much of the mathematics evident in their work came up in conversation with one another.

Despite being new to knitting, neither Tori nor Stephanie had as much trouble with developing fluency as did Amy. While they were not as fluent as Susie, and thus did not end up calculating their gauge in as many different ways, these two evinced spontaneous proportional reasoning around their knitting in unexpected ways. Early on, the researchers had encouraged participants to count their stitches after every row, as a way to help them notice mistakes –starting with twenty stitches and ending with twenty-four is a simple indication that something has gone wrong. Participants had also been encouraged to calculate row gauge and determine how many rows it would take to complete the project. Tori merged these two suggestions, and recounted both her stitches and her rows after nearly every row. Hearing Tori do this, Stephanie decided to count her own rows and remarked that Tori was "much faster" than she was, since she had more rows completed. Tori responded that she wasn't any faster, she simply had fewer stitches per row than Stephanie did, demonstrating her understanding of the inverse relationship between number of stitches per row and number of rows across a constant "amount of knitting."

Later, Tori was concerned that her original pattern was wrong– that she had miscalculated how many rows she would need to make her piece the size she wanted. Her conversation with Stephanie and then with one of the researchers about this problem is telling. At first, Tori seems to assume that she will have to execute her pattern as written– much as one might be tied to a procedure in math class. Stephanie suggests to Tori that she might be allowed to adjust it, and that she should simply ask. In response to her query, the researcher responds, "When we figured out your row gauge we, um, were wrong, and so ... but that's ok [...] I mean the nice thing about knitting is – if you think that's too short, make it longer!" After this exchange, Tori and Stephanie discussed how much more she would need.

Tori: I'm trying to get this to be... I don't think this is going to be big enough for a purse, because I'm on 24 now, and I have 28 [in my pattern]. It's gonna be way too small.

Tori: So, maybe I'll make it to 40?

Stephanie: Um, it depends on how long you want. Right now you have 24, and if you wanted it... like 48 would be double that size, wouldn't it? It'd be about here.

Tori: Yeah. That's pretty good.

Rather than abandoning the mathematics or returning to the original calculation (or even getting out a ruler), they used proportional reasoning with the existing material as the unit in order to predict how much more Tori needed to knit. This kind of proportional reasoning became a way to organize their emerging knitting work. In working together on the execution of their own designs, but also adjusting their designs –and even during the mid-week assessment– Tori and Stephanie used the spheres of mathematics and knitting to help inform each other. Some of the emergent questions from this group could be incorporated into future designs, including their focus on speed and on-the-fly adjustments to designs.

DISCUSSION

This paper explored the ways that a particular textile practice, knitting, could serve as a manipulative or resource for supporting mathematical reasoning. We considered cases of primarily novice knitters engaged in a design activity that had a core constraint: shape. Overall, all three cases showed engagement with mathematical reasoning around rate and proportion, though in significantly different ways. For Susie, her existing math knowledge was leveraged in service of knitting, and she saw new connections between the disciplines that might conceivably influence her dispositions toward math. Still, her relative fluency with the proportional reasoning made reasoning with the material of the project seem unnecessary –in terms of how the two spheres of math and knitting might help each other with solving specific problems, they remained effectively separate. In contrast, for Amy the genre of math initially blocked her view of the project at hand– when invoked, the sphere of school math eclipsed the work of knitting. Ultimately, however, the materiality of her own project offered her a place to anchor her thinking, which gave her a "way into" more sophisticated multiplicative reasoning than she seemed otherwise inclined toward. Tori and Stephanie spontaneously invoked proportional reasoning to help them think about how much knitting they had done or had left to do. In addition to the gauge calculations specifically included in the instructional design, the project of creating their bags offered even more opportunities to engage in proportional problem solving.

The material of student knitting projects served as an anchor for reasoning, helping younger students (Amy) to think about the meaning of mathematics, even beyond grade-level. The basic design of swatching and creating a pattern for a simple rectangular project promoted proportional reasoning in the calculation of gauge. Beyond that, however, the process of working through the execution also brought up opportunities for proportional reasoning (for Tori and Stephanie) in relation to their speed, and adjustments to their existing designs. Furthermore, the retrospective analysis suggests that discussions of precision and error in measurement would be a productive next step for beginning designers.

However, both disfluency with the physical requirements of knitting (for Amy), and familiarity with knitting practices (for Susie) served to undermine productive mathematical reasoning. For Susie, calculating gauge by counting across a single inch made sense as a beginning knitting practice, and there was nothing in the instructional design that motivated a more complex procedure except facilitator suggestion. Ultimately, this limited her engagement with some of the mathematical practices we aimed to promote, highlighting the importance of considering the learning ecology as a whole rather than simply the manipulatives as a fixed entity.

Next Steps

In addition to offering a window into knitting as a productive manipulative for multiplicative reasoning, the incident with Tori and Stephanie offers a preliminary view as to how knitting might be insulated from some of the influence of the traditionally male/masculine view of mathematics. The notion of both material and aesthetic judgement as the ultimate arbiter of what counts as "right" in knitting echoes many of our earlier expert interviews. Just as each knitter's gauge is different, so her taste and criteria may be different, and even different for certain projects or moods. In this way, binary notions of right/wrong, precise/imprecise, and even expert/novice are troubled. Part of the power of knitting, then, may lie in the capacity to trouble these boundaries –or as queer theorists might put it, to "stop the binaries all lining up" (Mendick, 2006). Both Amy and Sophie oscillated between math and knitting, where the rules may have been different, but did not conflict as long as the spheres were kept separate. When the math and knitting came into full alignment, however –as it did for Tori and Stephanie– so did the possibilities for self-direction and laying claim to authorship.

CONCLUSIONS

These findings suggest that textile design can be an important resource for mathematical reasoning, but as with all manipulatives, context matters. For students who are just beginning to think about new mathematical ideas, in this case, concepts of ratio and proportion, knitting can be a helpful model. Much like young children with blocks, modelling a situation that they cannot yet visualize allowed students to engage with core ideas and notice properties of number and operations that would otherwise have been invisible to them (Carpenter & Fennema, 1992). However, when students already understand core mathematical ideas that help them with their knitting designs, the knitting itself does not become a resource or even connect with their mathematical thinking. This is consistent with other manipulative use –when students are able to use efficient methods that rely on visualizations or algorithms, they do not need to use manipulatives, and doing so in fact can slow their thinking and progress. Finally, facility with the manipulative, in this case, knitting, is an important element of the overall thinking that can emerge; struggling with the practice of knitting can undermine opportunities to learn.

This leads us to wonder about the potential of textile in classrooms. Is it the case that the high bar for learning excludes these manipulatives from classrooms? Most of the manipulatives that are used in elementary classrooms involve stacking, clicking, and lining up, and use hard materials such as wood and plastic. The range of things that can be created with these manipulatives is quite limited, and as a consequence, so is their longevity in the classroom. These materials are easily used with little instruction by even small children. What is conspicuously absent from all classrooms is a form of manipulative that can be moved and shaped in multiple ways –malleable materials such as textiles. When considering the gendered histories of these materials, the conspicuous absence of this class of manipulatives –those traditionally associated with women's craft– seems more troubling. Beyond considerations of which gendered practices are reified in classrooms, there is a question of whether and why we would choose not to include manipulatives whose use can span multiple grade levels (from representing quantity and counting, through reasoning about rate and slope, even to modelling of hyperbolic space). Few manipulatives allow for the exploration of such an array of mathematical ideas as is afforded by textile work.

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DIVERSITY IN MEANINGS AS AN ISSUE IN RESEARCH INTERVIEWS

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Taking the social, political, and ethical dimensions of mathematics education seriously means not only researching these issues, but also designing and assessing research with these dimensions in mind. When designing an interview study about planning in mathematics, diversity in meanings was recognized and participants and their voices were foregrounded. In this paper, the design is related to perspectives on interviews, meaning as both durable and transient, and quality criteria such as reproducibility and bias. Theoretical assumptions had consequences for how meaning was seen, but also for relevance of the chosen quality criteria. Findings suggest that not only design, but also assessment of quality in interview studies have to be discussed in relation to the theoretical assumptions the studies build on.

INTRODUCTION

In times of worldwide crisis, research exploring and problematizing social, political, and ethical dimensions of mathematics education and mathematics education research is of great importance. Such research can be conducted in many ways, but it seems reasonable to say that social, political, and ethical issues should be taken seriously not only when choosing topics for study and formulating research questions, but also when choosing methods and encountering participants. A common concept within the research field is diversity, which can refer to diversity both in terms of language, sexuality, race, or ethnicity, and to diversity in how different groups and individuals understand and use common words or concepts.

When this diversity in meanings is taken into account, certain issues have to be considered. Whether the study is conducted through questionnaires or interviews, it is possible that words central to the research topic are used with different meanings for the researcher and for the respondents. As a researcher, I also need to be aware of the fact that when I enter a field my experiences and values will influence the design and implementation as well as the analysis.

The aim of this paper is to contribute to a theoretical and methodological discussion about interviews in mathematics education research. As a point of departure I use a study about what meaning planning in mathematics has for teachers and what they focus on when talking about their planning. The intention of the research design was to downplay the researcher's own preconceptions and the meaning that planning in mathematics has for her. For the critical discussion of the design I use three aspects: theoretical perspectives on interviews and meaning, Kilpatrick's (1993) quality criteria, and objections to research interviews from "the mainstream of social science" (Kvale, 1993, p. 167). Although the study itself is not directly related to social, political, and ethical issues, the design and the critical discussion of the design might be valuable for future research in the field.

BACKGROUND

All teachers in mathematics have the task of planning, and thereby the task of somehow deciding upon mathematical content for a specific group of students, in common. Despite this, it seems that the understanding of what planning in mathematics involves varies. Although individual teachers' ways of relating to and understanding this particular part of their work shift, teachers' decisions are made in a context of shared values specific to their culture (Hofstede, Hofstede, & Minkov, 2010), but also with values specific for the subject of mathematics and the specific school (Bishop & Seah, 2008). Thus, what meaning teachers attach to planning can both be described as situated within micro-contexts and in some sense structured and constituted by a larger mathematics education discourse (Alvesson & Karreman, 2000). Since planning has implications for what happens in the mathematics classroom and thereby also for what opportunities students have to learn mathematics (Clark & Yinger in Akyuz, Dixon, & Stephan, 2013), it is relevant to further explore planning in mathematics.

Different countries have different degrees of control in terms of teachers' planning. In the Swedish policy documents, goals and content are formulated not to govern details or restrict pedagogical freedom (U2009/312/S). Teachers in Sweden thus have a high degree of freedom to plan, form, and implement their teaching as they want based on the current curriculum. However, the Swedish National Agency for Education (Skolverket, 2011) has published a collection of advice on how teachers shall organize their planning. These guidelines as well as research models indicate that planning is a several step process (Gómez, 2002; Rusznyak & Walton, 2011), whereas other studies show that activities to be done during the lesson are in focus when teachers plan (Akyuz et al., 2013; Mathematics Learning Study Committee, 2001).

THE STUDY ABOUT PLANNING IN MATHEMATICS

The aim when designing the study was to arrange a situation where the teachers feel comfortable and respectfully treated and where they do not feel the need to think about what they are expected to answer. Asking questions about planning in mathematics could determine what meaning the teachers attach to the concept and what their story is going to be about. Without asking any questions, the conversation tends to be more of a casual conversation without focusing on the topic of interest. To overcome this dilemma, the study was designed as an interview with the support of stimuli (Hurworth, 2012). The inspiration for the stimuli came from studies in various fields where different stimuli have been successfully used (Alsup, 2006; Herbel-Eisenmann & Cirillo, 2009; Hurdley, 2006). In this design, teachers' notebooks served as stimuli.

RESEARCH PROCESS

The researcher initially met with the teachers individually to give them information, allow for opportunities to ask questions, and to give them their notebook. The participants were asked to write words or phrases or draw pictures of things related to their planning in mathematics for a period of two weeks. If a participant asked for clarification, she got the answer that it was what she thought was important that should be in the notebook. It was also pointed out that how much or how little that was documented was up to the teacher herself. No matter what, or how much or little the teacher had documented, the notebook could be a starting point for the interview.

The interviews were conducted after the two weeks of documentation. After initial small talk, the interviewee was asked to look in the notebook and talk about what was documented. During the conversation, the interviewer was deliberatively active by confirming that she was listening with nods, gestures, and confirmatory small words, by asking for clarification when something was unclear, and by asking follow-up questions on central themes. The activity from the researcher had dual purposes: on the one hand to make the interviewee feel comfortable and listened to, and on the other hand to keep the topic of the interview in focus. During the interview, the researcher was passive in that the interviewee was always the one to introduce new themes for the conversation by choosing topics from the notebook.

REPRODUCIBILITY AND BIAS IN RESEARCH INTERVIEWS

One of the common objections Kvale (1993) emphasizes is that interview results are biased. The biases can be either from the interviewee, who answers what she thinks the interviewer wants, or from the interviewer, whose experiences influence the questions as well as the interpretations of the research material. Interview questions as well as overall research questions determine what kinds of answers may be obtained, which means that questions in interviews to some extent always are leading. According to Kvale (1993), the researcher should make questions explicit so that the reader has a possibility to evaluate the influence questions have on findings and also assess the validity of the findings (Kvale, 1993). Also, Kilpatrick (1993) emphasizes bias and objectivity and argues that absolute objectivity is “an ideal worth working toward” (p. 23), although he also states that it is unattainable. What a researcher can do is try to identify biases and have “enough objectivity to rule out obvious bias” (p. 23) and also be open with how biases may have affected findings.

In this design, interviews were conducted without predetermined questions, which automatically makes it impossible to meet Kvale’s desire to make questions explicit. At the same time, the reason for not asking questions was that possible bias had been identified. Instead of assessing the validity of findings based on questions asked, the reader (as well as the researcher) has to be open to the interview’s different paths. I argue that validity in findings then becomes even more dependent on how well the analysis is conducted and what questions the researcher poses to the collected material. With openness in the analysis process, the reader has a possibility to assess validity in findings.

Another objection to qualitative research interviews is that they are person-dependent, that is, that two interviewers will not come up with the same result even if they use the same interview guide (Kvale, 1993). Hence, if an interview study is not reproducible in a traditional way, does this mean that the results are not reliable? And is it of interest to discuss reproducibility in a study conducted within a perspective where the interviewer is part of the context and co-creator of the situation? Kvale (1993) emphasizes that the non-formalized qualitative research interview has virtues and that the various results different researchers find would contribute to a more nuanced and deeper meaning of the research topic. On the other hand, Kilpatrick (1993) argues that not only the procedure of conducting a study, but also the findings of a study ought to be reproducible. Without the possibility of reproducing procedures and findings, it is not possible to draw generalizations from the study, and consequently the research is according to Kilpatrick (1993) of no use. A response to the demand for generalizations could be Kvale’s (1993) question, “Why generalize?” (p. 185), and the immediate answer to that question, “in order to predict and control, or because science aims at universal knowledge” (p. 185). However, instead of claiming it to be irrelevant to generalize, one can take note of Kilpatrick’s argument of usefulness and discuss what generalization in non-reproducible studies can be. Even when it is not possible to generalize in the formal way Kilpatrick argues for, findings may be transferable and contribute to the knowledge accumulation in a given field (Flyvbjerg, 2011). Eisenhart (2009) argues that the concept of theoretical generalization is useful in research and describes how Becker explains it: "the point of theoretical generalization is not to show that every site with the characteristics of a total institution produces the same results, but rather to show how each new site potentially represents different values of a generic process" (Eisenhart, 2009, p. 16). From these arguments, it becomes clear that findings from what Kilpatrick would call non-reproducible studies might be useful not only to understand the micro-context where they are produced, but also for understanding the larger context of mathematics teaching.

In this design, the teachers' notebooks guided the interview with the purpose of letting the teachers' meaning of planning in mathematics set the agenda. However, viewing it from a point of reproducibility, it can be problematic and has to be further discussed. Using Kilpatrick's (1993) definition of reproducibility, where not only the procedures of conducting the study, but also the findings should be replicable (Kilpatrick, 1993), one needs to think of what it means that "findings of the study –the observations, the patterns of results, though not necessarily the interpretations given them– ought to be reproducible too" (p. 29). In trying to reproduce the study, a researcher can use the same design and conduct the interviews in the same way. After the interview, the researcher would have the same kind of material: notebooks with some documentation and audio recordings of the teachers using that documentation to talk about planning. In that sense, the observations would be the same. If observations also include how the researcher looks upon the material, I argue with help from Kvale (1993) that observations never can be the same. Different researchers come to interviews with different experiences, which will influence how they make their observations. Either way, this is not unique to an interview with stimuli, but also applies to an interview with traditional research questions. Questions are biased and leading, and responses can be biased in the direction of what the interviewees think the researcher wants. This is, according to Kvale (1993), inevitable and should be addressed by describing the process transparently and thus allowing the reader to determine what importance these biases have for the conclusions. I share the view of transparency being important, but also argue that the design in which the interviewees determine what to talk about within a given topic reduces the impact of bias, which corresponds to Kilpatrick's demand that the researcher shall try to identify and rule out obvious bias.

PERSPECTIVES ON INTERVIEWS

As seen above, there are different positions when it comes to assessing quality in research interviews. Since theoretical assumptions play a key role in how data is treated as well as how the researcher looks upon the interviewee and the interview situation (Alvesson, 2003; Silverman, 2006), it is reasonable to assume that these different theoretical assumptions also play a key role when assessing quality in research interviews. Adopting a positivistic (Silverman, 2006) or a neopositivist perspective (Alvesson, 2003) implies that data are seen as facts, the settings in the interview situation do not matter, and the interviewee is randomly selected. Reality exists "out there," and the interviewee can tell the researcher about it. Hence, it seems reasonable that a study can be designed so that both procedures and findings are the same regardless of who conducts the interview. Consequently, from a positivistic/neopositivist perspective reproducibility is relevant to discuss. From an emotionalist (Silverman, 2006) or a romantic (Alvesson, 2003) perspective, on the other hand, the researcher wants to explore the "inner world" of interviewees. To do that, trust and commitment between the interviewer and the interviewee, particularly in the interview situation, are important. Also, from this perspective, there may be a point in discussing reproducibility. The "inner world" exists independently of the researcher, but since the relation between the interviewer and the interviewee is of importance, one can think that there always will be differences in how the "inner world" is brought out. The requirement that procedure ought to be reproducible thus falls. Still, it is possible that findings are the same, which makes an interview study in this perspective reproducible at least in one dimension.

From a perspective of constructionism, the interviewer and the interviewee are seen as co-creators in creating meaning. A special focus in this perspective is how interviewees construct their stories. Interesting data are *what* is being said, but also *how* it is said. In other words, how stories are constructed within the interview, but also how the stories relate to circumstances of the interviewee's life (Silverman, 2006). Alvesson's (2003) localist perspective has similarities with the constructionist perspective of Silverman. In the localist perspective, the interview is not seen as a method to collect material in order to say something about outside of the interview situation, but "an empirical situation that can be studied as such" (Alvesson, 2003, p. 16). This could correspond to Silverman's constructionist interest in *how*. To meet the interest in *what*, which is included in constructionism, Alvesson (2003) refers to the neopositivist and romantic view in which it is possible to use interviews to explore issues other than the actual interview situation, but with the addition that it is "without falling too deeply into the trap of viewing interview talk as a representation of the interiors of subjects or the exteriors of the social worlds in which they participate" (p. 17). In a perspective where the interviewer and the interviewee co-create meaning, each interview situation must be seen as unique. Each story is unique and the circumstances of each interviewee's life are unique. Hence, to discuss reproducibility and to require that procedures and findings are reproducible are not relevant.

Which perspective researcher has on interviews determines whether reproducibility is a relevant quality criterion. As shown in the table below a positivistic/neopositivist perspective enables reproducibility in terms of both procedures and results, an emotionalist/romantic perspective only in terms of findings, and in a constructionist/localist perspective neither procedures nor findings are reproducible.

Table 1: Reproducibility in relation to different perspectives on interviews.

Perspective	Procedures reproducible	Findings reproducible
Positivistic/neopositivist	X	X
Emotionalist/romantic	-	X
Constructionist/localist	-	-

The study about planning in mathematics would fit into the bottom row of the table. The theoretical assumptions of the study are substantially in line with the constructionist and localist perspectives. The interview situations are unique, and the interviewer and the interviewee co-create situated meaning. At the same time, stories constructed in the interview relate to the circumstances of the teachers' lives, including the mathematics education discourse with its specific shared values.

MEANING

The different perspectives on interviews and what knowledge they may generate also relates to how meaning is seen. From some perspectives, meaning is not an issue and discussions about it are avoided. From other perspectives, meaning is the key issue and what is worth exploring (Bryman, 2008). In this paper, the definition of meaning as "a (collectivity of) subjects' way of relating to –making sense of, interpreting, valuing, thinking, and feeling about– a specific issue" (Alvesson & Karreman, 2000 p. 1147) is used. Meaning is seen as constructed within contexts and influenced by such factors as politics, gender, economy, and ethnicity. Those who construct meaning are people interacting with each other (Cherryholmes, 1999). Meaning making can be understood and studied as a process, a meeting between the individual dimension of our past experiences, cultural dimension, and the social dimension with its relations (Quennerstedt, Öhman, & Öhman, 2011). Meaning can be described both as durable meaning, including cultural and individual ideas, and a transient meaning that is tightly and temporarily connected to language and emerges in interaction (Alvesson & Karreman, 2000). The durable meaning is stable enough to travel through discourses, whereas the transient meaning is constructed within the discourse of the interview situation. Since meaning emerges through individual interactions (Alvesson & Karreman, 2000), it is reasonable to think that the interaction has to be studied on several occasions to grasp both the durable and the transient meanings. Telling teachers to use the notebook before the interview was one way of letting them interact on several occasions. This adds an element of reflection normally not included in the planning process, but for this study it is a way for teachers to reconnect with previous interactions in the interview situation. This makes it possible to get ahold of durable as well as transient meaning and also streaks of shared values in the mathematics education context.

Since meaning is constructed and influenced by factors outside a person, meaning for each individual varies. Meaning is not only varying on an individual level, but also amongst people. In relation to some issues, there is a fairly large consensus, but regarding other issues, as in the example of planning in mathematics, meaning differs a lot. These positions are important when going back to the discussion about reproducibility and bias. If the diversities in meaning are taken seriously, reproducibility is not relevant to discuss at all. When meanings vary, both for the researcher and the respondents, the procedures as well as the findings will vary. Hence, no study is reproducible. Ruling out obvious bias may imply giving space for various meanings. On the other hand, in adopting a perspective where meaning is not an issue, reproducibility might be an appropriate quality criterion. In this perspective, ruling out obvious bias may imply the exclusion of variations of any kind.

Choosing to study meaning in relation to a concept means recognizing meaning as an issue. In the study described above the aim is to study what meaning planning in mathematics has for teachers. Meaning is understood as varying over time, but also varying among different contexts and individuals. Hence, traditional quality criterion of reproducibility is not relevant. Nevertheless, it is of great importance to rule out obvious bias, including giving space for diversity in meanings, and strive for high quality research. Using stimuli and letting interviewees choose concerns important to them at that point in time is meant to be a way of giving space for diversity in meanings and thereby reduce impact of researcher's meaning.

CONCLUSIONS

Interviews are a commonly used method in qualitative research (Silverman, 2006). However, research quality is often discussed in general terms and seldom in specific connection to interviews. What theoretical assumptions a researcher has when designing and conducting an interview study is of importance for how quality should be discussed. The determinations of what constitutes bias or which quality criteria are relevant differ within different perspectives and also depends on the research questions. A study where teachers' individual notebooks act as stimuli in guided interviews is not reproducible at all since the interviewees themselves choose concerns to talk about. However, this does not mean that the study automatically is of low quality. This method of collecting data is a way to rule out obvious bias and thereby meet quality expectations other than reproducibility, and findings from the study may be used for theoretical generalization (Flyvbjerg, 2011). If clear underlying theoretical assumptions and transparent decision making are communicated by the author, each reader has the possibility to assess the validity of the findings (Kvale, 1993). By broadening the horizons and assessing the quality of research in relation to the theoretical perspective underlying the study, there are opportunities for researchers' different results to contribute to a more nuanced and deeper picture of mathematics education, something that ultimately will benefit the students in mathematics classrooms.

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COMING TO UNDERSTAND THE BIG ISSUES: REMAKING MEANING OF SOCIAL JUSTICE THROUGH MATHEMATICS ACROSS THE SCHOOL YEAR

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This paper describes students' collective sense making about access to healthy food in their local community across two social justice mathematics projects, each of which incorporated different geometry content from the standard curriculum. I analyzed focus group interviews, which followed each project, and applied the theoretical framework of figured worlds to identify how students drew on their past experiences and the social justice and mathematics content of each project to understand issues arising from lack of access to healthy food. Findings demonstrate how students initially dismissed investigating the social justice issue but eventually ascribed value to using mathematics to investigate local access to healthy food and were able to discuss the social justice issue in more nuanced ways.

INTRODUCTION

In the face of economic and political crisis worldwide, some argue that teachers face a moral and ethical imperative to transform mathematics classrooms into spaces for the development of critical social awareness (i.e. critical consciousness) and social transformation (Stinson, 2014), and mathematics classrooms have increasingly become sites for critically investigating and reflecting on social justice issues (e.g. Enyedy & Mukhopadhyay, 2007; Esmonde, 2014; Gutstein, 2003; Pais, Fernandes, Matos, & Alves, 2012). Given the amplified effect of crisis on historically and systematically marginalized learners' educational experience, classrooms with Black and Brown students from poor families often serve as the target site for reforming mathematics teaching to include social justice goals (Brantlinger, 2013). Although efforts to integrate mathematics and social justice goals (i.e. teaching mathematics for social justice – TMSJ) aim to place the best interests of students at the forefront, tensions naturally arise when teachers attempt to translate theories of TMSJ into practice (Bartell, 2013; Pais et al., 2012; Skovsmose, 1985). Thus, understanding students' experiences using mathematics to investigate social justice issues is crucial to ensuring that well-intentioned TMSJ efforts do not further marginalize students in mathematics spaces.

To date, research provides seemingly contradictory accounts of historically and systematically marginalized students' experiences with TMSJ. Some practitioner-researchers found that TMSJ provided opportunities for students to develop more positive mathematics identities and to consider issues of social transformation in and through mathematics (e.g. Gutstein, 2003; Turner, Gutiérrez, Simic-Muller, & Díez-Palomar, 2009). Others practitioner-researchers, however, found that students resisted their efforts to integrate social justice investigations into the mathematics curriculum (e.g. Brantlinger, 2013). This contradictory evidence suggests the need for research that centers historically and systematically marginalized students' voices and experiences to understand how they make sense of TMSJ. This paper explores the following question: When reflecting on their experiences during TMSJ projects, how do students collectively make sense of social justice investigations that integrated mathematics?

THEORETICAL FRAMEWORK

In order to explore students' sense making about using mathematics to investigate a social justice issue in their local community (i.e. TMSJ), I borrow a framework from anthropology – *figured worlds* (Holland, Lachicotte Jr., Skinner, & Cain, 1998). Figured worlds provide a particularly strong theoretical basis for understanding students' experiences with TMSJ because the framework can account for student sense making across a range of topics (e.g., mathematics, social justice) and for the broader sociocultural and sociopolitical influences on interactions among teacher, students, mathematics, and social justice (Esmonde, 2014).

A *figured world* is a "socially and culturally constructed realm of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland et al., 1998, p. 52). For example, if a teacher (an actor) provides a solution to a mathematical problem to a student (another actor), the action might be interpreted as teaching. If the student acts similarly, providing an answer to a fellow student, it might be interpreted as cheating (Esmonde & Langer-Osuna, 2013). Figured worlds provide the context for what counts as mathematical engagement and for how students make sense of themselves as successful or not in relation to that mathematical engagement (i.e. mathematical identity) (Horn, 2008).

Figured worlds operate neither in isolation nor in rigid, fixed ways. Multiple figured worlds are at play in the mathematics classroom (e.g. mathematics, group work, gender), and various factors within these figured worlds (e.g. actors, actions, forces) relate to each other in ways that resemble a narrative or *storyline*. These socially and culturally constructed storylines provide a standard plot, or a taken-for-granted sequence of events, by which "the meaning of characters, acts and events in everyday life [are] figured against" (Holland et al., 1998, p. 54). Using the construct of storylines as a proxy for student sense making, this study asks:

- What storylines about investigating and understanding social justice issues through mathematics do students collectively construct?
- How do those storylines vary across two different projects focused on the same social justice issue but enacted at different times in the school year to incorporate different mathematical content?

METHODS

This study drew on a subset of data collected during a yearlong ethnography in a geometry class within an urban, secondary school in the United States. Ethnography is a particularly suitable methodology for understanding classrooms from students' perspectives (Anderson-Levitt, 2006), but as a white, female in her mid-thirties, I recognize the challenge of adequately interpreting the perspectives of Black and Brown adolescents. Throughout the larger study, my interactions with students and data were guided by my transformative worldview, meaning that I view research as power-and-justice-oriented, collaborative, and change-oriented (Creswell, 2014). Consequently, throughout the analysis and writing processes, I aimed to let students' words speak for themselves, as I questioned my own assumptions and interpretations. Rather than assuming to speak for students, I strove to speak alongside them (Anderson-Levitt, 2006).

Research context and participants

The study took place at an open-enrollment, STEM-themed magnet school located in a low-income area of a small city in the Midwestern United States. This newly founded magnet school enrolled approximately 400 7th-10th graders from the local community at the time of the study. The school's mission emphasizes technology-driven (1:1 student to laptop computer ratio) project-based learning, in which students collaborate on projects to explore and solve authentic, real-world tasks or problems, using ideas, knowledge, and skills across a range of disciplines. The ethnography took place during the 2015-2016 academic year in one 9th grade geometry class. Five of the students who participated in either focus group interview are male (3 White[1]: Blake[2], George, Antonio; 1 Latino: Tino; 1 Black: Dante),

1. Racial/ethnic identities are more complex than suggested in this paper due to space constraints. Several students are multiracial, or their racial/ethnic identities are misinterpreted by others (e.g., Latino student from Argentina perceived as White by peers.) Thus, I listed the racial identities based on student self-identification.

2. All names are pseudonyms. In most cases, students selected their own pseudonyms.

and seven of the student participants are female (Black: Simba, Nemo, Jane Doe, Carley, Kendra, Charlie; Korean American: Rosy). All students had the teacher participant as their 8th grade mathematics teacher the previous year (the school's inaugural year). The teacher participant is a White, female who was in her fourth year of teaching during the study, and she has taught in urban schools throughout her career. She and I have collaborated on TMSJ projects for three years.

Data sources

Across the 2015-2016 school year, I visited the class 3-5 times per week for a total of 85 classroom observations. Primary data sources included video, audio, field notes, and other artifacts collected during classroom observations and data generated by lunchtime interviews immediately following classroom observations. This study drew on a subset of data, namely, transcripts of audio from two semi-structured focus group interviews aimed at gaining insights into students' expectations for and interpretations of learning and doing mathematics through TMSJ. I conducted these focus group interviews in January and June immediately following the two TMSJ projects, which each focused on investigating the lack of access to healthy food in the local community using different geometry content from the standard curriculum (Table 1). One additional project, at the beginning of the school year, also focused on the same social justice topic, but I did not follow up with a focus group interview.

Table 1. Overview of three TMSJ projects across the school year.

	Social Justice Topic	Geometry Topics
Project 0: Sept-Oct	What are food deserts?	Distance Formula
Project 1: Jan	Alleviating food deserts	Triangle Centers
Project 2: May-June	Community Garden	Area & Volume

Across projects, a food desert was defined as a low-income area where residents have *low access* (more than 1 mile in urban areas) to a supermarket or large grocery store (United States Department of Agriculture (USDA), 2016). In Project 0, students determined whether or not they lived in a food desert by using the distance formula to calculate the distance between their home and the nearest supermarket. In Project 1, students used the USDA *Food Access Research Atlas* to locate broader areas of food deserts in their community and used triangle centers (i.e., incenter, orthocenter, etc.) to determine a possible location for a grocery store to help alleviate food deserts. In the final project, students wrote proposals and built scale models for a community garden at the school that would address some of the consequences of food deserts. Project 2 was the only interdisciplinary project, and it was completed across students' geometry, English, and biology/health classes.

Data analysis

First, I analyzed each focus group interview separately by segmenting the interviews by distinctive topics of conversation. Next, I carefully considered each speech turn for evidence of figured worlds that students were drawing on to interpret their experiences with each TMSJ project (Esmonde, 2014). Then, I coded speech turns at a more detailed level through iterative rounds of coding by collapsing and clarifying codes based on confirming and disconfirming evidence from statements by different students in order to identify storylines (i.e. who and what was given agency) and until storylines were exhaustive (i.e. each speak turn was linked to a storyline) (Erickson, 1986). Finally, I compared storylines from each interview.

FINDINGS

Although students invoked multiple figured worlds (e.g. mathematics classroom figured world; food desert figured world) to interpret and reflect on their experiences in the projects, storylines were overwhelming situated with a food desert figured world. In other words, mathematics was given little agency as students interpreted their experiences with the projects, and instead, they focused on food deserts even when prompted to reflect on their mathematical work. References to mathematics were rare and limited to naming (but not describing) a mathematical concept that was part of a project. Nevertheless, the food desert storylines provide insight into their students evolving understanding of social justice issues across two projects.

Project 1 storyline: Access to healthy food is what matters (more than distance to a grocery store), so food deserts are not a big deal in this community.

The main actors in the food desert figured world were the students and people in the community. Although this figured world was lived by students every day, in Project 0 and Project 1, they were new to interpreting their community as a food desert. They had significant experience with accessing food in their local community, but they had little experience considering the ease of access to food or evaluating whether not accessible foods were healthy.

Several students were able to describe a food desert as living more than one mile away from a grocery store (with some reference to the distance formula), but they placed more emphasis on access to healthy food than distance from a grocery store. During Project 0, most students decided they did not live in a food desert because they could easily access fast food restaurants or convenience stores within a mile of their home.

This storyline about whether or not students lived in a food desert was still being collectively constructed and negotiated during the interview, but most students recognized that fast food restaurants did not provide access to healthy food.

> Jane Doe: A McDonald's is not really a place to actually go and get groceries to bring home. You need a grocery store.
>
> Carley: But McDonald's does have kind of healthy food, like salads and stuff like that.
>
> Tino: That's not healthy. You see what they put on it?
>
> Rosy: I'm glad you brought that up [to Carley] - because yes [McDonald's] does have some healthy options and I live around the corner from a [convenience store] and you can get milk there and that's a healthy option, but I think the reason why I live in a food desert is because it's not like a real grocery store where you can get all different types of healthy foods.

Even though students began to recognize the importance of access to healthy food in Project 1, they continued to dismiss the significance of living in a food desert, as long as they could access a grocery store:

> Rosy: I don't think it's as bad as they make it seem, because if you have a car and just because you have to drive a little a ways don't mean you're going to die.

Other students in the interview agreed with Rosy that living in a food deserts was not a significant issue, and they found the topic boring and irrelevant to their lives and to understanding the mathematics of their geometry class (i.e. triangle centers):

> Tino: Well, a food desert has to do with health class and how to get yourself near a grocery store, this and that, then you want to put triangles in that, it's pointless.

Food desert storyline: Project 2: Food deserts are a big deal because eating healthy foods is important, and the community needs both greater access to healthy food and increased nutritional education.

By the completion of Project 2, students had significant experience interpreting the food desert figured. They had considered access to healthy food based on distance to grocery stores and possible consequences of food deserts in their geometry class; they had done research on community outreach to understand options for building a community garden in their English class; and they had evaluated whether or not accessible foods were healthy in their biology class. When reflecting on Project 2, students described the consequences of food deserts in more nuanced ways, supporting their claims with references to experiences in these classes, but they made almost no references to the mathematical work that involved area and volume.

Blake: It's actually a problem because 1 in 4 kids, 25% of all kids 12-19 are obese. Kids that are obese. That number's only growing.

Carley: In biology, we learned all the different things that vegetables do for you. Like what all the different colors do for your body. And everyone is just kind of focusing on food deserts [i.e. access to a grocery store, but we need to food on eating healthy, too].

Students described multiple ways in which a community garden could help alleviate problems associated with food deserts, and these were reflected in their proposals to build a community garden at the school. One idea was to use a community garden to directly provide food to people in the community. Blake expressed appreciation of planning to build a garden in geometry because he hoped to build his own garden as an adult (implying he might actually use his understanding of area and volume to do so). Students also saw a need for providing nutritional education through the community garden effort:

Rosy: One idea my group had is to make recipes [for each of the vegetables in the community garden]. That would be good, because when you're eating healthy, that doesn't always mean straight eating vegetables and fruits. You know what I mean? My dad can make a chicken and broccoli fettuccine and it still had vegetables in it. I think that's really important to include like a recipe so you're still eating your vegetables and it's not just straight vegetables.

Students saw their work on the community garden as benefiting them directly and also benefiting members of their community. The increasingly ascribed value of understanding and alleviating food deserts was also demonstrated through Dante's concern about the localized nature of the efforts to build a community garden:

Dante: Why do we just focus on [the community immediately around our school] having like a food desert? What about people that live 2 miles or more away from [our school] and don't have a store or nothing near them. I think we need to help everybody. Instead of just helping [our own community].

From Project 1 to Project 2, not only did students express increasing concern about the consequences of food deserts and the need for addressing these consequences in multiple ways, they also recognized food deserts as a larger problem even outside of their own community.

DISCUSSION AND CONCLUSION

Based on his experiences working with youth on social justice mathematics investigations, Gutstein noted, "There is the issue of just how much mathematics students need to read the world. It is my experience that youth (and adults) sometimes gravitate to shallow explanations and avoid subtlety and complexity" (2003, p. 68). Findings from this study demonstrate how students' explanations of the social justice issues related to food deserts increased in subtlety and complexity from Project 1 to Project 2, but their use of mathematics to describe these social justice issues remained superficial. Naturally, social justice issues are complex, and we might except that students would struggle to make sense of social justice issues through mathematics. This case suggests that allowing students more opportunity to think deeply about social justice issues across multiple projects and across multiple classes gives students an opportunity to make and remake sense of complex social issues in more nuanced ways. Yet, students might not recognize the role that mathematics can play in supporting their understanding these social justice issues.

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CATEGORIES OF CRITICAL MATHETATICS BASED REFLECTIONS ON CLIMATE CHANGE

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In this paper, we develop a theoretical framework on categories of critical reflections relevant for mathematics education. The framework is based on key concepts from critical mathematics education associated with critical citizenship. The framework adds ideas to, and combines, previous ideas and categories of critical reflections within mathematics and mathematical modelling and their applications. The framework can increase awareness of critical perspectives when addressing climate change in mathematics education research or when planning teaching contents on climate change. It can also be used as a tool for analysing critical reflections in empirical studies where climate change is the topic.

INTRODUCTION

Mathematics plays a central role in the *description*, *prediction* and *communication* of climate change (Barwell, 2013; Barwell & Suurtamm, 2011). Climate change is for instance described quantitatively based on measurements and statistical analyses of temperature, rainfall and sea level. The prediction of climate change within climate science relies on the application of advanced mathematical models while the communication of climate change includes various texts that typically make use of graphs charts and diagrams (Barwell & Suurtamm, 2011). Mathematics has a central role in shaping understandings of what climate change is and how society can respond to it. Mathematical concepts become part of the everyday language when talking about climate change (Barwell, 2013). For instance, when we talk about the climate getting warmer or colder we are in effect talking about mathematical constructs which corresponds to some measures for changes in global average temperature. These are computed through mathematical models where choices need to be made to reformulate the topic as a mathematical problem. Uncertainty in the knowledge base for climate change may be hidden in such reformulations and mathematical concepts may contribute in narrowing what is thought of as relevant for discussing climate change. The topic of climate change thus illustrates what Skovsmose (1992) refers to as the formatting power of mathematics (Barwell, 2013).

Skovsmose argues that mathematical literacy should include abilities that move beyond calculations and formal techniques and uses the notion of *mathemacy* to cover such abilities as well. Mathemacy can be defined as “a capacity of making responses and as reading the world as being open to change” (Skovsmose, 2012, p. 94). He defines mathemacy to consist of three different competences: *mathematical, technological and reflective knowing*. Mathematical knowing refers to mathematical skills and competences, technological knowing to being capable of applying mathematics and reflective knowing to recognising the formatting power of mathematics (Skovsmose, 1994).

Facilitating reflective knowing among students, associated with climate change, could include both developing mathematics based arguments in a climate change context or reflecting on mathematics based arguments produced by others, for instance by the Intergovernmental Panel on Climate Change (IPCC) or by climate sceptics in the media. A recent scoping survey on what teachers who work with climate change in mathematics classrooms do, and why, suggests that there are several ideas on how students can work with climate data to develop mathematics based arguments and critically reflect on these (Steffensen, Hansen, & Hauge, 2016). Barwell (2013) suggests that students and teachers can use data as a way to examine political issues, and look at the role mathematics can play in both creating the climate change (e.g. by facilitation of the technology), but also constructing our understanding of climate change. There are also ideas on how to critically reflect on experts’ computations (Hansen, 2012; Hauge et al., 2015).

Working with various aspects of climate change in mathematics classrooms can be valuable for the students’ present and future critical citizenship. The notion of critical citizenship can include citizen collaboration, concerns for social justice and motivation to change society (Johnson & Morris, 2010). In mathematics education this can be related to students’ knowledge and understanding of mathematics as means for self-empowerment to (re-)organize interpretations of social institutions and traditions, and for taking justified stands in social and political reforms (Skovsmose, 1994). We consider student engagement in important issues on which citizens disagree as crucial for building capacities for critical citizenship, where students for instance explore or develop mathematics based arguments. Critical mathematics education, seen for instance in the works of D’Ambrosio (2003, 2007, 2010), has further argued that being a critical citizen also requires mathematical literacy to be able to participate actively and critically in social discussions where mathematics is used.

The call for preparing students for critical citizenship matches ideas from post-normal science. This is a philosophy of science which argues that in situations where facts are uncertain, values in dispute, stakes high and decisions urgent, there may be no well-defined scientific solution to the societal problem (Funtowicz & Ravetz, 2003). Science-related policy processes, such as climate change, should also involve those who in fact have to deal with possible impacts. This idea implies that it is beneficial that non-experts are able to critically reflect on mathematics based arguments and their limitations. Taken together, there is a need to prepare students for critical citizenship, argued by both critical mathematics education and the philosophy of post-normal science. This means greater attention is needed to what such critical reflections might be.

The aim of this paper is to develop a framework of categories of critical thinking that complex societal issues may trigger when brought into mathematics classrooms. We use climate change as a case because the issue is complex, controversial and of global concern, where argumentation in the public space is often mathematics based. The involved mathematics may be too advanced for non-experts to understand, but our stance is that in a critical citizenship perspective, non-experts may have valuable capabilities in raising important questions on the assumptions embedded in the mathematics and on the role of mathematics in defining and solving the societal problem.

We build on ideas on critical reflection from post-normal science and from critical mathematics education on mathematics, mathematical modelling and statistics. In the following, we introduce some background information on literature on critical thinking related to mathematics before we introduce the framework.

IDEAS ON CRITICAL REFLECTIONS IN MATHEMATICS EDUCATION

Skovsmose (1992) developed a framework of six reflection steps, or six groups of questions, to relate types of questions to the three types of mathematical knowing. The questions listed here are examples related to each step: (1) Have we used the algorithm in the right way? (2) Have we used the right algorithm? (3) Can we rely on the result from this algorithm? (4) Could we do without formal calculations? (5) How does the actual use of an algorithm (appropriate or not) affect a specific context? (6) Could we have performed the evaluation in other way? The first two group of questions Skovsmose connected to mathematical knowing (see above) and are often answered in either a right or wrong way. Though this kind of question can suggest a true-false ideology, they are considered important for students' development of reflective knowing. The questions in the following two steps are associated with technological knowing and deal with whether an appropriate algorithm is used when solving a specific problem. These concern the relationship between tool and task, and can involve reflections on reliability. The last two groups of question focus on whether the use of a certain algorithm may shape the understanding of a problem differently than another (Skovsmose, 1992).

There are some empirical studies where the critical reflection steps are applied as analytical tools to categorize students' critical reflections in mathematics classrooms. Hauge and co-authors (Hauge et al., 2015) apply these steps when analysing preservice teachers' reflections on a graph produced by IPCC. In order to do so, adjustments were made to the group of questions. The first two groups were changed to concern the students' reflections on whether they understood the graph and underlying algorithms and critical reflections on how others, climate scientists, had applied mathematics when producing the graph. Also differentiating between the third and the fifth step has been shown to be challenging (Hauge, 2016; Hauge et al., 2015). The third step is on reliability of the approach while the fifth is on how it affects the context of the problem. These may be more or less inseparable because the context may trigger questions on the reliability of computations. For example, the attitude on how the global society should respond to global warming depends on the perception of reliability of computations.

Within statistics education there has been an emphasis on the need to understand and critically evaluate statistics used in the media and public space. This segment of the literature offers a range of arguments on why this is important, through an epistemological stance where democratic aspects of citizenship are promoted and linked to the use and misuse of statistics and statistical concepts (see for instance Gal, 2002; Mooney, & Jansem, 2011). In addition, examples from the media has been presented to offer ideas on how teachers can work with students to get experience in reading statistics in a critical way (see for instance Watson, 2004). Common for much of this literature is that it also points to specific statistical concepts and ideas and how these can mislead a reader. Although critical thinking is highlighted, there is little attention to characteristics of critical thinking similar to that of Skovsmose (1992).

Blomhøj and Jensen (2003) describe the modelling process as six sub-processes, of which each can be associated with critical reflections:

a) Formulation of a task (more or less explicit) that guides you to identify the characteristics of the perceived reality that is to be modelled.
b) Selection of the relevant objects, relations etc. from the resulting domain of inquiry, and idealization of these in order to make possible a mathematical representation.
c) Translation of these objects to mathematics
d) Use of mathematical methods to achieve mathematical results
e) Interpretation of the results regarding the initial domain of inquiry
f) Evaluation of the validity of the model by comparison with observed or predicted data, or with theoretically based knowledge.

We expect critical reflections associated with one specific sub-process to be of a different nature than another. Particularly the processes e) and f) will be in accordance with critical reflections on the formatting power of mathematics described by Skovsmose (1992).

Another framework we find relevant for our paper is Barbosa's (2006) perspectives on educational aims of working with mathematical modelling in classrooms, building on Julie (2002). Barbosa denoted these by the metaphors 1) *modelling as vehicle*, 2) *modelling as content* and 3) *modelling as critique*. These perspectives were transformed to fit the topic of climate change in a scoping survey where teachers who work with climate change in mathematics classrooms were asked what they and their students do and why (Steffensen, Hansen, & Hauge, 2015). The term *modelling* was changed to *climate change* in the educational aims. The aim within *climate change as vehicle* was to learn mathematical ideas and concepts, the aim within *climate change as content* was to learn about the topic of climate change in itself, while the aim of *climate change as critique* was to facilitate critical reflections on mathematics based discussions about climate change. We find the framework based on Barbosa relevant because each educational aim demand different categories of critical reflections, which we address in the next section.

THE FRAMEWORK

Our framework was initiated by the educational aims suggested by Steffensen, Hansen and Hauge 2015, illustrated in Figure 1. Since we, in the present paper, are particularly interested in critical reflections, we decided to investigate what critical reflections could be associated with each of the aims in Figure 1. We consider also the overlaps interesting, coinciding with Barbosas (2006, 2009) and Hansen and Hana's (2012) findings. Then we have tried to combine this with Skovsmose's (1992) six steps, or groups of questions, and critical reflections associated with statistics and mathematical modelling, as we consider all relevant for critical reflections on climate change. The resulting framework (Table 1) is thus a meta-level framework compared to that of the scoping survey, categorizing different types of critical reflections on climate change. It consists of three main

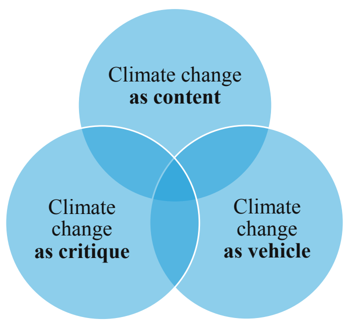

Figure 1: Visualization of different educational perspectives on climate change

categories with four overlapping categories. We have developed question groups within each category, borrowing the concept from Skovsmose's (1992) related to steps of critical reflections.

Critical reflections on *climate change as vehicle*

This category of critical reflections denotes those that are connected to the mathematics itself. Teachers may introduce the topic of climate change to learn and exercise mathematical concepts. In this case, it is valuable to reflect critically on the mathematical computations through the following question groups:

i. Are our calculations right?
ii. Are their calculations right?
iii. Was the right algorithm used?
iv. Did they use the right algorithm?
v. Have we understood the mathematical presentation?

The first and the third are the same sorts of critical reflections as Skovsmose's (1992) step i and iii. An example connected to climate change could be when students work with climate data (temperature, CO^2 emissions etc.) to learn average, spread, graphs, regression etc. Critical reflections within these group of questions, and crucial for this purpose, would be on whether the computation was correct, or whether the correct algorithm was used. Question ii and iv are similar to i and iii, except that the students consider other people's computations, for instance found in newspapers. Reflection question v concerns presentation of mathematics based arguments, as understanding a table or a graph. This is in line with what Hauge et al. (2015) found necessary in transforming Skovsmose's question i to become relevant for reflections on an IPCC graph.

Critical reflections on *climate change as critique*

The idea of this category is that working with climate change in mathematics classrooms can stimulate critical reflections on the reliability of a certain mathematical approach.

i. Is the mathematical approach reliable?
ii. Are the data or other input used in the computation of sufficient quality?
iii. Are there other ways of posing the mathematics based problem?
iv. Is the mathematics reliably presented?
v. V. Could the problem be solved without formal mathematics?

The first and last questions are the same as Skovsmose's (1992) steps three and four. We have included a question (ii) on data and other input because this is crucial for the reliability of statistical approaches and mathematical modelling. We refer to reflections on whether the mathematical problem could have been posed in another way as question iii. Questions ii and iii are in line with reflections on Blomhøj and Jensen's (2003) sub-processes b, c and d in mathematical modelling (see above). Question iv is often posed in literature on statistical literacy (i.e. Watson 1997; 2004; Gal, 2002).

Critical reflections on *climate change as content*

This area covers critical reflections which correspond to what Skovsmose's (1992) calls the formatting power of mathematics.

i. What is the role of mathematics in climate change?
ii. What are the consequences of a certain mathematics based argument for society?
iii. How can we respond to uncertainties in mathematics based arguments?
iv. What consequences are there for society when mathematics based arguments involve uncertainty?
v. How are controversies possible when arguments are mathematics based?
vi. Could we have reflected on this in another way?

The first and last questions are taken from Skovsmose's (1992) step v and vi, except that his step v, our question i, is articulated as being about climate change. The difference between question i and question ii is that question i is a more general question than ii, which refers to a certain computation of some kind. Question iii relates to reflections on how society should act on uncertainty. Because the issue of climate change is post-normal (complex, uncertain and conflicting interests), there are strong controversies. Mathematics based arguments have shown to support a range of claims, some conflicting.

Critical reflections on *climate change as vehicle/critique*

As described earlier, Skovsmose's steps are shown to be challenging to apply on recorded classroom discussions because critical reflections may seem to be in between steps or may be related to more than one step (Hauge, 2016). To address this challenge, we explore the areas which overlap the circles in Figure 1. We start with the area which covers critical reflections on climate change as vehicle and climate change as critique. We denote this overlap as critical reflections related to both the mathematics in itself and its relevance at the same time.

i. How do my choices related to data, input or defining the problem affect the results?
ii. How do their choices related to data, input or defining the problem affect the results?
iii. How does uncertainty affect the reliability of the measurement or result?

The first question is related to Skovsmose's (1992) step v, but it doesn't go all the way to reflecting on the consequences for understanding the problem, or the consequences for society. Question ii is similar to question i, except that it denotes someone else' computations. We have included the last question to differentiate between choices where the uncertainty can be discovered from exploring different choices, and actual knowledge of implied uncertainties.

The uncertainty from how a problem is posed, and its consequences, is a key issue in post-normal science, see for instance Walker et al. (2003).

Classroom activities that may foster critical reflections in this area include computations or modelling of some kind, where choices need to be made. An example could be computing the average temperature in your home town.

Critical reflections on *climate change as critique/content*

This heading we regard as involving critical aspects of mathematics based arguments in climate change that affects how climate change is understood. Such reflections concern the relevance and validity of mathematics in climate change

i. Do choices in the data, calculations or mathematical modelling affect how climate change can be understood?
ii. Would other ways of posing the problem mathematically affect how climate change can be understood?
iii. Would other ways of presenting mathematical information affect how climate change can be understood?

An example, taken from a classroom discussion on the IPCC graph on predictions of temperature change (Hauge, 2016; Hauge et al., 2015), is when one of the students comments that the idea of global warming might have been more convincing with more data back in time. Implicitly he argues that the high temperatures today would look more exceptional in a longer historical range. The student's comment could fit all questions. It could fit i as the student's topic may refer to the choice in selecting the data range, ii in that he may regard the data range as part in the posed problem, and iii in that he may refer to how mathematical information is presented in the graph. All three interpretations involve critical reflections on how choices in mathematics can affect how exceptional today's global warming is perceived.

This category of critical reflections resembles the category of critical reflections on climate change as content, but the latter reflections are rather at a meta-level of the former. For instance, the latter would be the case if the student reflected on the consequences of his suggestion for society or for decision-making.

Critical reflections on *climate change as content/vehicle*

These questions denote critical reflections in relation to learning about climate change when doing or reading mathematics.

i. How does developing my own mathematics based argument influence my understanding of climate change?
ii. How does interpreting tables, graphs or other mathematics based arguments influence my understanding of climate change?

In this case, students can collect data, work with emission time series, interpret tables or graphs, and critically reflect on what the results say about climate change. Or the students can interpret graphs etc. presented in reports or in the media.

DISCUSSION

In this paper we have developed a framework for distinguishing between various characteristics of critical mathematical reflections associated with climate change. We drew on ideas from Barbosa, Skovsmose, Blomhøj and post-normal science, which resulted in the categories of Table 1. The framework consists of three main areas, with four overlapping areas. We have not discussed the area which overlaps all three: critical reflections on *climate change as vehicle/critique/content,* but we think it is possible to imagine critical reflections sufficiently composite as to match all three criteria.

The framework adds new perspectives to categories of critical reflections, which we think are useful for understanding capacities on critical thinking in classrooms when climate change is the topic. This means that it can be used as a thinking tool both for raising awareness in research within critical mathematics education and awareness for what the teacher wishes to achieve when developing classroom activities related to climate change. In addition, the framework can be used as an analytical tool to characterize dialogues in mathematics classrooms when climate change is the topic.

The framework may be useful for a range of other topics besides climate change, but this lies outside the scope of this paper. Of course, to develop a rather detailed framework of critical reflections may be challenging as details may increase the rigidity of the categories. Also, the categories may be overlapping and confusing to apply. The framework is at its initial stage and needs careful attention in able to be developed further and examined in relation to concepts and empirical data.

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Table 1. Three main categories, and three overlapping categories, of critical reflections in relation to mathematics and climate change (seen in the headings). Each category is associated with an educational aim (second row). Groups of questions, exemplifying critical reflections, are suggested within each category.

VEHICLE	VEHICLE/ CRITIQUE	CRITIQUE	CRITIQUE/ CONTENT	CONTENT	CONTENT/ VEHICLE
Mathematics	Mathematics and relevance	Relevance of mathematics	Relevance of mathematics and climate change	Climate change	Mathematics and climate change
i. How does mathematics influence our understanding of climate change? ii. What are the consequences of a certain mathematics based argument for society? iii. How can we respond to uncertainties in mathematics based arguments? iv. What consequences are there for society when mathematics based arguments involve uncertainty? v. How are controversies possible when arguments are mathematics based? vi. Could we have reflected on this in another way?	i. How do my choices related to data, input or defining the problem affect the results? ii. How do their choices related to data, input or defining the problem affect the results? iii. How does uncertainty affect the reliability of the measurement or result?	i. Is the mathematical approach reliable? ii. Are the data or other input used in the computation of sufficient quality? iii. Are there other ways of posing the mathematics based problem? iv. Is the mathematics reliably presented? v. Could the problem be solved without formal mathematics?	i. Do choices in the data, calculations or mathematical modeling affect how climate change can be understood? ii. Would other ways of posing the problem mathematically affect how climate change can be understood? iii. Would other ways of presenting mathematical information affect how climate change can be understood?	i. What is the role of mathematics in climate change? ii. What are the consequences of a certain mathematics based argument for society? iii. How can we respond to uncertainties in mathematics based arguments? iv. What consequences are there for society when mathematics based arguments involve uncertainty? v. How are controversies possible when arguments are mathematics based? vi. Could we have reflected on this in another way?	i. How does developing my own mathematics based argument influence my understanding of climate change? ii. How does interpreting tables, graphs or other mathematics based arguments influence my understanding of climate change?

SOCIAL ASPECTS OF UNDERGRADUATE MATHEMATICS STUDENTS' LEARNING: PREPARING FOR THE COURSEWORK

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This study examines some social aspects in the undergraduate mathematics students' engagement with the preparation of their multitasked Abstract Algebra coursework. The results of this study suggest that an important element for enhancing their learning is the interchange between solitary and social learning activities. This includes studying the lecture notes and highlighting the definitions and theorems, attempting the given tasks, collaborating with peers, which allows them to improve their understanding, and getting support by their instructors. Finally, this study suggests that students follow a patterned approach during the preparation of the coursework, involving engagement with lecture notes, working in the seminars, making use of examples and communicating with other mathematicians.

INTRODUCTION

Learning mathematics in the university level is undoubtedly a multidimensional process that involves cognitive, affective, pedagogical and social aspects. This study aims to investigate university mathematics students' attempt to solve their Abstract Algebra coursework, from a social perspective. The choice of this mathematical field is grounded on its distinctive abstract nature, which students are to be engaged with for the first time in their university studies.

Research in the learning of Abstract Algebra proves significant, since novice students consider this module as one of the most demanding subjects in their syllabus. It "is the first course in which students must go beyond 'imitative behavior patterns' for mimicking the solution of a large number of variations on a small number of themes" (Dubinsky et al., 1994, p. 268). A typical first Abstract Algebra module requires a deep understanding of the abstract concepts involved, as well as the application of techniques in the preparation of coursework and final examination. An important element that causes students' difficulty with Abstract Algebra is its 'abstract' nature (Hazzan, 2001). The deductive way of teaching Abstract Algebra is unfamiliar to students and, in order to achieve mastery of the subject, it is necessary to "think selectively about its entities, paying attention to those aspects consistent with the context and ignoring those that are irrelevant" (Barbeau, 1995, p. 140).

Weber (2001) associates students' difficulty in this area of Mathematics, partly with the difficulty to construct proofs: "when left to their own devices, students usually fail to acquire optimal strategies for completing mathematical tasks and often acquire deficient ones" (Weber, 2001, p. 116). Alcock et al (2009) similarly point out that learning Abstract Algebra is challenging because of the abstract nature of its concepts and because it involves reading and writing proofs using various learning practices and beliefs. In addition, Gueudet (2008) suggests that many pedagogical issues emerging in undergraduate Mathematics Education are based on the transition from secondary to tertiary Mathematics, which can still occur after their first year. In fact, student difficulties in Abstract Algebra may be an indication of problematic transition, mainly due to the particular nature of this module (Ioannou, 2012). For the purposes of this study, I will use the Commognitive Theoretical Framework (Sfard, 2008), due to its great potential to investigate mathematical learning in both object level and meta-discursive level (Presmeg, 2016).

THEORETICAL FRAMEWORK

Commognitive Theoretical Framework (CTF) is a coherent and rigorous theory for thinking about thinking, grounded in classical Discourse Analysis (Yackel, 2009). It involves a number of different constructs such as metaphor, thinking, communication, and commognition, as a result of the link between interpersonal communication and cognitive processes, with commognition's five properties: reasoning, abstracting, objectifying, subjectifying and consciousness (Sfard, 2008). Epistemologically it is grounded on the participationist rather than the acquisitionist perspective about learning, which allows us to examine the learning process from a sociocultural viewpoint, emphasizing on the discursive characteristics (Nardi et al, 2014).

In mathematical discourse, unlike other scientific discourses, objects are discursive constructs and form part of the discourse. Mathematics is an *autopoietic system of discourse*, i.e. "a system that contains the objects of talk along with the talk itself and that grows incessantly 'from inside' when new objects are added one after another" (Sfard, 2008, p129). Moreover, CTF defines as discursive characteristics of mathematics the *word use, visual mediators, narratives*, and *routines* with their associated object-level and metalevel rules. In addition, it involves the various objects of mathematical discourse such as the *signifiers, realisations, realisation trees, primary objects* and *discursive objects*. It also involves the constructs of object-level and metadiscursive level (or metalevel) rules, along with their characteristics variability, tacitness, normativeness, flexibility and contingency (Sfard, 2008).

According to CTF, thinking "is an individualized version of (interpersonal) communicating" (Sfard, 2008, p. 81). Contrary to the acquisitionist approaches, participationists' ontological tenets propose to consider thinking as an act (not necessarily interpersonal) of communication, rather than a step primary to communication (Nardi et al., 2014). Interpersonal communication processes and cognitive processes are (different) manifestations of the same phenomenon, and therefore Sfard (2008) combines the terms cognition and communication producing the new terms *commognition* and *commognitive*.

Sfard (2008) identifies the commognitive capacities that depend on the human ability to rise to higher commognitive levels and involve an "incessant interplay between utterances and utterances-on-former utterances" (Sfard, 2008, p110). These capacities fall into two distinct categories: those related to commognitive objects (i.e. reasoning, abstracting and objectifying), and those who consider the thinkers or speakers, namely the commognitive subjects (i.e. subjectifying and consciousness).

Mathematical discourse involves certain objects of different categories and characteristics. *Primary object* (p-object) is defined as "any perceptually accessible entity existing independently of human discourses, and this includes the things we can see and touch (material objects, pictures) as well as those that can only be heard (sounds)" (Sfard, 2008, p. 169). *Simple discursive objects* (simple d-objects) "arise in the process of proper naming (baptizing): assigning a noun or other noun-like symbolic artefact to a specific primary object. In this process, a pair <noun or pronoun, specific primary object> is created. The first element of the pair, the signifier, can now be used in communication about the other object in the pair, which counts as the signifier's only realization". *Compound discursive objects* (d-objects) arise by "according a noun or pronoun to extant objects, either discursive or primary."

The (discursive) object signified by S in a given discourse is defined as "the realization tree of S within this discourse." (Sfard, 2008, p. 166) The realization tree is a "hierarchically organized set of all the realizations of the given signifier, together with the realizations of these realizations, as well as the realizations of these latter realizations and so forth" (Sfard, 2008, p. 300). Realisation trees and consequently mathematical objects are personal constructs, although they emerge from public discourses that support certain types of such trees. Additionally, realisation trees offer valuable information regarding the given individual's discourse. Moving with dexterity from one realisation to another is the essence of mathematical problem solving. Realisation trees are a personal construction, which may be exceptionally 'situated' and easily influenced by external influences such as the interlocutors. Finally, signifiers can be realised by different interlocutors in different ways, according to their own specific needs (Sfard, 2008).

The epistemological tenet of CTF described in the last sentence is cardinal in its development as theoretical framework. Due to this tenet Sfard (2008) describes two distinct categories of learning, namely the *object-level* and the *metalevel learning*. Moreover, according to Sfard (2008, p. 253), "object-level learning [...] expresses itself in the expansion of the existing discourse attained through extending a vocabulary, constructing new routines, and producing new endorsed narratives; this learning, therefore results in endogenous expansion of the discourse". In addition, "metalevel learning, which involves changes in metarules of the discourse [...] is usually related to exogenous change in discourse. This change means that some familiar tasks, such as, say, defining a word or identifying geometric figures, will now be done in a different, unfamiliar way and that certain familiar words will change their uses" (Sfard, 2008, p. 254).

Finally, CTF has proved particularly appropriate for the purposes of this study, since, as Presmeg (2016, p. 423) suggests, it is a theoretical framework of unrealised potential, designed to consider not only issues of teaching and learning of mathematics per se, but to investigate "the entire fabric of human development and what it means to be human." Moreover, CTF gives the possibility to the researcher to investigate not only cognitive but also social aspects of learning, as this study aims to do so.

METHODOLOGY

This study is part of a larger research project, which conducted a close examination of Year 2 mathematics students' conceptual difficulties and the emerging learning and communicational aspects in their first encounter with Abstract Algebra. The module was taught in a research-intensive mathematics department in the United Kingdom, in the spring semester of a recent academic year.

The Abstract Algebra (Group Theory and Ring Theory) module was mandatory for Year 2 mathematics undergraduate students, and a total of 78 students attended it. The module was spread over 10 weeks, with 20 one-hour lectures and three cycles of seminars in weeks 3, 6 and 10 of the semester. The role of the seminars was mainly to support the students with their coursework. There were 4 seminar groups, and the sessions were each facilitated by a seminar leader, a full-time faculty member of the school, and a seminar assistant, who was a doctorate student in the mathematics department. All members of the teaching team were pure mathematicians.

The lectures consisted largely of exposition by the lecturer, a very experienced pure mathematician, and there was not much interaction between the lecturer and the students. During the lecture, the lecturer wrote self-contained notes on the blackboard, while commenting orally at the same time. Usually, he wrote on the blackboard without looking at his handwritten notes. In the seminars, the students were supposed to work on problem sheets, which were usually distributed to the students a week before the seminars. The students had the opportunity to ask the seminar leaders and assistants about anything they had a problem with and to receive help. The module assessment was predominantly exam-based (80%). In addition, the students had to hand in a threefold piece of coursework (20%) by the end of the semester.

The gathered data includes the following: Lecture observation field notes, lecture notes (notes of the lecturer as given on the blackboard), audio-recordings of the 20 lectures, audio-recordings of the 21 seminars, 39 student interviews (13 volunteers who gave 3 interviews each), 15 members of staff's interviews (5 members of staff, namely the lecturer, two seminar leaders and two seminar assistants, who gave 3 interviews each), student coursework, markers' comments on student coursework, and student examination scripts. Moreover, for the purposes of this study, the collected data of the 13 volunteers has been scrutinised, with main focus on the student interviews. The interviews, which covered a wide spectrum of themes, were fully transcribed, and analysed with comments regarding the mood, voice tone, emotions and attitudes, or incidents of laughter, long pauses etc., following the principles of Grounded Theory, and leading to the "Annotated Interview Transcriptions", where the researcher highlighted certain phrases or even parts of the dialogues that were related to a particular theme. Naturally, all sources of data have been appropriately analysed, and the conclusions of the data analysis have been triangulated; yet due to limited space, only a limited number of excerpts from the students' coursework are demonstrated. The grounded-theoretic approach has been adopted, since the original research project had adopted an exploratory approach, regarding the investigation of the learning of Abstract Algebra.

Finally, all emerging ethical issues during the data collection and analysis, namely, issues of power, equal opportunities for participation, right to withdraw, procedures of complain, confidentiality, anonymity, participant consent, sensitive issues in interviews, etc., have been addressed accordingly.

DATA ANALYSIS

The following discussion on the strategies undergraduate mathematics students apply to solve the given tasks in the context of their coursework reveals combination of solitary and social activities, which aim to enhance their learning and performance. Data analysis suggests that, more often than not, students initiate the process of coursework solution by reading and condensing the lecture notes, by making record cards, shorter notes or highlighted notes. Students A and B have adopted this approach in a similar way, as the following excerpts suggest.

> I always write up my coursework notes, like I've got them on big A4 plain sheets and then with blocks of colour, cos I just can see them better and then I can find the definitions quicker, so I wrote up all my lecture notes like that, and then I used them to go through and find... the definitions, or –whatever in the question and try and pick the question apart. And then mark down if we've got any hints, I try and use that, and then try and work it through. Student A

> Well I read through the lecture notes a few times then I generally shorten them –which is a weird thing– I actually cut them down to notes to note and then I sometimes cut them down again so then I have got a really short key points and then I will basically learn them and then I will look at it... Student B

As the above excerpts suggest, in the newly introduced discourse of Abstract Algebra, students need to identify the important mathematical narratives (definitions, theorems) that will allow them to construct their new realisations and realisation trees of previously (in different mathematical discourses) introduced d-objects such as set, subset, isomorphism, etc. in the new context of Abstract Algebra. Additionally, focusing on the newly introduced routines (in the forms of proof or example) will allow them to enhance their metalevel understanding in the context of this new mathematical discourse. The need to identify and familiarise themselves with these notions is possibly more imperative due to the abstract nature of the particular module (Hazzan, 2001; Alcock et al, 2009)

According to the data analysis, the majority of students highlight the benefit of studying the lecture notes and attempting the coursework tasks, before any social interaction in the seminars. This is view is overtly expressed in the following excerpt.

> Yeah, cos if you haven't prepared, and you go in there, then you're wasting time, you can't ask them anything, cos you haven't got anything... You haven't done anything... Student C

Attempting the tasks beforehand allows the students to identify the gaps in their object-level understanding, regarding the newly introduced d-objects in the context of Abstract Algebra. The attempt of the mathematical tasks solitarily, will allow the commognitive conflicts (hurdles occurring due to the introduction of a new (to them) mathematical discourse) to emerge (for instance the treat of a group as a set, ignoring the role of the operation). Moreover, students will be able to identify these conflicts and will have the opportunity to resolve them through communication with the seminar staff. This is indeed a very beneficial practice for the undergraduate students since otherwise they cannot take full advantage of the opportunity to share their reasoning, expose their difficulties (expected in the construction of new realisations of the same signifiers, e.g. set with operation is now realised in the form of a group), and improve their understanding of the metalevel rules that govern the routines for solving the given tasks.

Social interaction and communication between novice and experienced mathematicians proves essential in the learning environment of university. The praxis of seminars is another characteristic of university mathematics education, which students need to accommodate and mostly make use of. Failure to do so demonstrates difficulty in transiting to the university education norms as well as delimits students to the full possibilities that have to enhance their object-level but most importantly their metalevel mathematical learning (in the sense of learning new mathematical norms and practices, such as proof). This last conjecture is in agreement with Weber (2001) and Gueudet (2008).

Data analysis suggests that there is a rather patterned way in which undergraduate students approach the solution of mathematical tasks in the context of coursework. The first step is the lecture note taking in the context of lecture, followed by an individual revisit of these notes at home. More often than not students condense these notes either by highlighting the main results or by preparing short reference notes. A fourth step includes the participation in the seminar, during which students have the opportunity to get valuable guidance, in order to proceed in their first engagement with the coursework. Usually students after their first attempt to solve the coursework, decide to revisit the examples in the lecture notes, and then they revisit the coursework with a new perspective. Occasionally many students require further guidance, in order to be able to finalise their coursework. The aforementioned process is represented in Figure 1.

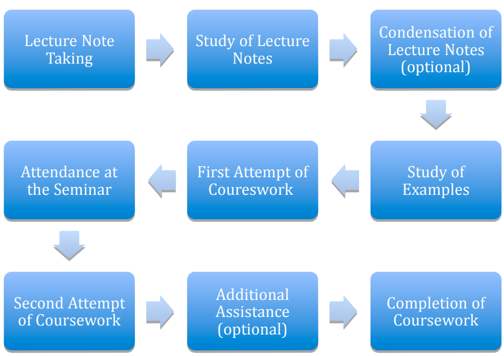

Figure 1: Coursework Preparation Process

A particular pedagogical practice applied regarding the coursework tasks was that students were given a set of 8-10 mathematical tasks that needed to attempt before seminar, yet only 3-4 were going to be assessed. The assessed tasks were announced to them after the seminar sessions. According to data analysis, this practice was not well received by the students, as this view is expressed it the following excerpt.

> Oh, this one stresses me out! There's a lot of questions to do, and I always tend to find – because obviously you go to seminars and they launch questions and it's – and then – I always find that like I end up doing the ones that aren't the actual coursework questions, and it's so annoying, because it's like oh, I've put all this effort into them... and it's not wasted, because it's good practice anyway, but – sometimes I prefer to know which my questions are, and then I can save the other ones for nearer the exams, to practice? Student D

Although Student D realises the benefit of attempting all the questions of the coursework, she prefers to work on the assessed ones. Not knowing the assessed exercises possibly encourages students to attempt all the questions without exception and therefore widens their experience in different tasks, achieving in this way broader object-level understanding and variety in the contexts in which metarules are applied. Accepting this learning technique would require a mature approach towards learning from the students' viewpoint. Knowing though the exact questions beforehand would possibly allow students to take more advantage of the seminar assistance, and focus, at this stage of their learning, on the assessed ones. It would, perhaps, enhance the effectiveness of social interaction between novice and experienced mathematicians, by discussing the commognitive conflicts that need to be 'urgently' addressed.

Finally, Student B has raised the role of the syntax of mathematical narratives with which the mathematical tasks are expressed in the coursework, and the impact that this might have on novice students' learning and guidance to the process of solution. In particular, he expresses an interesting view about the role of language as part of effective communication between experienced and novice mathematicians, in the social context of university mathematics seminar.

> I think the coursework puts you off really [...] this subject has been vicious and horrible, but how they phrase the coursework you are more inclined to go into it and start adapt or you just look at it and you can try and don't want to start trying it as such... you have to force yourself to go and try... but some of the other ones which I haven't liked I have quite happily started trying them even if I don't like them, because of how they phrase the question or they break it up more so the more they break it up I feel like it's more bite. Obviously the questions further down the page they do it all in one bit but at the beginning they sort of bite size it which is kind of I find really useful because it gets your brain into the mode. Student B

The above excerpt leads to the conclusion that well phrased questions, from the students' perspective, are considered to be the ones that are constructed in a way that each step of the solution is linked to one subsection of the question, guiding the students towards the application of the particular object-level and metalevel rules and moreover the correct direction for solving the question, but also encouraging them to attempt it. In agreement with Weber (2001) and Alcock et al (2009), the last view expresses the urge of students (novice mathematicians) to communicate in an effective way with the experienced ones and adjust effectively in the social norms of mathematical community. Ioannou (2012) suggests that the need of students to enhance their mathematical communication and understanding, very often is achieved through learning interaction with their peers who often face the same difficulties and 'speak the same language'.

In sum, this study has focused on certain social aspects related to strategies that undergraduate mathematics students have adopted for the preparation of the coursework, as well as some other interesting perspectives. The interchange between solitary and group activities demonstrates the importance of the social aspect of mathematical learning in the university level. Summarising the lecture notes and highlighting the important mathematical narratives, namely definitions, lemmas, and theorems, is a practice that occurred often in the student interviews. In addition, and according to students that do so, it contributes favourably in their object-level learning and facilitates the solution of the coursework. Many students have expressed their belief about the importance of attending the seminars in the process of solution of the coursework and share their difficulties with the other members of the local mathematical community. Four students wished to know the assessed questions before going to the seminar, although they expressed their concern about the possibly negative effect this might have in their mathematical learning. Finally, the way coursework questions are phrased is conjectured to affect students' performance.

DISCUSSION

In agreement with Dubinsky et al (1994), and Alcock et al (2009) the learning of Abstract Algebra is an arduous task for novice students, not only from a cognitive, but also from a pedagogical perspective (Ioannou, 2012). Mathematical proof in the context of Abstract Algebra is particularly demanding and requires successful application of certain learning skills (in agreement with Weber, 2001). This study suggests that regarding the strategies for the preparation of the coursework, there has emerged that summarizing the lecture notes and highlighting the important mathematical narratives such as the definitions of the involved d-objects and the related theorems and lemmas who describe the respective object-level rules, is a solitary yet important first step for many students in the preparation of the coursework. This method possibly allows the students who apply it to improve their object-level learning as well as make more practical and efficient notes that allows them better grasp of the mathematical notions that need to use.

In addition, this study suggests that most of the students follow a rather patterned approach towards the completion of their coursework. Namely, after lecture note taking during the lectures, students revisit these notes individually and, more often than not, they condense them. Later on they attend the seminars, during which they have the opportunity to get valuable guidance, in order to attempt the coursework for the first time. Usually students after their first attempt to solve the coursework, they decide to revisit the examples in the lecture notes, and then they revisit the coursework with a new perspective. Occasionally, many students require further guidance, in order to be able to finalise their coursework.

The aforementioned process allows students to overcome any disengagement with the object-level and metalevel rules related with the d-objects of Abstract Algebra, and also identify any commognitive conflicts that may possibly emerge. In agreement with Weber (2001), collaboration with experienced or other novice mathematicians allows students to overcome these problems. Not doing so, students will have significantly more limited opportunities to get effective assistance, expose their difficulties, and improve their mathematical knowledge and capabilities.

An emerging pedagogical implication stemming from the above discussion would be that, regarding the coursework, it would be more beneficial for the students to hand in smaller pieces of coursework spreading throughout the semester, and perhaps after each cycle of seminars instead of submitting one long piece of coursework at the end of the semester. In this way, students could get their solutions (and the model solutions) for each coursework before handing in the next one. In this way, they would have the chance to reflect on the previous coursework, pinpoint their mistakes and localise and improve their mathematical learning.

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GAMIFICATION, STANDARDS AND SURVEILLANCE IN MATHEMATICS EDUCATION: AN ILLUSTRATIVE EXAMPLE

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There has been an impressive growth in the number of online games and apps for mobile devices, which attempt to integrate school type mathematics tasks into game environments. These are based on a pedagogic tactics that is often referred to as "gamification". This paper looks at gamification from a perspective that draws on Foucault's ideas of surveillance and normalisation. It explores the pedagogic discourses promoted by gamification, how the mathematics learner and teacher are constituted, and how records of students' conduct and performance may potentially be incorporated into larger infrastructures of accountability. One example of a "serious game" is analysed in detail with a focus on how the game regulates student players, what forms of assistance for self-discipline or self-awareness are promoted, and what micro-strategies of normalisation may emerge.

GAMIFICATION

Surveillance and normalisation are central in education. School mathematics discourse that is based on curriculum standards inevitably attempts to categorise the learners in terms of their performance in relation to what "normally" is expected, which creates both, criteria for what it means to be successful in mathematics and students who achieve below and above a minimum standard or at a range of levels. As then there are always students with low marks, grades, or test scores as well as a range of "truths" for explaining the reasons for their low attainment, there are also a range of pedagogic tactics for overcoming what for instance is framed as "lack of engagement", "emotional disturbance", "behavioural problems", or "lack of scholastic aptitude". What has become known under the label "gamification" is a comparatively recent example of such tactics.

Gamification strategies for developing so called "serious games" (Ulicsak and Wright, 2010) are based on tracing data produced by the user that are reworked into feedback (that may include numbers and diagrams) in order to modify their action. The production of this feedback may include voluntarily or unwittingly contributing their data into connected databases, which can be searched for patterns within groups or populations (McRae, 2013; Zuboff, 2013a). Gamification combines this mode of surveillance with design features and mechanisms taken from computer games, such as 3-D virtual environments seen from a first-person "shooter-perspective", surprises to increase attention, fast-paced game environments that demand quick responses, beeping sounds, partly self-designed avatars, an overall narrative about a mission to be accomplished, level-ups, and badges (DeBurr, 2013; Kebritchi, 2008; Whitson, 2013).

Gamification includes the introduction of a "token economy". Intervention programmes based on a token economy were developed in the 1960s and disseminated in the 1970s for disciplining psychiatric patients, prisoners, juvenile delinquents, or "disadvantaged" primary school children (Kazdin, 1982). These interventions make use of the idea of operant and conditional reinforcement; accordingly, research articles include reports about the successful functioning of token economies with rats (Malagodi, 1967) or chimpanzees (Sousa & Matsuzawa, 2001), where exchangeable tokens and food were comparable in reinforcing behaviour. In hospitals or educational institutions the reward consists of the opportunity to engage in activities that are chosen by many when freely allowed, with the tokens used to bridge the delay between the adaptive behaviours for functioning in the institution and that opportunity. Subjects exposed to this treatment are expected to "purchase" the activity with their tokens and consequently to engage in token-earning behaviours (Kazdin, 1982). This "economy", however, where the tokens may be interpreted as wages for the labour of engaging in adaptive behaviour, is obviously restricted as there is no choice on the side of the presumed pleasure-seeking "consumer" who pays for activities they are already entitled to pursue in the institution.

In contrast to these examples of token economies, in gamification playing the game itself functions as the activity assumed to be chosen over others when freely allowed; and the collected tokens (such as points and level-ups) are exchanged against other rewards only in the virtual setting. If aimed at controlling the acquisition of skills, "cascading information" by de-composition into a series of steps, which is one aspect of the gamification tactics, is clearly inherited from behaviouristic learning theories in addition to the use of quests and reinforcement. In companies, gamification is employed for increasing efficiency by means of controlling the emotions of employees, such as the motivation to work overtime via engagement in multiplayer games for training purposes or finding problem solutions under surveillance of the management. The absence of "punishments" as well "key performance indicators" in relation to an overall task function as techniques of "disciplinary power" (in the sense of Foucault, 1977), often visualised as percentages, traffic light colours or charts.

The features described above are also included in the gamification of acquisition of skills in educational contexts, as for example in online games and apps for mobile devices, which attempt to integrate school type mathematics tasks into game environments. The idea of course is not new as the use of computer or calculator "games" in school mathematics has already been promoted in the 1970s. The concomitant knowledge discourses concentrate on their benefits with a focus on the games' production of favourable affects, such as increased motivation and positive attitudes (Bragg, 2007). In addition, playing mathematics-related games has been reported to affect some achievement measures (e.g., Kebritchi et al., 2010, and Kolovu et al., 2013, cited in van den Heuvel-Panhuizen, Kolovou, & Robitzsch, 2013).

While unmediated surveillance is essential in classroom practice (e.g., Walshaw, 2010), gamified surveillance tools often provide functions for (hidden) observation and record-taking of students' conduct and performance and their potential incorporation into larger infrastructures of accountability. In Foucault's (1994) conception of surveillance, the possibility of observing others only implies power if they "use their own presence in order to watch over others" (pp. 349–364). The former may not be the case when students work on-line and access different sites, even if they are in the same physical space. Advancement in technology, however, offers "opportunities to monitor students while working online" (van den Heuvel-Panhuizen et al., 2013, p. 285). Not only student performance data during online gaming can be surveilled and stored, but also process aspects of their behaviour.

The example analysed in this paper is typical with regard most of the features that are central in gamification tactics. It is designed for primary school teachers, students, and their parents. Notably, gamified mathematics pedagogy is not restricted to (primary) school. Kallweit and Griese (2014) present an example that has been used with first year engineering students, who are portrayed as lacking the ability to autonomously organise their (mathematics) studies.

EXAMPLE: "LEARNING ENGINE"

The label 'Learning Engine' is part of the subtitle of a site entitled 'Sumdog', which contains 'games' and a 'progress hub': "Our new learning engine for the National Curriculum in England, Years 1–9"

The analysis of this example was part of an earlier presentation (Jablonka & Bergsten, 2016). As empirical data we used information about the features of the tools and associated knowledge discourses, such as promotion videos, 'witnesses' (teachers or students) and texts aiming at an academic audience, if available. We also used our reading of the texts (in the widest sense) this game produced while we were engaging with it in the role of students and teacher. We interacted with the free version of the site by setting up a 'school-class' with three students and a teacher.

Promotion discourse: fluency, achievement reports and happy students

School mathematics is configured as a set of skills, in which students acquire "fluency". The system is portrayed as an agent that can read the traces produced by each individual student-player during the games. The system is also said to be able to rework these traces into numbers and display speed, accuracy and time spent, "proficiency charts", "improvement charts", "maths reports" and "diagnostic test reports", numbers for "top five students", and "class reports" in relation to progression in standards:

"It [the system] gets to know each student, leading them through the standards, and reports their progress to their teachers", "building a precise picture of their fluency", "skill by skill, and day by day" and "can tell that the whole skill is mastered."

In addition to a teacher being eager to get quantitative reports about each student's 'progress' with regard to 'standards', the system also suggests a teacher who is interested in live-surveillance, as they can log in "while your students play, and you'll see their scores live on your dashboard. The live data is great for pinpointing students who are working well, or those who need a little help."

Another promotion topic is the production of happy students: "We turn fun into fluency, Happy students learn more". Due to absence of direct teacher surveillance "students have a great time –but their teachers retain control over their work" while they perform an activity called "play" or "game" or "diagnostic testing…while students play".

In addressing the students, the fun is attached to making it "easy": "Every few games, to keep things fun, you'll take a break, and revise skills you've already mastered." Furthermore, there is a 'natural' category of student who invests effort, which apparently is a moral virtue to be rewarded, but does not necessarily lead to progress: "For the first time, we're rewarding progress as well as effort. Click your pet to see the tricks it has learned, and then choose one to play it."

Practices and discourses within the games

To proceed in a game the player has to answer multiple-choice mathematics questions that suddenly appear on the screen. The games, however, point at a mix of different practices and discourses at different levels of interaction with the site.

Computer or console games. The games' opening images and names (e.g., cake monsters, pop tune, junk pile, soccer, submarine, dress town) hint at their apparent main action. Some (only superficially) resemble some simulation of an activity outside the game; most scenarios are fantasy.

Each player battles in real time three other players currently inside, either from the class or the "world", or plays against a selected robot at a particular mastery level (e.g., "challenger" or "destroyer"). One feature these games share with multiplayer online-games is that players can select and partly compose (gender, skin colour, hair style, etc.) an avatar in the form of an image of a person. The players cannot, however, choose their name as these are set by the teacher and are displayed beside the avatar image, in addition to their school name.

Selection of the correct answer is rewarded with a 'coin' across all games and in many games linked to a repetitive action that relates to the title of the game (e.g., flicking a ball into a goal, balancing junk falling from the sky on a growing pile, feeding monsters with pieces of a raising cake). These skills contribute to the game-score. There is game-style background music and sounds evaluating the skills in the activity (e.g., making a goal or not when flicking a ball towards in the game 'soccer'). Furthermore, there is some element of chance responsible for variation in the fluid animation of the activity and occasionally there comes a bonus (e.g., an extra kick etc.). After each game, a ranking of all four players appears. The player can also look at their own ranking in relation to the scores achieved by the friends' best, class best and the world's best.

The player's action does not change any part of the unfolding scene except the invisible 'level' of the mathematics tasks presented and does not have any bearing on the other players' course of action. Given this rather closed nature of the games, there is not much room for freedom of inventing interesting ways of engaging with the scene. The activity establishes children as more or less proficient individual computer game players, who find some pleasure in accumulating 'coins' and enjoying the sounds, badges, scores, and images that associate appreciative evaluation of a mastery of isolated arbitrary repetitive skills, such as shooting monsters, balancing a pile, flicking a ball, and hitting the goal. The children are in competition with a virtual community of changing school students or their classmates who play the same game and against whom they will be ranked, with a new chance in each game to be ranked first.

School mathematics and diagnostic testing. The mathematics tasks appear on the screen in written form as multiple-choice (four alternatives) in front of the animated scene (mostly top or bottom of page). The type of tasks changes in relation to the number of correct answers selected in previous tasks, clearly recognisable as ranging from recognition of number words and small set cardinality, basic arithmetic and geometry, to elementary algebra and reading simple representations of statistical data. The tasks were mainly about procedures (some quite technical –such as selecting the correct long division); only very few included some interpretation (e.g., place value, comparing fractions, simple 'word problems').

The player initially plays a couple of games to enable the identification of a 'level', if the teacher has chosen this feature. The machine announces, "We are finding your level" and some animals in a still image explain in speech balloons that the students should play games as they always do, promise a pet and a free picking of skills to work on as soon as the test is finished; the animals also advice to guess if you do not know the answer. It is also stated (in a smaller font at the bottom of the page), "teachers and parents can return the test if your level is wrong". When playing in training mode, the system occasionally also gave commands, such as "Eva: keep practicing your tables. See your progress here", or "Congratulations, you have finished a skill." Indeed, one can look up visualisations of the number of correct answers ('progress') in a range of mathematics skills.

Here each child is constructed as being on a level associated with an examination about answering the mathematics tasks, a level that remains hidden to them but can be checked by their teachers or parents. Their examination outcome is derived from training and answering multiple-choice questions as quickly as possible under distractions. The interaction with the system establishes a relation with the computer-examiner that produces reports independently from one's teacher (although with their teacher/parent still keeping some authority).

Earning, shopping and trading in a token economy. The tokens are virtual coins earned for correct solutions of mathematics tasks. Accumulated coins translate into the player's rank name (a species' name). This rank is independent from the curriculum mathematics 'level' identified by the system at which one is made to play. These coins can be used for buying furniture for the room the avatar inhabits, outfit for the avatar, or gifts for a player-friend. Bought items can be resold for a given lower price. This more stable rank constructs a child-consumer with a level of wealth achieved by effort, with wealth of other players being visible through their rank-name. For example, a player at level-1 is a Common Rat; the skilled player proceeds in order of decreasing estimated population of the name-giving species towards a the highest rank of Baiji (level-31). When searching the web, one also finds an informal community of hackers and cheaters who propose tactics for quickly maximising coins in the game.

Social media. One can join a family (when parents sign up) and be-/defriend other players; yet, one cannot communicate with them. From the system one receives messages that congratulate to a "level-up" regarding the wealth-rank.

Normalisation and surveillance: double standards and wishful identification

In the game there are two layers of competition and normalisation, one open and visible and the other covert. The first is derived from the gaming actions and the second from solving the mathematics tasks. Hence the game establishes two categories of player: a child-player and a student-player.

For the child-player there are fluid relative categories in terms of the ranks achieved in the games from defeating other players and the levels-up from accumulating coin tokens. But these are independent of the invisible mathematics curriculum 'level' and the performance profiles of the student-player, only visible to the teacher or parent. As the child-player performs the avatar's mathematical action as a student identifiable by the teacher, the mathematical game-performance and associated subject position is not short-lived, but incorporated into school performance, and becomes part of it.

The avatar may provide an escape exactly from a 'real' low level reported to the teacher offering the vicarious experience or wishful identification with the wealth earned by extensive playing with a high number of correct solutions, even when being at a low 'real' mathematics skills level. The construction of the mathematics student-player works via the invisible ranker's eye on their own and the other players' mathematical skills.

Both as a child-player and student-player they are objectified as sources of data for the calculation of performance profiles, because in the game there are no choices that influence the outcomes or actions of the other players other than correctly answering the tasks, which pop up in an order to the principles of which the player has no access.

Naturally, it is difficult to know the nature of the children's awareness of being surveilled. When clicking 'parents/teachers' they can read that the page is for teachers and parents only and they will not be able to "use the tools on this page". But they can still see the categories (Live Data, Assessments, Contests, Reports). In addition, there is an example for teachers and parents on 'live Accuracy Data', which shows a table with student names and percentages ranked in decreasing order, some with green and other with red dots.

DISCUSSION

As illustrated by means of the example, the 'gamification' of mathematics skills acquisition creates a hybrid of a range of practices and discourses. The game intends to use seduction as a tactics for controlling the emotions of the child-player in order to regulate the student-player's allegedly unpleasant mathematical activities. This discourse conditions a particular way in which fun is related to school, or 'playing' a multiplayer online-game to 'learning mathematics'. Individual competition within a logic of outdoing of one's own previous performance and that of others in terms of quantitative metrics (speed, number of tasks, coins, levels) is the basic principle for the construction of the "game", which appears strongly regulated. The gamification tactics, however, follows the same selling point in establishing an opposition to something that is not fun as found in other pedagogic discourses that focus on "play". Yet, gamification has little to do with the pedagogic discourse Bernstein (2000) saw associated with play, based on the re-emerging liberal romantic philosophy of education and developmental psychology, which he referred to as competence model (as opposed to a performance model). For the gamification in the learning engine is not based on psychological theories of cognitive development, but on behaviouristic psychology. Despite the apparent emphasis on the subject as self-regulating, the focus is on absences of performance in relation to standards and the requirements for accountability. This and similar games may be seen as a development of "Learning Machines", such as those by Pressey (1926) and Skinner (1954). In contrast to these early attempts of constructing learning machines, however, an explicit management of feelings, motives, and intentions is included in this new surveillance.

As McRae (2013, n.p.) observes, a new generation of more sophisticated adaptive learning systems "still promote the notion of the isolated individual, in front of a technology platform, being delivered concrete and sequential content for mastery". According to McRae, this type of platforms not only provokes a revival of behaviourism in education but also facilitates data accumulation by large corporations involved in their development. "At its most sinister, it establishes children as measurable commodities to be cataloged and capitalized upon by corporations" (McRae, 2013, n.p.). This development reflects what Zuboff (2013b) refers to as surveillance capitalism.

By the machine's calculations, the game-performance of a range of mathematics skills is transformed. In contrast to classical 'performance models' (in Bernstein's 2000), the teachers cannot easily read or interpret the performance without the machine. The categories for students at different 'levels' are initially empty. Only with the help of the machine the 'levels' are interpreted in qualitative terms in a discourse about progress in standards, which Llewelyn (2015) sees in the discursive space of educational policy as producing the 'normal' mathematical child as a functional automaton. Neither the players nor the teacher are aware how mathematical relations materialise in the machine and have been used as a resource for programming the game and for producing reports.

Whitson (2013) argues that with the aid of new surveillance technologies which enable recording and linking of different bodies of big data, 'gamification' fosters a "quantification" of the care of the self (Foucault, 1988) "enabled by increased levels of surveillance (self-monitoring and otherwise)" in projects that use "incentivisation and pleasure rather than risk and fear to shape desired behaviours" (p. 167). But as argued above, the forms of incentives and reinforcement used by gamification tactics conceive the subject in entirely behaviouristic terms. Furthermore, as illustrated by the example of the "learning engine", the accumulated data about the individual student-player remains largely hidden from them as child-player but are subjected to the displaced teacher's gaze. This constellation then is a panopticon without disciplinary power (in the sense of Foucault, 1977), and dissociated from the direct control of space and time.

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'SETS 4 AND 5 WERE STUFFED FULL OF PUPIL PREMIUM[1] KIDS': TWO TEACHERS EXPERIENCES OF 'ABILITY' GROUPING

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Nowadays in England 'ability' grouping is taken as a given in most English secondary schools and increasingly in many primary schools. However the overwhelming mass of research in this field shows that 'ability' grouping always disadvantages somebody, invariably from the less privileged sections of society. Many teachers care about social justice issues but there are only a few who are not prepared to accept the 'common sense' notion of 'ability' and are prepared actively to resist, even to the extent of making their own lives less comfortable. In this paper I consider the stories of two such teachers and ask: can such stories help in advancing a social justice agenda?

INTRODUCTION

In this paper I begin by defining social justice in mathematics education. I argue that mathematics education as organised in most secondary schools in England is incompatible with this conception of social justice and further that one of the most inequitable organisational strategies is the grouping of children by 'ability'. 'Ability' grouping in mathematics classrooms in England is now rarely challenged. Despite this there are still a few mathematics teachers who are prepared to resist the current orthodoxy. In this paper I introduce you to two teachers, Bob and Sarah[3], who are passionately committed to social justice, so much so, that they are prepared to move many miles in search of schools which do not group children by 'ability'. I end with a short discussion.

1. Pupil premium is extra funding allocated to children who are looked after by the local authority, those who have been eligible for free school meals at any point in the last six years and for children whose parents are currently serving in the armed forces. Schools can choose how to use this additional funding.

2. I use the vocabulary of 'mixed-attainment' in this paper because this is the language used by Bob and Sarah rather than the more common 'mixed ability' to avoid endorsing so-called 'ability thinking (see, for example, Boylan and Povey, 2014). In other papers I use 'all attainment' instead of 'mixed-attainment'.

3. Bob and Sarah are pseudonyms.

SOCIAL JUSTICE AND MATHEMATICS

Social justice is a contested concept. One of the main thinkers on social justice in the last hundred years is John Rawls whose major work, *A Theory of Justice* (1999), is based on the concept of justice as fairness. However Sen argues that Rawls' theory is not based sufficiently on reality, that Rawls is seeking ideal justice. Sen has developed an alternative theory in *The Idea of Justice* (2009) which is based on a social choice model and can operate even in conditions where not all information is available. Others argue that a theory of justice must say something about the choices on offer; whatever freedom we have must enable us to pursue whatever we are able to choose reflecting the various human positions and interests (Bird-Pollan, 2010; Gray, 2009).

In the current neo-liberal climate the 'common sense' concepts of social justice as understood by most people in a liberal, social democratic society are portrayed as not politically feasible including in the field of education. Unsurprisingly the educational discourse in Britain is not impervious to neo-liberalism and consequently the hegemonic ideas of the dominant class in society are now seen as 'common sense' (Bambery, 2006). Schools are a particular locus of power (Apple, 1995) and as such are a focus for social justice issues. Reay (2012), drawing on speeches by the then Secretary of State for Education, suggests that

> the vision of a socially just education system that is currently dominant is the dystopian vision of the Right in which the economic ends of education are transcendent and competitive individualism is seen to be a virtue. (p 589)

Qualifications in mathematics are used as a filter for access to an increase in life chances both in the United Kingdom and the wider world (Martin, Gholson, and Leonard 2010, Stinson 2004, Woodrow 2001, Noyes 2009); and mathematics teaching in England and indeed in much of the world is inherently conservative reflecting a western view of the world (Gutierrez 2002).

Gramsci argued that what was needed was not common sense but 'good sense' (Bambery, 2006; Gencarella, 2010). I argue for a conception of social justice in mathematics education that is best expressed by Reay (2012) writing about Tawney

> In Tawney's terms a socially just educational system is one in which a nation secures educationally for all children 'what a wise parent would desire for his own children' (p. 590).

In contrast to the neo-liberal view of social justice and education Reay, drawing on Tawney, says a socially just education is communitarian, is not about increasing social mobility but is an end in itself enabling people to be who they are rather than someone they should aspire to be. It rejects the special privileges of the upper and middle classes as 'the liberty of the working classes depend[s] on the restraint of the middle and upper classes'. It rejects the neo-liberal language of choice as the choices for the working classes are those left by the upper and middle classes. Reay thinks that the current orthodoxy should be turned on its head and, rather than working class families needing to change, the current attitudes and values of the middle and upper classes are the ones that need to change as

> A socially just education system is one premised on the maxim that a good education is the democratic right of all rather than a prize to be competitively fought over (p 594/595).

SOCIAL CLASS, MATHEMATICS AND 'ABILITY' GROUPING[4]

The 'common sense' view that academic ability is fixed, easily measurable and unchanging leads directly into the current practice in most English secondary schools of putting children into groups of the same predetermined 'ability' in order to teach children effectively; current research is investigating the reasons for the lack of impact of research on this (Francis et al., 2016). In addition, in Britain, many teachers are philosophically opposed to mixed 'ability' (Barker Lunn, 1970). However, research from the OECD (2013) indicates that where students are highly stratified, as in the case of setting, there is a wider range of achievement than when they are taught in heterogeneous groups. Grouping students heterogeneously is supported by a very large body of research which indicates that it improves educational outcomes (e.g. OECD, 2013).

Most of the benefits of 'ability' grouping are benefits for teachers and schools whereas most of the disadvantages concern the negative effect on students (Hallam and Ireson, 2003). 'Ability' grouping sends a clear message that only some can do mathematics and that this is due to some type of 'natural ability' (Marks, 2013), a message some children (currently about a third) receive as early as age 4 (Dixon, 2002). Early grouping by 'ability' has long term implications for children's educational opportunities (Boaler, 2005).

Research reviews (e.g. Sukhnandan and Lee, 1998) state that studies on 'ability' grouping have produced few conclusive or consistent findings while, for example, Slavin (1987, 1990) reports that 'ability' grouping has little effect overall on attainment across the attainment spectrum. Despite radically different schooling systems in England and the USA the discourse is remarkably similar: it has changed little since the introduction of mass schooling and it is framed in terms of what is perceived to be better for the students.

4. Both in this section of the paper and in the corresponding section of Jackson and Povey, 2016, I draw substantially on Jackson, 2016 where there is a more extended discussion.

The allocation of children to 'ability' groups is claimed to be objective in that children are allocated on the basis of their prior performance. However in English secondary schools, although perceived 'ability' is found to be the main predictor of set, it is a relatively poor predictor (e.g. Muijs and Dunne, 2010; Alexander, 2008). In reality schools have multiple reasons for allocating children to particular groups and they are allocated informally on often insubstantial evidence. Children with higher socio-economic status (SES) and/or ambitious middle class parents are more likely to be assigned to higher sets as are children with low SEN. Children seen as disruptive or poorly behaved, which might be seen as linked to social class, are more likely to be in bottom sets (e.g. Muijs and Dunne, 2010). Boaler and colleagues (2000) found that working class students tended 'to be placed in a lower group than would be expected on the basis of their attainment alone' (p. 130) and this was a result of the school's desire not to 'alienate the most powerful (and highly valued) constituencies of parents' (p. 130).

The set a pupil is in can be crucial to their attainment. High level content is only made available to students in top sets (Porter, 1994) who are expected to work faster covering work in more depth while pupils in low sets have a reduced curriculum with low level work where there is less discussion, more repetition and more structured work including merely copying off the board (e.g. Boaler, Wiliam and Brown, 2000) with lower attainers being deprived of role models of more successful learners (Hornby and Witte, 2014).

The teachers of children in low level sets talk *about* them differently and talk *to* them differently, adopting a more authoritarian mode (Watson and De Geest, 2005). Behaviour is constructed very differently depending on the group's 'ability' level which leads to groups being treated very differently. Bartholomew (2001) reports on one secondary teacher's reaction to similar behaviour in a high attaining group and a lower attaining group. The teacher's concern with the high attaining group was about their *learning* while his concerns with the low attaining group were about their *behaviour*.

According to Duncan and Magnuson (2011), writing about the USA, on arrival in school, many middle class children are already academically more advanced than working class children. Researching in England in 1970 Lacey found education-conscious middle class parents endeavour to ensure their children secure a place in the best performing junior schools (p 35), further widening the academic achievement gap. In addition, teachers' beliefs frequently lead to lower expectations of working class children (Zevenbergen, 2003). Hence many working class children start both primary and secondary school at a disadvantage compared to many of their middle class counterparts. One pattern of response to this is that of rebelling against a system that predisposes them to do badly, committing themselves to behaviour patterns which means that their work will stay poor (Lacey 1970, p. 58). Writing some 40 years later Duncan and Magnuson (2011) note that antisocial behaviour similarly increases in the USA.

One important effect of grouping by 'ability' is that middle class children have minimal contact with those working class children who are less well behaved (Ireson, Clark and Hallam, 2002). Students who 'lack the social knowledge for what is seen to be appropriate behaviour' (Zevenbergen 2003, p. 146) by teachers will tend to populate the lowest sets. Those who do succeed in making it into the higher 'ability' groups soon discover that in order to succeed in school they must conform to the accepted middle class behaviour norms as failure to do so causes a descent into the lower attaining groups. Thus there is a self-correcting mechanism for dealing with children who do not conform.

Underlying the issue of 'ability' are issues of power and culture and hence whose ways of knowing are dominant. 'Ability' grouping is not a neutral disembodied organisational practice. Oakes, Wells, Jones and Datnow (1997) maintain that the 'common sense' conceptions of 'ability' and intelligence that are at the heart of schooling and the 'ability' discourse are part of an ideological battle defining children from lower SES groups as being expendable, that is, attainment grouping serves purposes in schools other than that of teaching and learning. Schooling is designed to reproduce the current social, political and economic systems and not to provide a meritocratic route to success in adult life (Oakes, 2005).

Contemporary English society assumes that middle class values are superior to working class values and hence the working classes need to 'aspire' to join the middle classes (Jones, 2011). The values that working class children bring to school are neither recognised nor valued by schools while the abilities they bring to school are ignored at best and indeed are thought to be detrimental to a good education (Delpit, 2006). This feeds into the informal judgements about intellectual 'worth' noted above. In addition, in general, working class students will not understand the mechanisms required to succeed in the curriculum as they will not have the cultural capital to 'play the game' that is involved in the learning of mathematics (Bourdieu, 1992). Separating them from their middle class peers by putting them into lower sets reinforces the cultural capital gap.

CONTEXT OF THE STUDY

The research, on which this paper is based, is a small scale study which focuses on that small group mathematics teachers in England who are interested in mixed-attainment teaching and who, in some sense ‘resist’ the ‘common-sense’ view of grouping students by ‘ability’. These teachers have either chosen to teach in departments that do not practice grouping by ‘ability’, according to some measure of prior attainment in, at least, some year groups or they are teachers who are interested in doing so. Hence the main research question in the study which I was seeking to address was:

How do some mathematics teachers (in England) resist grouping by ability?

a. How do they resist the rationale given for grouping students by ability when it is understood as 'common sense'?
b. How do they describe and understand their teaching practices?
c. What is the role of their local context?

In carrying out this study I visited five very different schools in differing regions of England and interviewed eleven teachers. Some of the teachers interviewed had actively sought out mathematics departments with some mixed-attainment teaching and some had had the opportunity to introduce mixed-attainment teaching into their schools. Not all teachers in the latter group were the teachers in overall charge of mathematics. These teachers are not following the prevailing discourse and it is their stories I am interested in. It is my intention, in drawing on these stories, to understand how they have come to stand outside contemporary discourses on grouping by ‘ability’ in mathematics in England. I hope that this, in turn, may enable us to understand how other teachers could be encouraged to reconceptualise their views on ‘ability’ grouping and so be empowered to argue for changes in the system; but this lies outside the scope of this paper.

METHOD

In the main I conducted individual semi-structured interviews although I did conduct one paired interview. Semi-structured interviews provided the opportunity to ask non-specific questions which through my prompts and probes allowed the interviewee to give depth to their responses (Cohen, Manion, and Morrison, 2000). In many ways the interviews were more akin to conversations and had something in common with life history interviews, enabling me to understand better the circumstances which led them to their current situation and their current thinking (Brewer 2000).

The teachers interviewed were chosen through opportunistic sampling. I initially identified three of the teachers through personal contacts and the Association of Teachers of Mathematics (ATM.) This snowballed leading to several additional interviewees recruited via Twitter. I had intended to interview only one teacher per school but in two of the schools more than one teacher volunteered to be interviewed.

The interviews were recorded electronically: some parts of the interviews were subsequently transcribed by me.

This paper focuses on two of the teachers, Bob and Sarah, who are both passionate about mixed-attainment teaching and have taught together in two different schools. They were both teaching in the same school at the time of the interview but both were leaving because the school had decided to return to grouping students by 'ability' in mathematics.

DATA ANALYSIS

I have chosen to use narrative analysis to analyse my data as one purpose of narrative enquiry is the construction of knowledge through lived experience in order to capture events so as to bring them close up (Mulholland and Wallace 2003, Gudmundsdottir 1997). In the case of Bob and Sarah narrative analysis helps to draw out their individual but intertwined stories (Povey and Angier with Clarke 2006) and so helps to understand 'what is going on' (Brewer 2000) at a quite deep level.

THE TEACHERS

I now offer the stories of two teachers, collected in 2016, who have resisted this dominant discourse: Bob and Sarah.

BOB

Bob believes mixed-attainment teaching is rooted in social justice and as such he has made a positive choice to work in schools which are fully comprehensive and which have a substantial cohort of children from socially disadvantaged backgrounds. This is his story.

Bob always thought he might do something in education but after his first degree he was just drifting about with no definite plans. However his mother spotted a two year PGCE in mathematics at Midland University and encouraged Bob to apply. In Bob's words

> ... she was pushing at an open door. I was quite happy with that idea. It wasn't necessarily something I would have gone for but ... in terms of social conscience that's what I wanted to do.

Within a week he had had an interview and was on the course.

During his time at Midland University Bob encountered a tutor who greatly influenced his thinking as a mathematics teacher, both at that time and throughout his subsequent teaching career. This tutor[5] espoused the discovery method of learning which Bob's fellow students either loved or hated, there being no middle ground.

On finishing his PGCE Bob was fired up with enthusiasm for teaching mathematics and like many young teachers thought now he could 'change the world' so he looked for and obtained a post in a challenging north

5. Bob did not mention this specifically in his interview but the tutor in question is a very strong proponent of mixed-attainment teaching and this would undoubtedly have featured in the course.

London comprehensive. However at this point in Bob's story things started to go awry. Bob's teaching experiences on his PGCE had been in 'some very lovely schools in the local ... area'. While this may have been a good preparation for teaching in the local area it was not such a wonderful preparation for teaching in a school in challenging circumstances which can be demanding both emotionally and physically.

Bob describes an incident which illustrates the difficulties he encountered, one which clearly made a considerable impression on him.

> The year 10 class I taught snapped all of the pencils in the lesson before, all the coloured pencils ... [and] came in with them in plastic bags, and every time I turned around to write on the board, bits of pencil rained down on me (laughing), and it went on like that for a year, and the second year got a bit better and then I left.

Having decided to leave Bob went for a complete change. He obtained a post abroad in a private English medium school and spent six years there teaching history to comparatively mature students using discovery approaches. It was during this period he began to think about how this type of approach might be adapted for teaching mathematics in a state comprehensive in England.

Returning to England in the late 90s he eventually settled down in a medium sized town in the south east of England. After a relatively short spell he applied for and obtained a post as head of mathematics in Chancellor School, a quite prestigious school for the area. He stayed at this school for several years and during this period he began to develop and put into practice his ideas on using investigative approaches to learning and teaching in mathematics. He describes this time as being very important in developing his ideas. The school environment encouraged creativity whilst the headteacher was supportive, leaving Bob to get on with running the mathematics department as he saw fit.

Having run the mathematics department successfully for several years Bob decided to change schools. However, he wasn't overly happy with his new school so after two years he moved back to his previous school. Unfortunately, for Bob, the headteacher changed and he did not see eye to eye with the new one so he decided it was time to move again. He obtained a post at Shortvalley, a school with a similar demographic to the school he was working at; it was comprehensive with a substantial pupil premium intake (>40%). Before accepting the post Bob negotiated an agreement with the headteacher that he could introduce mixed-attainment teaching in the first year and, providing this was successful, it was agreed it could be rolled forward to later years.

Despite this agreement Bob had to overcome a number of obstacles before mixed- attainment teaching could be successfully introduced. The first obstacle took the form of a deputy head who was sceptical about introducing mixed-attainment teaching in mathematics. A second obstacle was introduced a few months after Bob took up his post in the form of an OfSTED[6] inspection which placed the school in the *Requires Improvement*[7] category. The senior leadership team in the school responded by imposing a number of extra demands on the teachers including additional monitoring and target setting for the students. This additional burden possibly delayed the introduction of mixed-attainment teaching as it temporarily shifted Bob's focus away from the preparations for its introduction. However, one obstacle was removed as the OfSTED inspection resulted in the deputy headteacher leaving. A further obstacle Bob faced was that the mathematics teachers at the school when Bob arrived had little experience of teaching in anything other than a very traditional manner, something that served many working class students very badly. The pedagogy of the mathematics teachers at Shortvalley was very text book based. Bob based his evidence for this on his observations of the teachers in his first year at the school and on the piles of textbooks lying around the department.

Despite his desire to implement mixed-attainment teaching in year 7 Bob knew he could not simply impose this as he needed the cooperation of at least some members of the department. Several of his teachers were opposed to mixed-attainment teaching. However, he could not afford to alienate them as mathematics teachers were difficult to replace and they indicated they would leave if they were required to teach mixed-attainment classes. Despite this Bob succeeded in implementing mixed-attainment teaching in year 7.

The ability grouping practices that had been used in the school prior to Bob's appointment were markedly different from the more usual 'ability' grouping practices, indeed some were definitely counter to the norm. In most schools that group by 'ability' the groups with the 'more able' students are usually bigger than those with the 'least able'. At Shortvalley the most 'able' students in set 1 were in a group of twenty while the 'lower ability' students, e.g. set 4, were in groups of twenty-eight. This goes against one of the usual justifications for putting students into ability groups in that it is argued grouping students by ability enables the 'lower ability' students to be put into smaller groups so they receive more individual attention. A second unusual practice was that the same teacher would teach the same set in each year so that, for example, the teacher who taught the top set in the first year would also teach the top set in the other four years.

Many of the mathematics teachers had been there quite a long time

6. OFSTED are the government inspection agency

7. Requires improvement is equivalent to a grade 3 from OfSTED. This was formerly known as satisfactory but all schools must now be good or outstanding.

and, in addition, two of them had been at Shortvalley as students. These two in particular would have had limited experience of teaching methods other than those they had been exposed to as students, and indeed one of these teachers was 'philosophically' opposed to mixed-attainment teaching. This teacher had been given a guarantee by Bob that he would not have to teach any mixed-attainment classes in the year mixed-attainment teaching was introduced to the school. This teacher left at the end of the second year but interestingly at the time of the interview was returning as he had heard that Bob was leaving.

When the school was re-inspected by OfSTED after two years it failed to get out of the *Requires Improvement* category and the headteacher left shortly after. At this point Bob decided to move on as he was aware that the “writing was on the wall” as the governors were seeking to appoint a headteacher to sort out the 'mathematics problem'. During Bob's time at the school mixed-attainment teaching had only just progressed up to the second year and had not begun to affect the public school examinations. The GCSE results had not changed much which was hardly surprising as moving away from ability grouping and implementing mixed-attainment throughout the mathematics department was a long term strategy and could only come to fruition after several years. However the climate today in education is one of instant fixes and instant improvement.

Bob left at the end of the school year moving to a new school about a hundred miles away where he anticipated being able to teach mixed attainment in at least some years.

SARAH

Sarah has come late to enjoy mathematics. Her school experience, unsurprisingly, had been typical of most secondary school children in England in that

> Maths wasn't taught creatively, it was very text book, it was like … the teacher would model how to do something and we'd all answer thirty questions on how to do it … I wasn't interested in it.

She had achieved less in mathematics than in any other subject and said

> I began to convince myself I'm not a maths person, I'm an English person. I'm English and Humanities and creative stuff. I'm not maths so yeah …I really do think that … I never would put my hand up to answer a question in maths class, never …

She describes herself as a creative person. She had come to believe she wasn't any good at mathematics,

> I had told myself I wasn't any good at maths … ridiculous but I genuinely believed that … I would say, 'oh I'm no good at maths'.

Sarah's background is working-class; she had free school meals when she was at school. She said

> I was a pupil premium child and that's again, a bit like the maths thing. I was free school meals, so was my husband. He left school with no GCSEs at all, he's got a PhD now. I think the school system lets down [ ] ... I was also a young carer, so I think the school system lets down certain types of student which is also why I feel strongly about the mixed-attainment.

Sarah had stumbled into mathematics teaching rather by accident. She had 'never wanted to be a maths teacher'. Her first degree was in sociology and she applied for a behaviour inclusion job at Chancellor School but

> they said 'Oh you didn't get that job but we have got a space [for a teaching assistant] in the maths department' and I hated maths at school, absolutely hated it so I ... don't know but then I thought I need this job so I took the job.

Her work and experiences supporting the mathematics teachers at Chancellor School transformed her views on mathematics and mathematics teaching.

> ... after supporting Bob's lessons and other teachers that he'd kind of, you know, that had bought into his ethos ... I wish I'd been taught maths this way when I was at school ... never in a million years did I expect to be a maths teacher ... I definitely thought it was my worst subject at school, definitely.

Sarah is interested in mindsets and the work of Carol Dweck (for example Mindset: How you can fulfill your potential (2012); Self-theories: Their role in motivation, personality and development (1999)) and thinks it is really powerful particularly with regard to mathematics. Having experienced the teaching and learning of mathematics differently as an adult she understands that believing that that you cannot do mathematics is a misconception but a misconception that is still too dominant.

As a result of her experiences as a teaching assistant supporting mathematics teachers at Chancellor Sarah decided to become a mathematics teacher. To achieve this she enrolled in a degree course in mathematics with education at a local university continuing working in school as a teaching assistant while studying part-time and attending university, initially one day a week but increasing to two days in the final year. Subsequent to getting her degree she enrolled as a trainee on a school based initial teacher education programme continuing to work while she gained Qualified Teaching Status (QTS).

After gaining QTS, Sarah taught mathematics at a number of schools in challenging circumstances in her Local Authority. This was a deliberate choice on her part; it was what excited her and what she wanted to do. Apart from one school when she was a main scale teacher, all of the schools she chose had some mixed-attainment teaching. Eventually Sarah became Head of Mathematics at Knapper School. She was 'really, really happy there ... [it] was going to be my forever school'. She was teaching mixed-attainment at Knapper School but a new headteacher decided to re-introduce ability grouping. She had been forced to move on because she did not agree with setting.

Sarah had moved on to work as a Lead Mathematics Teacher at Shortvalley School because Bob, who had been the Head of Mathematics at Chancellor School, had just been appointed as the Head of Mathematics.

When I interviewed her, like Bob, Sarah had just resigned and was moving to a new school in September. Sarah had resigned for much the same reasons as Bob. Sarah is passionate about mixed-attainment and mathematics teaching. She had hoped to stay at Shortvalley more than three years but

> I do feel strongly about it. I won't teach maths in a school ... I can't lead a department as Head of Mathematics and promote setting to a team because I am fundamentally opposed to it so I think I'd always be saying I don't agree with this, I don't agree with this and I don't feel you can lead a team successfully if you don't agree with what you're doing and it feels the same being a lead practitioner.

The new headteacher had been appointed by the governors to "sort out mathematics": she had previously been a head of mathematics and had decreed that the school was returning to setting prompting Sarah to leave. Sarah had investigated all the secondary schools in the Local Authority in which she was working and further afield but had found no schools teaching mixed-attainment mathematics in the vicinity. Consequently she had decided to get a job at a school a considerable distance away, a school to which she would not be able to commute but where she would need lodgings during the week, only travelling home at the weekends. The school where Sarah has got a new job is led by the ex-headteacher of Knapper, her old school, leading Sarah to believe that she will be able to stay at her next school longer than three years as she believes that the Headteacher and Vice-principal will be more in tune with her beliefs.

FINAL COMMENT

These two teachers' stories are important not because they are extraordinary people, but rather they are ordinary people who believe, as Rawls maintains, that an accident of birth should not decide life chances. Bob and Sarah are committed to social justice in mathematics teaching and are not prepared to work in a system that they believe does not serve all children's interests. What separates them out is the connection they make between this and grouping students by 'ability'. In addition, we see their willingness to act. As we have seen above, when the governors decided to recruit a headteacher to "sort out mathematics", Sarah and Bob both felt they had no option other than to leave: Sarah because she'd had a similar experience before as Head of Mathematics at Knapper School and Bob because he knew that the "writing was on the wall".

In this paper I have narrated part of the stories of Bob and Sarah. Their stories are important because they reject 'common-sense' notions of 'ability' that disadvantage working class children. They are not prepared to work in a system that treats some children as dispensable but are prepared to resist the current orthodoxy even if it means making their own lives less comfortable.

In this paper I have gained a small insight into what can still be done even now and I hope that having done so that it enables some teachers to ask the question 'what can I do?' and, further, to go some little way in answering it.

I would welcome any suggestions that indicate paths I might explore both to use such stories to understand the social world better and also to support other teachers who are interested in the issue of 'ability' grouping and social justice.

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DEVELOPING 'QUALITY' TEACHERS IN REMOTE INDIGENOUS CONTEXTS: NUMERACY LEADERS

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Remote Indigenous education is challenged by many factors one of which is the attraction and retention of quality teachers to work in hard-to-staff schools. In this paper, I explore how successful remote schools have worked with the significant challenge – ensuring quality and rich mathematics learning for Indigenous students Building the skills of beginning teachers to work in these schools is the focus of this paper. Building a culture across the school in which early career teachers negotiate their new context and develop a sense of identity as beginning teachers has been achieved at many schools through the deployment of a numeracy leader.

THE CHALLENGES OF REMOTE INDIGENOUS EDUCATION

Remote education is fraught with many challenges, most of which are documented across many years of research. For the purposes of this paper, I will provide a brief summary of the diversity of research with the intent to provide a context. In this background, I focus on those issues associated with teacher quality in this context. The development of teacher quality within the context of remote Indigenous schools is the focus of the paper. I draw on data from a national study across nearly forty schools where many of the schools have developed a middle leader role whose primary task is the development of quality practices and quality teachers in those schools. While the term "quality" is a contested one, it is used here to highlight the characteristics of good educators who work in challenging contexts.

I am creating a term –pedagogical capital– as a reference to Bourdieu's framing of the forms of knowledge and dispositions which he refers to as capital (Bourdieu, 1983). These knowledges and dispositions have particular exchange value within a particular field. In the context of this paper, pedagogical capital refers to the knowledges and skills that teachers need to be successful in remote Indigenous education. These skills and dispositions may resemble some of those that are found in urban settings, but there are peculiar demands in remote settings that require different practices if there is to be success in learning mathematics.

Teacher Quality: Transient, Tourist Teachers

Many of the teachers who come to teach in remote areas are early in their career so they lack the experience of both teaching (mathematics) and are often in their first position in a remote/Indigenous context. Most employers recognise the importance of mentoring for early career teachers with most statutory-employing groups offering some form of mentor to beginning teachers. This is not so easy in the remote context where often all teachers are at the early start of their careers, and in some cases the principal is equally early in her/his career. This begs askance as to how, at a very practical level, can beginning teachers develop the repertoire of skills, knowledge, dispositions and resilience need to survive and thrive in remote contexts. For early career teachers to lead others can be problematic when they do not have a extensive toolkit for professional learning of others (Borko, Koellner, & Jacobs, 2014). Teaching in remote schools places considerable pressure on teachers and school leaders as they negotiate the environmental and emotion challenges of living in remote isolated areas (Jarzabkowski, 2003). There are some authors who question whether too much is asked of early career teachers in remote contexts and that, in fact, employers may be putting too much reliance on the personal resilience of teachers as they enter these 'hard-to-teach' schools (Sullivan & Johnson, 2012) rather than building the skill set of teachers to be able to work effectively and productively in these contexts.

The pressure on teachers in remote (and rural)[1] settings often results in a high turnover of teachers. In some states, the contract for teachers may between 1 and 3 years. This high mobility or transience results in perceptions held by community members of the teaching staff (Mills & Gale, 2003), often where there is a high degree of scepticism as to the teachers' commitment to the school and community.

There are many motivations as to why teacher seek to work in remote areas. In a study of teachers working in a remote region of northern Australia (R. Jorgensen, Grootenboer, & Niesche, 2013) it was found that the motivations varied from adventure, travel and missionary with only one teacher (out of 32) identifying a socially-just motivation to working in the context. Similarly others (Schulz, 2015) have found the unwitting complicity to the three Ms[2] and tourist discourses for motivating white teachers to work in remote desert contexts. As some (Hickling-Hudson & Ahlquist, 2004) have argued, the inexperience of neophyte teachers places them at greater risk of implementing reproductive pedagogies, vis a vis neo-colonial approaches and thus expose students to a Eurocentric curriculum which may contribute to the alienation and marginalisation of Indigenous learners.

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1. In the Australian context, remote settings are those which are geographically isolated, while rural settings are those often found in farming areas where there is often some sense of isolation, but without the considerable geographical isolation of remote settings.
2. The three Ms are a reference to "missionaries, mercenaries and misfit" as the people who opt out to live in remote, harsh contexts.

Culturally Inclusive Practices

As the contexts within which the study is being conducted are very remote, the culture/s and language/s often are still very traditional. For many students, coming to school represents a strong cultural dissonance between the home and school. There are numerous studies and philosophical writings of the value of including approaches that advocate a culturally inclusive approach. Such approaches are quite diverse ranging from those that are ethnographic in standpoint and seek to build the cultural knowledges and practices into the existing mathematics curriculum, or in some cases to become the mathematics curriculum. Examples of this type of work are evident in the ethnomathematics tradition where there is a celebration of the mathematics embedded in cultural practices of non-dominant cultures (Rosa & Orey, 2015). There have been explicit attempts to seek the mathematics undertaken by Indigenous Australian communities and then incorporate this into a revised mathematics curriculum (Watson & Chambers, 1989). Other approaches have sought to identify more subtle aspects of culture and recognise how these impact on learners as they negotiate the taken-for-granted social and cultural norms of classrooms (Malin, 1990). These approaches adopt a strong care factor and seek to build into the programs elements of culture/s that will enable students to feel validated and included in the classroom practices (Savage et al., 2011) and, in so doing, sustain cultural pluralism (Paris, 2012). The culturally inclusive/responsive approaches often lack strong, effective and practical examples for educators and often at risk of not having the potential impact that the theory suggests (Griner, 2012). There is risk within these approaches as cautioned by Nakata (2003) that can engender the educational context being subverted for the cultural or anthropological discourses and thus serving as a convenient rationale for the failure of those intended to be beneficiaries of the approach. The vast literature on mathematical content knowledge and pedagogical content knowledge has shown that teachers who have strong knowledge in one or both of these areas is more likely to produce better learning for the students (Baumert et al., 2010; Campbell & Malkus, 2014).

One of the major issues in remote education is the tyranny of distance and how this impacts on the possibilities for teachers' learning (Parding, 2013). It has been found that teacher support is critical for beginning teachers and the resultant quality of their teaching (Blömeke & Klein, 2013). Most communities do not have access to relief teachers who could come into the school and relieve a teacher to undertake external professional development. The distance itself also represents a significant issue. At best, there is a day travel each way to attend a professional development outside the school. Alternatively to bring in external people to conduct professional learning, requires additional travel costs for the consultant –both temporal and fiscal. As most remote schools are isolated, it is just as problematic to link schools to provide professional learning opportunities. Finally, accessing on-line resources may seem to be a good option but most schools have unreliable satellite internet which will fall over on cloudy/rainy days to the point of not even working, the cost is extremely high for downloading, and the band width is limited so that high resolution video is almost an impossibility to download. Collectively, these issues provide challenges for schools in terms of professional learning, particularly for new graduates, and/or teachers new to remote education.

Numeracy: Key Learning Area

For most remote and very remote schools, literacy and numeracy are key learning areas that take a priority in curriculum offerings. Most schools in the Remote Numeracy Project (which is the basis of this paper) structure their day around three sessions. The order may vary, but it is predominantly the first session of the day is literacy, the second is numeracy and the third is all other curriculum areas. This process not only gives a high priority to literacy and numeracy but in most cases the lessons are in the first part of the day so that quality learning time is allocated to the two key areas.

THE OUTCOMES OF REMOTE INDIGENOUS EDUCATION

There is widespread recognition of the educational chasm in achievement for Indigenous and non-Indigenous students. It is not possible to make sweeping comments since other factors impact on success including geographical location, social status, gender, language etc. What is very apparent is for Indigenous students living in remote and very remote locations, there is a marked gap in achievement. To this end, successive Federal governments from 2007 have implemented the "Closing the Gap" initiative which seeks to lessen the gap in health, education and housing for Indigenous people in comparison to non-Indigenous people (Australian Government: Prime Minister and Cabinet, 2016). Despite considerable funding being allocated to education through the funding associated with Closing the Gap, it appears that there has been little change in educational achievement (Taylor, 2016). While educational outcomes are important, other authors (Yeung, Craven, & Ali, 2013) have explored the nexus between academic scores in literacy and numeracy with self-concepts, self-ratings of schoolwork and learning-related factors for Indigenous and non-Indigenous students. They reported that Indigenous students reported much lower scores than for non-Indigenous learners thus suggesting that schools need to focus on academic as well as factors associated with enjoyment of school life.

MOVING FORWARD: BUILDING PEDAGOGICAL CAPITAL

Building scholastic capital, that is the capital that has value with in the field of education (R. Jorgensen & Sullivan, 2010), through education underpins the purpose of schooling. Investing in education allows students to build better lives in the future. Whether this is seen as an overt principle or a tacit assumption, it is without doubt the key purpose of schooling. Yet, what is known is that the gap between Indigenous students and non-Indigenous students, most notably those living in remote and very remote settings is alarmingly worrying. Many strategies have been developed, some of which were discussed earlier but mostly emphasise the importance of quality teachers (Pearson, 2009; Penfold, 2014). Winheller, Hattie and Brown (2013) have concluded that "the perceived quality of learning is connected with 'confidence in' and 'liking mathematics', which in turn predict students' mathematics achievement" (p. 49). Their work across a number of publications emphasises that the teacher is the most important variable in students' success despite some criticism around methods as to how the Hattie and co-researchers were able to make such claims (Ingvarson & Rowe, 2008). It is generally accepted by employers that investing in teachers is a positive step in building capacity of both teachers and students. To this end, it is invaluable for teachers to have access to practices that will allow them to build their pedagogical knowledge unique to remote Indigenous contexts, that is, build their pedagogical capital.

THE REMOTE NUMERACY PROJECT

The project has been described elsewhere (Robyn. Jorgensen, 2015) but, in brief, it (to date) has consisted of nearly 40 case studies of remote and very remote schools that have a population with more than 80% indigenous students attending. The schools have been successful in the teaching of numeracy. The study has been conducted across 5 states/territories and includes all sectors and systems of schooling. The study is ethnographic in design and seeks to develop case studies of each school (Jorgensen (Zevenbergen), 2016) that describe the practices adopted by the schools. Data consist of interviews with leaders, teachers and other staff at the school, classroom observations and document analysis. All interviews are recorded, transcribed and coded using NVivo (QSR, 2010). The data presented here draws on the node relating to middle leadership.

BUILDING PEDAGOGICAL CAPITAL THROUGH MIDDLE LEADERSHIP

Many schools across the study have adopted a role within the school whose task is to build the expertise or capital of the teachers in mathematics; to foster the development of a whole school approach; to provide support for the teachers in many areas including feedback on lessons, advice on assessment, interpretation of data; build a whole school plan for mathematics; and to liaise between the leadership team and the classroom teachers. Across the schools, the title of this position varied, but for the purposes of this paper, I have opted to adopt the term 'numeracy leader' for this role. In the following sections, I draw on teachers' voices to highlight the role and value within this context of education, which in turn, helps to identify the characteristics of pedagogical capital – the skills and dispositions that are needed and valued in remote Indigenous settings.

ROLE OF THE NUMERACY LEADER: IN-CLASS SUPPORT

Across the schools that had adopted the numeracy leader, there was a general consensus that the in-class support was a valuable role in building the culture of the school and the expertise or pedagogical capital of the teachers. The types of support that could be offered in the classroom varied across the study, and included feedback on lessons, co-planning with the teacher, developing tests/assessments and then interpreting the data to inform subsequent teaching, and modelling teaching, along with tasks that the teacher and/or school saw as valuable. While various terms are used in different schools –such as numeracy specialist, support teacher, numeracy coordinator, mathematics specialist etc– the terms are used to describe a role where there is a dedicated teacher who is tasked with supporting teachers to develop their numeracy practices within the contexts of their classrooms.

> Numeracy leader: They [teachers] had a support teacher every day for maths. We also had a numeracy specialist that would be coming in and that was part of my role as a year 1 support. I would take out a group of the lowest children and I'd be responsible for doing their numeracy learning for the year.
>
> Teacher: [name] used to be our maths specialist but now we don't have that any more. That was good having her because she was timetabled in to help you as well during maths. During the term she'd be like, 'alright for the next two weeks I'm going to support you and help you with your programs' and she'd move around the school ... She'd sit down with me and we'd write our whole term program together and pick out what we needed to do. We'd look at the kid's data that we'd take from diagnostic tests and stuff and decide what we needed to target and look through the curriculum and come up with our plans. She used to do that with everyone.
>
> Numeracy leader: I was going out to [name of community], they've got an early year's centre out there as well so worked with those kids as well. So helping the teachers plan and assess lessons and then I'd also go in and support them. Collating the data and analysing data to keep passing on to the teachers the following year.

Co-Planning and Co-Teaching

The numeracy leaders often worked very closely with the teachers to build their planning documents and assessments. The numeracy leader often would team teach with the teacher. In some cases, this was as a support person in the classroom to help with the diversity within a classroom, in other cases to model teaching for the teacher.

> Teacher: So we'll sit down and we'll do it together. Like, so she knows that, you know, we'll work off my term planner that we've got, and I know that on those 2 days I wanted to do time and yeah, so that's what, so I use that. And so we'll sit down and we'll just go through the First Steps books and we'll find some activities that will help the kids reach it. .. Well, we're meant to do it weekly, and then it used to be, and it's meant to be, but it hasn't happened lately, on Mondays and Tuesdays is when I generally do number. Only because there's so many kids in the class and they're quite needy, [name] usually comes in and we team teach.

Building Deep Mathematics

As is well known from the research literature, many primary school teachers have low Mathematics Content Knowledge (MCK) and often are fearful of teaching mathematics. Building MCK in both teachers and students is empowering and has been a part of many schools' professional learning. Many workshops have been held that focus on the learning of mathematics, and this in turn has helped teachers build the mathematics learning for their students.

> Teacher: …we've got our numeracy coordinator, … but she works very closely with teachers to ensure that mathematical understanding has been developed in the kids not just, like I was saying about the fractions, not hollow, there's a depth to it.

Professional Learning

The numeracy leader has a role in the professional learning of the teachers. This was undertaken in many different ways across the schools –after school sessions, in-class in real time, professional reading, mathematics activities, and so on and largely based on the needs of the teachers and the vision of the school.

> Teacher: We've had a lot of PD and how to develop appropriate, well not appropriate, it's sort of like a bit of a developmentally-appropriate maths lesson to really get these kids moving from what they were doing before [the numeracy leadership team] got here to now and it really has deepened the whole understanding.

Depending on the school, the Numeracy Leader often worked with the Aboriginal Education workers[3] as well to build their knowledge –both mathematics and pedagogy so that they would be able to be a valuable resource in the classroom.

Building a Whole School Approach

There is strong sense across the participating schools of the need for a whole school approach to teaching numeracy/mathematics. The middle leader has an important role in building that culture and the knowledge within the teachers on how to teach mathematics at this school.

> Principal: I think because [name] is spread across 2 coordinator roles, literacy and the numeracy roles. So she might be being stretched a bit thin in that way. I think the whole school has to work on being on the same, have the same vision and we got new staff so perhaps that will take time.

> Principal: Teachers are aware when appointed [to the school] what program we use. They get lots of info about the program, and support. Numeracy coordinator gives less time to experienced teachers, and more time to new teachers, initially.

SUMMARY AND CONCLUSION

In summary, the role of the numeracy leader is quite diverse. Having a person based within the school ameliorates many of the issues identified in the literature in terms of supporting teachers in remote contexts. The role is diverse as shown in the previous sections and summarised below.

> Numeracy leader: [it's a] Mentoring role. I'm not expert in anything. Try help them develop further understanding in all areas of maths; providing them with good assessment items; showing them how to use it to inform teaching; keep them enthusiastic; be ready to go in and model (not just talk the talk); trying to show staff the way you can show kids how to pick up patterns (because maths is all about patterns).

While the role is overall seen as a very positive one for so many reasons, the characteristics of the person in the role is very important. While in most cases, the teachers and leaders were very positive about the role and the appointees, there was a case where the teachers were somewhat circumspect about the person. This was largely due to the person also being early career (3 years since graduation) and did not have the repertoire of skills, knowledge and classroom experience to be able to support the teachers in a genuine and deep way. Overall, however, the numeracy leader role has been instrumental at some schools to build a whole school approach but also to build a positive learning culture among the staff.

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3. Aboriginal Education Workers is a term used across the project to refer to local First People who live in community and take various roles to support teachers in the classroom.

Principal: So you're seeing similar practice being used across the board. And a lot of it is good discussions too. You know, we'll often have that chance, let's just have a brainstorm on sharing some good practice together. Or after our staff meetings, we're all held in our meeting room, and after we developed the, um, data wall in March this year, we found that that's really added to some wonderful discussions and people hanging around after staff meetings to talk.

Having the right person in the role as a numeracy leader has enabled schools to address many of the issues that are commonplace across remote schools. The schools in this project have taken a proactive stance, often being quite creative in how they manage to fund the role, to ensure that teachers are able to access the support they need to build a comprehensive and cohesive approach to teaching mathematics. The processes described by the participants in the project elucidate the ways in which the pedagogical capital of the teachers and Aboriginal Education Workers can be built up (and sustained). Having particular skills and dispositions, that is, pedagogical capital, is paramount to building the success in numeracy learning for Indigenous students. This paper has explored some of the features of that capital that have enabled success in the contexts of this study.

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PEDAGOGICAL AND MATHEMATICAL CAPITAL: DOES TEACHER EDUCATION MAKE A DIFFERENCE?

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This paper explores teachers' pedagogical content knowledge and mathematical content knowledge for both primary and secondary school teachers. Using established questions from other reviewed research, the research team posed a series of tasks for teachers to complete. The results surprised the researchers as they contradicted our expectations. Primary teachers scored lower on both Pedagogical Content Knowledge (PCK) and Mathematical Content Knowledge (MCK) while secondary teachers scored (significantly) higher on both scales. Preservice teachers scored similarly to primary teachers. To explore this further we subsequently added a further cohort – engineers who were expected to have strong mathematical knowledge but little to no pedagogical knowledge. The results reported pose serious questions in the current political and employment contexts. The data suggest to us the capital building that may be facilitated through teacher education programs needs to be questioned as do many of the assumptions that permeate the field.

TEACHER KNOWLEDGE

Creating an understanding of what makes for a good teacher in mathematics is a very vexed question. In a forum such as MES, there is a heavy emphasis on issues around access and equity –how do teachers cater for the diversity found in contemporary classrooms? Many researchers in this forum take issue with how teachers create learning contexts that enable students from socially diverse backgrounds to be able to access and succeed in mathematics. This paper digresses from this literature to focus critically on dimensions of teachers' knowledge that dominate the field –content knowledge and pedagogical knowledge. These two concepts have been appropriated by researchers in the field who have assigned various terms to the two broad constructs. There is a vast literature in this area and with competing findings. The research described in this paper raises some serious questions that we seek to pose based on the outcomes of part of a large research project conducted in Australia. Teaching mathematics is a complex process that goes beyond the simple collection of activities that keep students busy or engaged. Teachers need to have a complex contingency of sophisticated professional knowledge that unites the knowledge that must be taught/learned and effective ways in which that knowledge can be created for students. Knowledge of the discipline and effective and quality pedagogical practices are the touchstone to high quality teaching and learning.

Mathematics education, as a field (Bourdieu & Wacquant, 1992), has certain discourses and practices that are widely accepted as cultural truths. Some practices, over time, gain certain credibility and ultimately operate as powerful truths within the field, conveying power to those who accumulate and promulgate such truths. Within the field of mathematics education there is a large literature around the knowledge, practices and beliefs that teachers hold in relation to the teaching of mathematics. For example, a google search for mathematics pedagogical content knowledge yields 763 000 hits, while mathematical content knowledge yields 16 700 000 hits. Much of this writing has been founded on the seminal work of Schulman (1986) where he made the clear distinction between what to teach (discipline knowledge) and how to teach that knowledge (pedagogical knowledge). His work has been taken up in mathematics education, with heated debate in contemporary times as to the importance or not of discipline knowledge. There has been an historical and on-going debate as to the primacy of either discipline knowledge or pedagogical knowledge. This is most evident in the maths wars in the U.S. (Schoenfeld, 2004) where mathematicians predominantly declare the primacy of discipline knowledge as essential to good teaching in mathematics. Other mathematics educators such as Boaler (2002) advocate strongly for the pedagogical knowledge of teachers. In their comprehensive study of the importance of mathematical knowledge, Hill and colleagues (Hill, Blunk, & Charalambous, 2008) argue for the importance of mathematical knowledge in fostering and supporting quality instruction in mathematics. At the same time, they recognise that this is mediated by other factors, one of which is pedagogy. There is considerable tension in the field as to the importance of the two constructs. Increasingly researchers are proposing other terms to reflect nuances within these broad constructs. It is beyond the scope of this paper to provide a comprehensive account of this literature so a broad albeit condensed overview, recognising the limitations of a conference paper, is undertaken.

The importance of teachers' mathematical content (or discipline) knowledge has been linked to student achievement (Hill, Rowan, & Ball, 2005) where it was found that there was a positive relationship between teachers' knowledge of mathematics and student achievement. Part of the reasoning for this relationship is that teachers with a deep knowledge of mathematics are better able to see relationships and networks in mathematics and build the mathematical understandings in their students. In contrast, in a large study of German teachers, researchers (Staub & Stern, 2002) found that not only was pedagogical content knowledge important but teachers' beliefs impacted significantly on the end performance of students. These authors found that teachers taking a particular pedagogical approach (cognitivist constructivist orientation) produced better outcomes than teachers with other approaches in the pedagogy.

Ball and Bass (2000) have suggested that pedagogical content knowledge often consists of routines and practices that are commonly used across mathematics –such as ways of teaching number, fractions and integers– which produces regularities in the teaching approaches commonly used in schools. However, they contend that much of teaching mathematics is also uncertain. For teachers to be able to cope with this uncertainty, they need to have strong discipline knowledge. It is one thing to know the regularities, the 'tricks', in teaching mathematics, but it is another thing to know how to deal with the uncertainties and application of mathematics.

In an international study of graduating preservice teachers (Blömeke, Suhl, & Kaiser, 2011) it was found that there were remarkable differences in how countries prepared their prospective teachers in relation to pedagogical knowledge and content knowledge in mathematics. They also reported that there were marked gendered differences in mathematical content knowledge (MCK) but not in mathematical pedagogical content knowledge MPCK). We (Lowrie & Jorgensen, 2015) have reported elsewhere the particular findings of our preservice cohort to include the intersection of pedagogical knowledge, discipline knowledge and teachers' beliefs as beliefs impact on teaching in profound ways.

Within this context, we draw on a number of Bourdieu's concepts to make sense of practices within the field of mathematics education. In the current context, at least in Australia, there is now a growing recognition that teachers must have strong discipline knowledge. There are now entry and exit requirements for preservice teachers who must demonstrate their competence in numeracy tests. A graduate who cannot pass the test (for literacy and numeracy) will not gain registration as a teacher. While this is somewhat contentious, it highlights a recognition that teachers should have fundamental knowledge in the disciplines –literacy and numeracy– that underpin the core work of educators. The field of teacher education is now recognising the importance of discipline knowledge. Teachers who can engage with the practices of the field *vis a vis* content and pedagogical knowledge and embody these practices into the repertoire of teaching skills into their teaching habitus are likely to be seen as better teachers or quality teachers. In so doing, these skills, as represented through their teaching habitus, can be converted to other benefits such as higher salary for being an advanced skills teacher, or assuming leadership roles in mathematics within the school and so on. Thus, these skills are embodied into the teacher habitus to become forms of capital that can be exchanged for other goods within the field –such as salary, certificates, status. One of the key features of teacher education, whether preservice or in-service, is to build the capital of the teachers so that they become valued members of the field. But fields are not static and change. At this point in time there are valued forms of knowledge that convey status (and capital) and these are centred around teacher knowledge which is seen to create quality practices.

This paper reports on the findings from part of a much larger study. Here we discuss the findings of a survey in which we explored teachers' knowledge of mathematics content and pedagogy. We anticipated that primary school teachers were more likely to have strong mathematics pedagogical content knowledge (MPCK) and would not be as strong in mathematics content knowledge (MCK), with the reverse being the case for secondary school teachers. The results surprised us. This paper is intentionally reflexive as we track through the data and the implications for our thinking about the relationship between MCK and MPCK.

METHOD

This paper is part of a much larger study funded through the Australian Research Council's Discovery Grant system (DP1200101495) and reports on the findings from the online survey. The survey was the first phase of the project where we initially sought to identify teachers' and preservice teachers' backgrounds in the teaching of mathematics. We sought to explore the backgrounds of teachers, where they taught, where they grew up, and their current levels of knowledge in terms of mathematics content and pedagogical content knowledge, as the larger project was concerned with the socio-geographic implications of mathematics education. In this paper we draw on the MCK and MPCK elements of the survey.

Participants

The participants are teachers from all over Australia. The teachers were invited to participate in the on-line instrument described below. Participants were solicited through various means that included approaching principals in schools to pass on the request for participation, advertising through social media including Facebook, advertising through various mathematics organisations, and through personal contacts through previous research and consultancy work. Teachers identified their teaching background via the profile section of the survey. Initially we only sought to include teachers and preservice teachers. As an addendum to the study, we included a cohort of engineers. The rationale for the inclusion of the engineers will be discussed later in the paper.

Table 1. Numbers of participants undertaking the survey.

Participant	Preservice teachers	Primary teachers	Secondary teachers	Engineers	Total
Number	162	100	139	31	432

The instrument

The 'Social and Geographic Dimensions of Mathematics Education' questionnaire comprised four main parts –the first is demographic data that broadly included location and qualifications; the second section contained 11 mathematical content knowledge questions (MCK); and the third section 9 pedagogical content knowledge questions (MPCK), while the final section focused on beliefs and dispositions to teaching mathematics. The second and third sections were timed sections, which supported an effectiveness and efficiency measure. This paper reports on data from section two (MCK) and section three (MPCK). The knowledge sections consisted of items sourced from national and international tests designed for middle high school attainment levels. All items were trialled by other researchers or assessment corporations (such as Australian Curriculum, Assessment and Reporting Authority, Australian Mathematics Trust, The Mathematical Association of America, and Educational Testing Service) so that we can assume reliability and validity of the test items, and thus can be used for comparative purposes. The test items have all been published in peer-reviewed research papers within the mathematics education research community. Permissions were sought from authors and organisations to be able to use the test items in the survey. The selection of items was on the basis of ensuring a spread of content areas as well as a range of levels of mathematics. The mathematics content knowledge questions should be reasonably answered by junior secondary students, with some of the items being below this level. Questions were all multiple choice. For example, one question asked the respondents to select from four different representations of MABs of the number 32 which showed the underlying concepts of base 10 numeration (see appendix for test item).

FORMS OF CAPITAL

From here we discuss the MCK and MPCK as forms of capital that teachers have. We anticipated that primary teachers were likely to have more MCPK than secondary teachers as primary teachers are experts in pedagogy as they teach across many curriculum areas. Conversely, we anticipated that secondary teachers would have greater MCK than primary teachers as they (should) have been qualified in the discipline of mathematics whereas this is not the case for primary teachers. However, our results yielded a contradictory pattern which has resulted in us rethinking the results and building another dimension into the study, the inclusion of engineers. We write this next section as a reflexive piece as it was of considerable concern to the research team as to what we uncovered. To assist in the analysis of the data, we employed an external consultant to analyse the survey data.

As part of the survey, teachers were asked to rate themselves in teaching mathematics. On a scale of 1-10 where 10 was excellent and 1 was poor, 90.1% of the teachers rated themselves above 5, while 19.3% rated themselves on 9-10 on the scale. This suggests to us that the teachers, overall, had a very strong sense of themselves as being good teachers of mathematics.

In terms of their background in mathematics discipline, the level of mathematics studied at school varied between different teaching groups ($p<.001$). A majority of pre-service teachers had their highest level of maths at Year 12 General Maths (56.4%) while only 30% of primary practising teachers and 20% of secondary teachers reported highest level of maths studied at Year 12 General. Similarly, almost half (46.5%) of secondary teachers studied maths at Year 12 Specialised level, followed by primary teachers (34.7%) and last pre-service teachers at 8.0%.

MATHEMATICAL AND PEDAGOGICAL CAPITAL

The results for the survey are presented in the two areas –MCK and MPCK. The participants were assessed with 12 MCK and 11 MPCK questions. Each correct answer was given a score of 1, the responses were then summed up to give final score for each test domain. For each group of interest, we calculated the group's mean MCK and MPCK scores and the corresponding standard deviation. To identify if the mean between groups were statistically significant, we performed Turkey post-hoc tests. Initially we compare the MCK results for the primary teachers and secondary teachers using a Turkey post-hoc test and found that there were significant differences between primary teachers (6.27 ± 3.12), and secondary teachers (7.57 ± 2.62) ($p<0.015$). This was not a surprise as we can see from the studies of mathematics reported in earlier research, primary school teachers were less likely to study high levels of mathematics whereas this is not the case for secondary teachers whose expertise should be in the area of mathematics. Our background data also supported this, so it was not a surprise to obtain these results.

When considering primary and secondary teachers' MPCK we were surprised to find a counter-intuitive finding. Here we found that secondary teachers scored significantly higher than primary school teachers where the Turkey ad hoc analysis showed that the difference between junior teachers (5.64 ± 2.27), to senior teachers (7.02 ± 2.12), showed a difference of 1.38, p<0.003. What was alarming for us was that this component had nine items. The mean score for the primary teachers was 5.64 which suggested that they were only, on average just reaching a pass equivalent – assuming that a pass is 50%. All of the items in the MPCK section related to concepts taught in the primary school curriculum. In contrast, secondary teachers, who do not teach these constructs, were able to obtain a mean score greater than the primary teachers despite not being responsible for teaching this content.

At this point, we questioned our results, wondering why primary school teachers were scoring significantly worse on both measures than secondary teachers, but more concerning is why the teachers were scoring counter intuitively on MPCK. We questioned how teacher education may have been implicated (or not) so then compared these data sets with our data from the preservice cohort. We anticipated that teacher education should be adding capital to participants and thus, teachers would score significantly better than preservice teachers who were only commencing learning the craft of teaching. We also anticipated that as practicing teachers both MCK and MPCK would be enhanced as teachers worked through their craft and gained more experience and confidence in teaching.

For MCK, we found that this increases with the different cohorts. Mean scores increase from pre-service teachers (4.94 ± 2.80) to primary teachers (6.27 ± 3.12), to secondary teachers (7.57 ± 2.62) and the difference between each group is statistically significant using the Turkey post-hoc test (p<0.015). Similarly we found that there was an increase in MCPK across the cohorts pre-service teacher (4.95 ± 2.18), to primary teachers (5.64 ± 2.27), to secondary teachers (7.02 ± 2.12). Interestingly (or alarmingly) the only significant differences were between the secondary teachers with the other groups (p<0.001). That is, there is no significant difference in MPCK between primary school teachers and preservice teachers.

So what does this suggest? In many team discussions we perplexed, contemplated, wrestled with these findings. Do they suggest that preservice teacher education is not adding capital –particularly in MPCK– to prospective teachers seeing that there are no differences in the MPCK between preservice and practising primary teachers? If this is the case, what is happening in preservice teacher education? Or is it the case that there is no growth in MPCK as teachers move into their positions in schools. One would hope that as building the teaching capital of teachers is the primary purpose of teacher education that there would be a marked growth in the MPCK between teachers and preservice teachers. Or is it the case that teacher education is so successful that teachers exit their programs with most/all the knowledge they will demonstrate as a practicing teacher? Or is there a link between having strengths in MCK that flow to MPCK which we see in the secondary teachers but not in the primary teachers?

ENGINEERS: MATHEMATICAL AND PEDAGOGICAL CAPITAL

It was our assumption (to be read as hope) that preservice teacher education and in-service education does make a difference. To explore this further, we modified our research design to incorporate a cohort of engineers. These are professionals who must, by the nature of their work, have strong MCK but as they are practitioners in their fields, it is reasonable to expect that they have no teacher education. Including this cohort would help us better understand the role of MCK in supporting (or not) MPCK. To this end, we secured a cohort of 31 engineers who worked across many fields of engineering so it was a diverse group. As we were concerned with only the MCK and PCK of the engineers, they were only asked to complete the first three sections of the survey and any questions relating to beliefs about teaching and their experiences as a teacher were removed from the survey instrument. We did ask if they had had teaching experience but this was a negative response.

Unsurprisingly for the research team, the engineers scored the highest score in MCK (9.42 ± 1.67) and were statistically different from the other three cohorts using the Turkey post-hoc test ($p<0.015$). As part of the engineering qualification, engineers must study high levels of mathematics so this was not a surprise. However, what was of most interest to the research team was the score in MPCK. It is noted that none of the engineers had undertaken any studies in teacher education. All test items in the MPCK phase of the survey related to aspects of teaching mathematics. The mean scores for the engineers on MPCK was 4.67 ± 1.96 whereas pre-service teachers scored 4.95 ± 2.18 and primary teachers 5.64 ± 2.27. There was no statistical difference between these three cohorts. The only statistical difference in MPCK was between secondary teachers and the other three cohorts. This says to us that despite never having undertaken any formal study of mathematics teaching and learning, engineers performed close to preservice teachers and primary school teachers in MPCK.

So what do these data tell us? Preservice, primary and engineers can be seen to have relatively similar MPCK, despite one cohort never having studied teaching. To try to understand what these data reveal, we will refer back to some of the research in MCK and MPCK. For this paper, we intend to conclude the paper with questions and implications of these findings and ask what they mean for the field of mathematics education and mathematics teacher education.

How important is MCK?

Krauss and colleagues (Krauss et al., 2008) have argued that high expertise in a domain provides much more opportunity for stronger integration of other domains of expertise and, consequently, strong MCK supports the development of MPCK. Wu (2011, p.381) makes a stronger assertion when stating 'what must not be left unsaid is the obvious fact that, without a solid mathematical knowledge base, it is futile to talk about pedagogical content knowledge.' These positions have been supported by the comprehensive work of Hill et al. (2008) where they argued that students' gains were most strongly correlated with teachers' MCK. What we see from the data in this study is that MCK may be critical for teaching.

From our analysis, we were compelled to ask ourselves whether or not the observed outcomes were a reflection of MCK knowledge. Is it the case that if a teacher (or engineer) has strong MCK that they have a better understanding of how to teach/learn mathematics. Such a proposition might help to explain our outcomes. Those teachers who have strong MCK were stronger in MPCK, and those (engineers) who never studied teacher education yet had strong MCK were similar to those teachers who had studied teacher education and/or had been involved in the teaching of mathematics. This could suggest that strong MCK ameliorates the effect that teachers with poor MCK spend learning how to teach. It does beg askance of how teacher education is adding pedagogical capital to teachers when outsiders (engineers) can perform comparatively similar to those who have studied educational practices. It begs the question to be asked as to whether or not MCK is sufficient for being a good teacher in mathematics?

How important is teacher education?

There are some researchers who question the possibilities of teacher education to actually make a difference to the professional work of teachers (Brouwer & Korthagen, 2005), in part, through the conservatism that is entrenched in schools (Brouwer & Korthagen, 2005), and mathematics departments (Guitterez, 1990). But our data suggest something potentially more sinister.

What was observed was that there was no significant difference in the MPCK of preservice teachers and primary school teachers. Initially we though that this might be a reflection of the high quality of initial teacher preparation and that graduates were exiting with pedagogical skills relatively commensurate with primary school teachers. However, the inclusion of the engineers suggested something else. The engineers' MPCK was not significantly different from the preservice teachers who would have 2-4 years studying pedagogy and primary teachers who varied from new graduates to long standing practitioners with ≥16 years practice. While their mean score was lower than teachers who had studied pedagogy and worked in classrooms, they were scoring relatively comparably to these cohorts despite never having formal qualifications in teacher education nor being a practitioner in schools. This begs askance of how much capital is being added to teachers as a result of preservice, in-service and service in the classroom. Further it questions whether preservice teacher education may be better served through the inclusion of mathematical content courses rather than mathematics education courses.

How important is MCK for educators' MPCK?

It would seem to us that the inclusion of the engineers into this study raises very serious questions about teacher preparation and the importance of discipline knowledge in the quality of teachers. These findings suggest that MCK impacts significantly on MPCK. This finding poses serious challenges to teacher education programs. As researchers and teacher educators, these findings have challenged many of our assumptions about the content in teacher preparation courses. Our faith in mathematics education courses has been challenged and we have been caused to rethink the role of mathematics as a discipline and its impact on teaching. These findings suggest that MCK has significant influence on the capacity to teach mathematics well.

CAVEATS AND LIMITATIONS

In the study we have worked from the assumption that our test items are reliable given that they have a strong research basis to them. Our findings are reported against this assumption.

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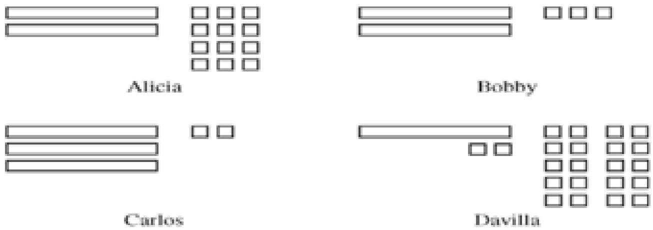
APPENDIX: EXAMPLE OF A TEST ITEM

1. The illustrations below show how four students – Alicia, Bobby, Carlos and Davilla – used base 10 blocks to represent the number 32.



Which of the students used the blocks to represent the number 32 in a way that does not indicate an understanding of the underlying concepts of the base 10 numeration system?

Tick the circle beside the correct answer

(A) Alicia O

(B) Bobby O

(C) Carlos O

(D) Davilla O

Source: Educational Testing Service: The Praxis Series (2011). Elementary Education: Curriculum Instruction, and Assessment (0011/5011)

INVESTIGATING PARENTAL INFLUENCES ON SIXTH GRADERS' MATHEMATICAL IDENTITY: THE CASE OF ATTAINMENT

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In this paper, we present aspects of a broader study aiming to investigate parental influences on sixth graders' mathematical identity. We concentrate on students' attainment: In which ways do sixth graders perceive themselves (identifying the other, being identified and self-identification) concerning their mathematics attainment? The study was conducted in a typical Greek primary school where sixteen children were interviewed. The analysis results revealed that two student groups could be identified with respect to their self-identification about mathematics: Group A included those who are self-identified as 'good' at mathematics and Group B included those who identified themselves as being 'average' at mathematics. In the 'good' students group, family seems to function as a consistent system with respect to attainment, contrasting the more diverging 'average' group.

INTRODUCTION

Mathematics education research has relatively recently focused on the relationships between in-school mathematics learning and the broader socio-cultural environment. This interest is related to the socio-cultural and socio-political approaches to mathematics education and is in line with the attempt to gain deeper understanding of the identified variation in the students' learning mathematics. Sfard and Prusak (2005) argue that the notion of identity can be used as a tool for investigating learning as a culturally shaped activity. The notion of identity has been widely discussed in mathematics education research (for example, Abreu & Cline, 2003; Anderson, 2007; Crafter & Abreu, 2010; Wenger, 1998). The main characteristics of identity include its constant construction through interactions amongst people, connected with questions concerning the links between personal worlds and collective discourses. Mathematical identity is constructed through the individuals' participation in different communities of practice, including school classrooms, family context and the broader community. From this point of view, the family practices could provide different opportunities to the students' construction of mathematical identity, a diversity that constitutes a socio-political issue for mathematics educators. In this paper, we present aspects of a broader study that investigates parental influences on sixth graders' mathematical identity. Parents were chosen as the 'significant others' in the students' life, as studies in Greece support that mathematics are at the crux of the parental involvement about their children's learning, since the Greek parents appear to spend more time with their children for mathematics, when compared with the time spent about other courses in primary school (Kafoussi, 2009).

THEORETICAL FRAMEWORK

The last decades, many researchers have investigated the role of the students' family in the students' developing positive attitudes and better achievement in mathematics (Cao, Bishop & Forgasz, 2006; Galindo & Sheldon, 2012; Jacobs & Bleeker, 2004; Moutsios-Rentzos, Chaviaris & Kafoussi 2015; Wang, 2004). These studies suggest that the parents' views about the value of mathematics affect their children's attitude towards mathematics, whilst the parents' high expectations about their children mathematical attainment appears to positively affect both their attainment and their self-estimation or self-confidence (Jacobs & Bleeker, 2004; Wang, 2004) and even more from their early years of schooling (Galindo & Sheldon, 2012). Moreover, the broader family support (such as number of books at home) has been found to be positively linked with the students' attainment (Hyde, Else-Quest, Alibali, Knuth & Romberg, 2006; Wang, 2004). Furthermore, the quality of the collaboration between parents and students –rather than the mere amount of the time spent– crucially determines the effectiveness of parental involvement on the students' preparing their homework (Hyde et al., 2006; Pezdek, Berry & Renno, 2002). Finally, concerning the similarities between mother and father's involvement, though it has been found that within the family unit exists an internally consistent value system about mathematics, the students' perceived role of the mother about mathematics seems to to differ from the role of the father (Moutsios-Rentzos et al., 2015); the children's perception of their mother as being good at maths was positively linked with her helping with difficult mathematics problems at home, while their perception of the father as being good at maths had broader positive links (including checking homework frequently, asking about assessment results, encouragement; Moutsios-Rentzos et al., 2015).

From a socio-cultural aspect of parental involvement about mathematics, mathematics education research has suggested that the students' perceived parental involvement about mathematics is linked with the students' age, country of residence and language (Cao, Bishop &

Forgasz, 2006). Moreover, when considering students who are not in the dominant sociocultural community of a country, parental involvement is a crucial factor for their appropriate social integration (Crafter, 2012). Investigating the resources that parents from different cultural backgrounds used in order to understand the mathematical achievement of their children, Crafter identified three dominant resources: the teacher, exam test results and constructions of child development. However, all these resources were interpreted by the parents in different ways according to their own cultural models and this issue could cause misunderstandings between home and school. Furthermore, Abreu et al. (2002) studied the parental support to their children' transitions between school mathematics and mathematics at home revealing that the parents' views about their own involvement or not in their children's learning varied depending on the ethnic origin.

Crafter et al. (2010) drew upon the theoretical framework of Abreu and Cline (2003) to investigate ethnic minority students' identities about mathematics learning within home and school communities. Abreu and Cline (2003) posited that the students' mathematical identity incorporates three complementary processes: a) *identifying the other*, b) *being identified* and c) *self-identification*. "Identifying the other" concerns individual's understanding of social identities of others, "being identified" concerns individual's understanding of identities extended to themselves by others and "self-identification" refers to the internalized and individual level of identity. Describing the results of two case studies, Crafter et al. (2010) concluded that these three processes allow the study of students' identity within school and home community.

Based on the above research results and theoretical considerations, it seems that a fundamental question for mathematics education nowadays concerns the basic characteristics of the family context (mother, father) that the students is part of which will support the development of a creative relationship with mathematics. In our broader research, we employed the theoretical context of Abreu and Cline (2003) in order to investigate parental influences on sixth graders' mathematical identity. In this investigation, we focused on six axes in order to search for these influences: attainment, child-parent collaboration for homework, nature of mathematics, attitudes, values and self-confidence. In the present paper, we concentrate in the first axis: In which ways do sixth graders perceive themselves (identifying the other, being identified and self-identification) concerning their mathematics attainment?

METHOD

The study was conducted in May 2015 with students attending the 6th grade (12 years old, last grade of primary school in Greece) of a typical primary school in Athens, Greece. We used narrative theory of identity (Sfard & Prusac, 2005) in order to collect our data through the use of structured interview. Sixteen children were interviewed in their school by their teacher, but only fifteen were included in the study (one student had learning difficulties). Each interview lasted about 30 minutes. The interview was structured through a series of questions for each axis, in line with the tri-focussed model of Abreu and Cline (2003). The attainment axis, reported in this paper, included the following questions: a) *Self-identification* (Do you believe that you are good at mathematics? What makes you believe that?), b) *Being identified* (Does your mother believe that you are good at mathematics? What makes you believe that?; Does your father believe that you are good at mathematics? What makes you believe that?), and c) *Identifying the other* (Do you believe that your mother is good at mathematics? What makes you believe that? Do you believe that your father is good at mathematics? What makes you believe that?)

The analysis of the results drew upon phenomenographic analysis (Marton, 1981; cf. Newton & Newton, 2006) concerning two axes: a) students' positioning about their attainment in mathematics, and b) students' resources with respect to their positioning.

RESULTS

The analysis results revealed that two student groups could be identified with respect to their self-identification about mathematics: Group A included those who are self-identified as 'good' at mathematics (N_A=7) and Group B included those who identified themselves as being 'average' at mathematics (N_B=8). No student was self-identified as 'bad' at mathematics. This grouping proved to be a useful differentiation as it was linked with qualitative differences about the experienced parental involvement.

The students' positioning about their mathematics attainment

In Table 1, we summarise the students' positioning about their mathematics attainment through self-identification and being identified by their parents.

Table 1. Self-identification and being identified by parents (father and mother).

Self-identified		*Being identified by*					
Group		Mother			Father		
		good	average	don't know	good	average	don't know
A (good)*	7	7			5	1	
B (average) *	8	3	2	3	2	1	4

* Two single-parent families were included in the study.

Five of the students in Group A ('good') reported that both of their parents also consider them to be 'good' at mathematics, whilst only one student stated that his father consider him as 'average': "[My father] doesn't believe that I'm good, he thinks I'm ok" and "Yes, in general she [my mother] thinks I am doing fine considering mathematics". On the contrary, five students of Group B ('average') stated that they did not know about at least one of their parents' perception of their child's mathematics attainment. In specific, two of the students reported both of their parents as 'don't know', two only their father and one only their mother. Only one self-identified 'average' student reported that both of their parents think of him as being good, whilst another reported that only his mother considers him as 'good' (and his father as 'average'). In both single-parent families, we found complete agreement between the daughters and their mothers.

Consequently, though the 'good' students experience a consistency in their positioning about mathematics attainment between self-identification and being identified by their parents, the 'average' students do not experience such a consistency. Importantly, the 'average' students appear to not know what their parents think of their children's mathematics attainment.

'Self-identification' resources

Considering the students' self-identification resources, we identified four dimensions (summarised in Table 2), with two of them reported only by Group A students ('good'):

- *Dealing with school tasks*. The students linked their success (or not) in dealing with school mathematics tasks with the mistakes made, the difficulty of solving problems and the lack of understanding. For example, "I believe that in general I don't have 'gaps' [referring to the maths knowledge], it is just that sometimes I make mistakes because I do not pay attention [...] few are the problems that I cannot solve" (stud9, Group A); "in problems, I have difficulty in thinking solutions" (stud3, Group B); "I am not sure why sometimes also depending on the topic I may get confused, not understand well" (stud7, Group B).
- *Individual practices and characteristics*. The resources of this dimension refer mainly to the time spent studying, to the time that they devote to the specific course at home, or to their lack of memory or attention. For example, "Because there are times that I study and there are times that I don't study much" (stud7, Group B); "I devote time to mathematics, I sit down many hours and study" (stud1, Group A); "I don't remember well theories, that is I cannot remember staff that we were told three years ago and I may mistake one for another sometimes" (stud2, Group B); "Many times in class I don't pay attention... and sometimes I am not focused in at home" (stud14, Group B).

- *Affective reinforcements*. In this dimension, we included the students' affective resources that included the students' experiencing pleasure, ease and usefulness of mathematics. It should be stressed that only students of Group A ('good') reported this resource. For example, "I think that it [mathematics] is easier than the other courses" (stud1, Group A), "I have liked it [mathematics] since little girl" (stud6, Group A); "Because my favourite subject is mathematics and I only work with that" (stud5, Group A).
- *School performance*. This dimension was reported by only one Group A student: "Because I have seen how well I go with maths at school" (stud8, Group A).

Table 2. 'Self-identification' resources.

Resources	Group A	Group B
Affective reinforcements	4	
Dealing with school tasks	2	5
Individual practices and characteristics	2	4
School performance	1	

Consequently, the 'good' students consider as their main resource the affective reinforcement they experience from mathematics, whilst the 'average' students perceive themselves mainly through their (in)ability to cope with the school tasks or through their lack of studying at home and their lack of paying attention.

'Being identified by their parents' resources

Considering the students' being identified by their parents' resources, we differentiated seven dimensions (see Table 3), with one of them reported only by Group A students ('good') and one reported only by Group B students ('average'):

- *Interaction with parents at home*. The child-parent interaction at home concerned three aspects: parents' checking through mathematical activities, parents' checking about school activities, and parents' discussing their child's school behaviour. For example, "Yes, she gives me exercises [to solve] and checks how well I do, I do it quickly, correctly (stud1, Group A); "Yes average … because when he helps me sometimes I get stuck" (stud16, Group B); "Because when she sees me doing mathematics at home with the exercises, she sees that 'I've got it' [meaning he is good] with mathematics" (stud11, Group A); "I tell her that most of the exercises are correct, my hand is raised in mathematics" (stud1, Group A).
- *Lack of or 'surrogate' interaction with parents at home* (Group B only).

For example, "because I don't study with her mathematics and she doesn't know" (stud3, Group B); "I don't know … because my mother has brought a person named George to tutor me" (stud4, Group B).
- *Parental expectations*. Parental expectations were linked with their trust towards their child. For example, "Because, she believes in me that I can do more in mathematics" (stud5, Group A); "… she believes that I can do it and even if I don't ask for her help (stud9, Group A).
- *Teacher assessment*. The teacher's assessment was linked with school grades. For example, "Because she has seen the mathematics grades of the two trimesters" (stud8, Group A); "so from the report card and only" (stud3, Group B).
- *Comparison with parents* (Group A only). A student mentioned her comparison with her mother. For example, "Yes, because she's also good and she sort of knows what I can do and she compares with her own [mathematics knowledge] and she thinks that I am good" (stud12, Group A).
- *Individual practices and characteristics*. Including, Time ("because he sees me and asks 'what are you doing?' mathematics … 'what are you doing?' mathematics. I work a lot and my father sees it"; stud1, Group A), Mistakes ("because I don't make the common mistakes, I am making the so-called 'careless' mistakes"; stud9, Group A), and Study ("in exercises she tells me that I do them in a kind of a rough draft, just to finish them"; stud10, Group B).

Table 3. 'Being identified by the parents' (mother and father) resources.

Resources	Group A		Group B		N of Students
	Mother	Father	Mother	Father	
Interaction with parents at home	4	2	3	3	9
Lack of or 'surrogate' interaction with parents at home			2	3	4
Parental expectations	2	1	1		3
Teacher assessment	1	1	2	1	3
Comparison with parents	1				1
Individual practices and characteristics		2	1		3

Drawing upon the above, considering the students' constructing their parents' perception of their mathematics attainment, it appears that the main resource mentioned is the interaction with their parents. For the 'good' students this interaction has positive attributes, whilst for the 'average' students this interaction has either negative attributes or it is missing. Notably, school assessment does not seem to constitute important resource for the parents according to the students.

Furthermore, Group A was characterised by the fact that the mother and the father are described as having the same resources, whilst in Group B resource differences were identified between the mother and the father. In specific, three students reported the lack of collaboration with one of the parents at home. This may suggest that the 'good' students experience that their family functions as a consistent system concerning their attainment, contrasting the identified divergences in the 'average' students' families. Finally, the lack of child-parent interaction was linked the students' 'don't know' about their being identified by their parents.

'Identifying the other'

Most of the students think that both of their parents are good at mathematics (see Table 4). Nevertheless, it should be noted that the characterisation 'average' predominantly referred to the mother and was the results of the students' spontaneously comparing the two parents. For example, "My father is definitely better ... because if I don't know something, I ask him, he answers. My mother may not reply immediately" (stud4, Group B).

Table 4. 'Identifying the other' (mother and father).

Student	Mother good	Father good	Mother average/not good	Father average/not good
good	5	6	2	1
average	5	8	3	

Considering the resources employed in the students' identification of their parents, five different resources were identified (see Table 5):

- *Interaction with parents at home*. The child-parent interaction concerned the quality of parental help at home. For example, "She explains it, whatever I haven't understood is like having a lesson ...yes I understand it well" (stud1, Group A); "He showed me an easy way to do divisions without having difficulties" (stud5, Group A).
- *Lack of interaction with parents at home* (only Group A). For example, "When I ask him to help me with the exercises, he talks about other issues to get away with it" (stud8, Group A).
- *Parents' self-declared mathematics attainment*. This resource was connected mostly with the identification of their mothers. For example, "Every time I ask for her help she tells 'look 'I don't have it' that much with maths'" (stud9, Group A).

- *Parents' profession*. For example, "My father because of his profession I believe that he is good. He uses measurements every day, he remembers it in general. Ok mathematics he is good" (stud9, Group A).
- *Everyday practices* (only Group A). For example, "They both sit for hours doing the calculations, they don't use calculator (stud1, Group A); "When she does the operations with big numbers she does it in less than a minute, very very fast and she is very very good (stud12, Group A).

Table 5. 'Identifying the other' (mother and father) resources.

Resources	Group A		Group B		
	Mother	Father	Mother	Father	N of Students
Interaction with parents at home	5	4	4	3	10
Lack of interaction with parents at home		1			1
Parents' self-declared mathematics attainment	3	1	3		6
Parents' profession	1	2	3	4	6
Everyday practices	1	1			1

Regarding the students' perceptions of their parents it appears that the employed resources qualitatively differ from the resources employed when being identified by their parents. Though the child-parent interactions seem to be dominant again, several students seem to be affected by their parents' profession or by their parents' self-declared mathematics attainment. Moreover, profession appears to be more dominant in Group B ('average'). Finally, no significant mother-father differentiations for each child were noticeable with respect to 'identifying the parent'.

CONCLUDING REMARKS

Drawing upon the findings discussed in this paper, we posit that by studying the students' identity with respect to their mathematics attainment we may differentiate between two self-identified groups that are linked with qualitative differences in their experiencing the parental effect in mathematics: the (self-identified as) 'good' at mathematics students and the (self-identified as being) 'average' at mathematics. In the 'good' students group, family seems to function as a consistent system with respect to attainment, contrasting the more diverging 'average' students group (notably, the father-mother 'being identified by the parent' divergence). The good students seem to experience a consistency in their declared positioning about their mathematics attainment (through both self-identification and being identified by their parents), as well as in the resources that their parents employ. This is in contrast with the 'average' students, who experience differentiations between their self-identification and their being identified by their parents.

Furthermore, it must be noted that the students' resources about their mathematics attainment are crucially affected by the child-parent interaction at home (or the lack of such interaction), especially for the processes of 'identifying the other' and 'being identified'. On the contrary, school assessment does not seem to constitute an important resource for the students. This implies that in the case that the construction of attainment is powerful for the formation of students' identity (Crafter & Abreu, 2010), the basic characteristic of the family context (mother, father) that the Greek students notice for the development of a 'positive relationship' with mathematics is the existence of a fruitful interaction with their parents. As 'everyday homework' is a dominant cultural characteristic of Greek educational system the role of family seems to become crucial for students' mathematical experiences. This means that the educational design should be reformed to include practices that facilitate the teachers' and the parents' awareness of this issue.

Consequently, it is argued that the employed research framework (theory and method) seems to be useful in the identification of parental influences on students' mathematical identity, helping in our gaining deeper understandings of the complementary multiple constructions of the students' identity. The analysis results of the remaining five axes investigated in our research is expected to provide us with a clearer picture about parental influences on students' mathematical identity.

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WHAT IS THE ROLE OF VALUE ALIGNMENT IN ENGAGING MATHEMATICS LEARNERS?

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In an economic and political crisis, people are crossing national borders in search of a better life. Students from diverse cultures are entering mathematics classrooms and teachers are adopting strategies for value alignment to nourish mathematics teaching and learning practices. Teachers are striving to create harmonious learning environments so that diversity is embraced positively and is utilised in a manner that will enrich mathematical learning. Value alignment allows diversity to be rethought as an empowerment tool for effective mathematics learning. Vignettes from a recent PhD study are used to exemplify strategies that teachers used for value alignment and student engagement in mathematics classrooms.

DIVERSE MATHEMATICS CLASSROOMS AND VALUE ALIGNMENT

The movement of people across country borders is taking place in unprecedented levels due to reasons such as armed conflicts, globalisation and regionalisation of trade and business (Seah & Andersson, 2015). Students from diverse cultures are creating diversity in mathematics classrooms through their values. In countries such as Australia and Singapore, culturally diverse classrooms are part of the 'norm' due to the in-take of refugees and historical immigration. A finding from international comparative studies such as TIMSS (Trends in International Mathematics and Science Study) and PISA, is that effective teaching is more about responding to and valuing the socio-cultural aspect of the learning environment than it is about adopting particular teaching methods (Hollingsworth, Lokan, & McCrae, 2003; OECD, 2004). A teacher who facilitates value alignment in the classroom promises to strengthen the relationships between individuals and will nourish teaching and learning practices (Seah & Andersson, 2015).

Students make sense of and construct mathematical ideas in different ways, drawing upon their own unique experiences in life and mathematics learning (Seah & Andersson, 2015). Values in mathematics education are inculcated through the nature of mathematics and individual experience, and thus become the personal convictions that an individual regards as being important in the process of teaching and learning mathematics (Seah & Kalogeropoulos, 2006). In a culturally diverse mathematics classroom, a teacher has the opportunity to travel the world with her/his students and return with a metaphorical suitcase filled with diverse thoughts, opinions and values in a single day. In this paper, we consider how these ideas or values empower a teacher to enrich students' mathematics learning?

Teachers and their students enter their classrooms with their personal values. The decisions and actions that teachers and students make in a mathematics lesson reflect their respective valuing. As diversity in mathematics classrooms around the world increases due to reasons such as crisis, development and growth, so does the demand for the mathematics teacher to become flexible in embracing difference. Given the stable nature of values, it is reasonable to argue that teachers and students cannot expect that the other party will naturally share their valuing. However, in an organisation such as a classroom environment, it is reasonable to assume that teachers and students will want to co-exist harmoniously and therefore they will adopt strategies to exhibit values such as tolerance, respect and acceptance without compromising their own values. In particular, this paper addresses the question how is this value alignment achieved in a mathematics classroom?

In a recent PhD study (Kalogeropoulos, 2016) that was conducted in Melbourne, Australia, the role of value alignment in engagement and (dis) engagement in mathematics learning was investigated. Four year 5/6 teachers and sixteen year 5/6 students from two different schools participated in the study. Teacher and student questionnaires, classroom observations and interviews were used as instruments to collect data. Critical incidents that arose in the observed mathematics lessons reflected value conflicts and how these were resolved to restore harmony and engagement within the mathematics lesson. In this paper, four vignettes are offered as examples of how conflict can be resolved through value alignment in situations of cultural diversity.

FOUR VALUE ALIGNMENT STRATEGIES

Vignette 1 – The Scaffolding strategy

The scaffolding strategy is adopted by a teacher when they come to their mathematics lesson with some type of preparation to scaffold the learning of the intended learning objectives (Kalogeropoulos, 2016). In one episode noted in the PhD study and one that is rather commonly encountered in mathematics classrooms, a graduate teacher asked their students to complete a challenging worded problem independently. Certain students attempted the task but soon complained that the question was "too difficult" and began to disengage with their mathematics learning. The teacher responded by directing the students to complete similar but simpler word problems with a peer. The teacher's initial values of *independent work-style* was in conflict with their students' valuing of *small group work*.

Student: It's better to be working together, sometimes you might not know something, your team mate can help you.

The teacher adopted the scaffolding strategy to reengage the learners by offering worded problems that were more suited to the learner's abilities and by allowing them to work in small groups for support. This practice was successful in this situation but it may not be 'equally' effective in another. For example, in a culturally diverse mathematics classroom, the option to work in small groups may appear daunting to a student who is limited in speaking/understanding the spoken language. In catering for students with language needs, a teacher may decide to offer additional time and support. Therefore, depending on the situation, other value alignment strategies could also be preferred.

Vignette 2 – The Equilibrium strategy

The equilibrium strategy is adopted by teachers when students unexpectedly refer to values that are not being catered for in their lesson (Kalogeropoulos, 2016). A critical incident arose when students requested a calculator to check their answers in class. The teacher's response outlined a conflict in values.

Teacher: You won't have a calculator during NAPLAN [National Assessment Program Literacy and Numeracy] testing, so don't use it now.

For value alignment, the teacher decided that she/he would collect and correct the students' work, acknowledging the students' value of *accuracy*. The teacher maintained her/his initial values but also accommodated the students' expressed value, acknowledging *accuracy* as an important value in mathematics learning. This is an example of a classroom interaction between the teacher and the students representing a site of contestation and conflict which naturally occurs in mathematics learning but with the teacher facilitating the students' values, the student learning of mathematics is optimised (Seah & Andersson, 2015).

Vignette 3 – The Intervention strategy

There are times in mathematics learning when teachers put most of their values aside and respond to the students' values that are being exhibited (Kalogeropoulos, 2016). As an example from the PhD study results, a student described their feelings when they were unable to independently complete mathematics tasks:

Student: I am just picturing myself in the 5/6 unit, I wish I had something else to work on in Maths and then it got even harder because Ms Belinda gave us even harder ones.
Interviewer: And did you cry that day?
Student: Yeah sort of…sometimes embarrassed

This conflict situation was resolved when the teacher became attentive to the anxiety of the student and decided to offer her/him one-to-one assistance. The student reengaged with their learning because her/his teacher had intervened and provided the help that she/he required. In a 'multicultural' classroom, this strategy could be deployed frequently due to the diverse learners but is it the most effective but time-consuming strategy? Further studies would be required to determine this.

Vignette 4 – The Refuge strategy

Finally, the refuge (not to be confused with the term refugee) strategy is when the teacher puts most (if not all) of her/his values to the side and uses her/his authority in a manner that postpones her proposed lesson planning and succumbs to the value orientations of the students (Kalogeropoulos, 2016). In this situation, the teacher usually finds new values that are aligned between her/himself and the students. In a lesson observed, the students became stumped by a problem solving task that the teacher had planned. Even after three attempts of the teacher trying to explain the steps required to solve the mathematical problem, the students remained confused, became agitated and disruptive. The teacher spontaneously made a decision to play a mathematical game with her students. The chaotic classroom reformed to an enthusiastic environment as the teacher and the students' value of *fun*, was embraced. In a culturally diverse mathematics classroom, a teacher must be prepared and skilled when students demonstrate a particular interest in learning about a mathematical concept that they are unfamiliar with. For example, students may become more inquisitive during the introduction of new number systems and currencies. A teacher may need to display flexibility and make detours from their intended lessons to accommodate these areas of interest.

The four value alignment strategies described above were classified, based on the extent to which the mathematics teacher retained her/his values after value negotiation had taken place (see figure 1). These have been called, in decreasing order of teacher values retained: scaffolding, equilibrium, intervention and refuge (Kalogeropoulos, 2016).

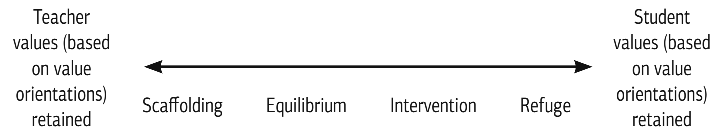

Figure 1: The four value alignment strategies.

The adoption of these strategies does not indicate that a mathematics teacher is losing control of their lessons. Far from it, it is suggesting something of the opposite. A teacher is conscious of triggers that lead to student disengagement and therefore uses her/his professionalism to adopt a value alignment strategy in an attempt to avoid a state of disengagement (Kalogeropoulos, 2016). In fact, such a mathematics teacher would be capable of recognising student values explicitly or implicitly and respond to these through an appropriate value alignment strategy. Could this be characteristic of an effective mathematics teacher?

OTHER RELATED STUDIES

Teachers' and students' values concerning effective teaching play a subtle yet influential role in the learning and teaching process (Zhang & Seah, 2015). Recently, Chinese students have achieved outstanding performances in PISA (Thomson, De Bortoli & Buckley, 2013). A recent study on the underlying values of effective mathematics teaching perceived by teachers and students in Chinese Mainland suggested that both students and teachers value *fun, involvement, multiple methods,* and *examples* (Zhang & Seah, 2015). Are these four shared values of students and teachers in Chinese Mainland common to other places around the world?

In a different but related study, investigating the value structures of three Chinese regions, the results revealed six dimensions in the students' value structure, namely *achievement, relevance, practice, communication, information and communication technology* as well as *feedback* (Zhang, Barkatsas, Law, Leu, Seah & Wong, 2015). If 'values in mathematics education are the deep affective qualities which education fosters through the school subject of mathematics' (Bishop, 1999, p. 2) then how do teachers help students develop these values especially if the students have received minimal exposure to formal schooling? It has been suggested that the reason why East Asian students perform better in international comparative tests is due to their cultural value of *achievement* that has been internalised over the years as learners and provides them with the ambition to succeed in school mathematics (Seah & Andersson, 2015). So, how could the value of *achievement* be developed in children who are still fighting to survive?

'Students not only bring to school prior achievement…but also a set of personal dispositions that can have a marked effect on the outcomes of schooling' (Hattie, 2009, p. 40). Home factors can significantly influence the educational development of a student through parental support e.g. showing interest towards one's child's learning, with empathy towards their feelings and frustrations. Also, by providing students with resources such as calculators, to support their learning can also achieve positive educational outcomes (Bishop & Kalogeropoulos, 2015). For example, immigrant children need to adjust to a new set of values in their new environment and run the risk of conflict between home and school values.

> A different approach to the mathematical learning of minority students emphasises the crucial role of classroom discourse in shaping and building their learning identities; acknowledges the distance between the social and cultural frames of reference of pupils and the ones implicit within the school; and attempts to use cultural diversity as a resource, considering the contributions of ethnic minorities as a source of richness to be maintained and shared (Chronaki, 2005, p. 61-62).

In order for students to embrace mathematics learning positively, it is necessary to address the links between specific mathematics practices and the broader social values that the students assign to these mathematical practices (Chronaki, 2005). This way, mathematics becomes interesting, relevant and a solid foundation for value alignment.

The actions that teachers take (or choose not to take) may support the flourishing and well-being of learners and others or impact negatively on them (Boylan, 2016). For example, during a critical incident in a mathematics classroom, the teacher is provided with an opportunity to adopt a value alignment strategy or ignore the students' portrayed values and persevere with the planned task. These situations could entail ethical choices that are ambiguous and cannot be resolved through applying a principle of set rules (Boylan, 2016). Instead, teachers could use the value alignment strategies (mentioned previously) to help them determine the student values that are being portrayed and how a negotiation can take place to ensure student engagement in mathematics learning.

Student engagement is a highly complex and multi-faceted construct (Fielding-Wells & Makar, 2008). The composition of a culturally diverse classroom involves students who bring with them personalities, values, and attitudes that are influenced by their own cultures. The schooling of immigrant children is a transition process where they are required to cope with many changes involved in moving from one culture to another (Gorgorió, Planas & Vilella, 2002).

This study has shown that it is important to:

- acknowledge the student as an individual.
- understand the meanings that students attach towards people and their environment.
- acknowledge the student as a member of the classroom community.
- recognise the meanings as social products developed from social interactions between members of the classroom community.
- acknowledge the student as an individual with a sociocultural identity (Gorgorió et al., 2002, p. 33)

Students attach values to situations, to actions, to themselves, and to others through an interpretive process, which is revised and controlled through the acquisition of new experiences. As mathematics classrooms become increasingly culturally diverse in times of crisis, there is a growing need for teachers to consciously engage their students through value alignment.

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COLLEGE INSTRUCTORS' ATTITUDES TOWARD STATISTICS

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This study explores the attitudes of college instructors of statistics, which are known to have an impact on their teaching and on the learning of their students. To address instructors' attitudes to current statistics teaching guidelines, we employed a classification of attitudes into three previously established pedagogical components of attitude: affective, cognitive, and behavioral. The quantitative data were collected from 92 college instructors of statistics. The study compares the participants' responses in terms of the three pedagogical components across three groupings of teachers: by gender, by academic background, and by statistics teaching experience.

INTRODUCTION

Research has demonstrated the importance of non-cognitive factors in cognitive performance in mathematics learning (Schoenfeld, 1992). Findings consistent with those in mathematics education have been reported in the field of statistics education as well. For example, non-cognitive aspects of teachers' learning of statistics impact how they implement statistics curricula, their beliefs on the usefulness of professional development programs, and their students' statistics learning (Estrada, Batanaro, & Lancaster, 2011; Lancaster, 2008). Recent studies, however, point to the ubiquity of negative attitudes toward statistics among pre- and in-service primary teachers (Estrada, 2002). For instance, many teachers believe statistical methodologies can manipulate results, mistrust statistics presented in the media, and avoid using statistics in their daily lives. Such findings regarding primary teachers suggest the need to investigate teachers' attitudes toward statistics beyond the primary level, as noted by Martins, Nascimento and Estrada (2012). Hence, this study explores the attitudes toward statistics held by college instructors who have taught or potentially will teach introductory statistics. For brevity, we refer to college level introductory statistics as "introductory statistics," and instructors who have taught (or could teach) statistics as "college statistics instructors" (CSIs).

Teaching introductory statistics differs from teaching statistics at the primary school level in requiring pedagogical content knowledge for higher level statistics as well as upper level content knowledge of statistics. However, introductory statistics courses are commonly offered in math departments and taught by math instructors, whose educational backgrounds may not include enough statistics to enable them to teach college level statistics effectively. This discrepancy could be problematic in terms of the distinctness of statistics as a discipline –statistics is unique as a discipline, and is distinct, in particular, from mathematics (Kim & Fukawa-Connelly, 2015).

Theoretically, this study employs a classification of attitudes into three pedagogical components –affective, cognitive, and behavioral– which is widely accepted in studies of non-cognitive factors in psychology and education (Estrada, 2002; Gómez-Chacón, 2000; Martins, Nascimento, & Estrada, 2012). In practical terms, this study focuses on five teaching recommendations in the *Guidelines for Assessment and Instruction in Statistics Education [GAISE] College Report* (Aliaga et al., 2005), which provides guidelines and goals for introductory college statistics courses. The current study is part of a larger study on CSIs' attitudes toward statistics, which considers a range of teachers' biodata (gender, academic background, and statistics teaching experience) and each of the five teaching recommendations individually. In the current study, we focus on how the three instructor characteristics (biodata) might explain their attitudes toward the GAISE teaching recommendations (GTRs), as well as their attitudes toward statistics. The specific research questions are: What features do CSIs show in their attitudes toward statistics and the GTRs? In particular, how can their attitudes be characterized in terms of the three aspects of attitude (affective, cognitive, and behavioral) and how do CSIs' personal characteristics (gender, academic background, and statistics teaching experience) explain their attitudes?

BACKGROUND

In this section, we review the literature on non-cognitive aspects of statistics education; discuss the study's theoretical perspective; and describe the features of statistics teaching at the college level that led to the rationale and design of the study.

Non-cognitive factors in statistics pedagogy

The *Statistics Attitude Survey* (SAS; Roberts & Bilderback, 1980) and *Attitudes Toward Statistics* (ATS; Wise, 1985) are widely used to measure college students' attitudes toward statistics. Based on these two instruments, Estrada (2002) developed the *Scale of Attitudes towards Statistics* (EAEE), which she used to identify areas of negative attitudes among Portuguese primary teachers. Martins, Nascimento, and Estrada

(2012) explained, in a follow-up study, the nature of the negative attitudes that Estrada had reported in 2002. Also, Lancaster (2008) reported preservice primary teachers' current self-efficacy to learn statistics in the future as a moderate predictor of their beliefs on the usefulness of future professional development in statistics in their classroom teaching. These studies have contributed to the literature on attitudes toward statistics by identifying areas of negative teacher attitudes, the nature of those attitudes, and the correlations between affect and beliefs, but they consider only primary teachers. This study extends this line of research to college level instructors.

Pedagogical components of attitudes toward statistics

The classification of attitudes into three pedagogical components has been widely used in studies of teachers' attitudes (e.g., Estrada, 2002; Gómez-Chacón, 2000; Martins, Nascimento, & Estrada, 2012). In particular, we draw on Estrada's (2002) and Martins, Nascimento, and Estrada's (2012, p. 27) work on primary teachers' attitudes toward statistics, in which they used the "affective" component to address feelings about objects in question; the "cognitive" component to address self-perception or beliefs about objects in question; and the "behavioral" component to address inclinations to act in a particular way about the objects in question.

Teaching recommendations of the 2005 GAISE college report

The GAISE college report (Aliaga et al., 2005) outlined teaching recommendations for introductory statistics courses. They are: (1) emphasize statistical literacy and develop statistical thinking; (2) use real data; (3) stress conceptual understanding; (4) foster active learning; (5) use technology; (6) use assessments to improve student learning. To address attitudes toward statistics teaching practices in this study, we refer to the first five GAISE teaching recommendations (GTRs), as described in Table 1. In sum, to address CSIs' attitudes toward statistics and the teaching practices of statistics, we classify attitudes into the three pedagogical components, and to address the teaching practices we refer to the five GAISE teaching recommendations.

Table 1. The teaching practices addressed in this study

Practices	Description
Statistical Literacy & Thinking	Promote understanding of the basic language and fundamental ideas of statistics, and methods to approach statistical problems
Use Real Data	Use real data as opposed to the fake data in classroom

Conceptual Understanding	Stress conceptual understanding, rather than mere knowledge of procedures.
Active Learning	Foster active learning in the classroom
Technology	Use technology for developing concepts and analyzing data

Characteristics of CSIs

In this study, we group the CSIs by their academic background and statistics teaching experience in addition to the commonly used category of gender.

Academic Background: Many statistics educationists (for example, Garfield & Ben-Zvi, 2008; Kim & Fukawa-Connelly, 2015) hold that statistics is a discipline distinct in nature from mathematics. They differ, for example, in problem-solving goals and whether the problem-solving context is essential. Aligning with this view, the authors of this paper believe statistics instructors must be equipped with teaching strategies and pedagogical content knowledge specific to the teaching of statistics as well as statistics content knowledge. The reality is, however, that introductory statistics courses are often taught by instructors with an insufficient statistics background.

This fact points to potential problems in statistics education at the college level: CSIs without a degree in statistics are likely to have less experience, less content knowledge, and/or less familiarity with the pedagogical methods of statistics than instructors with a degree in statistics. For this reason, this study explores whether the attitudes and heuristic approaches that CSIs take toward statistics and statistics teaching might differ depending on their educational background. We therefore divide the participants into three groups by the teacher's highest degree: math, statistics, and others (including math education).

Statistics Teaching Experience: There is a wide disjuncture between the practices recommended by credential programs and the practices actually employed by teachers, partly due to the fact that novice teachers find it difficult to implement practices they learned in such programs (Gainsburg, 2012). As teachers become more experienced, they tend to form their own teaching methods that incorporate practices that fit the culture of their work environment (Gainsburg, 2012) as well as aspects of the teachers' own dispositions toward learning, teaching, and specific subjects (Kim, 2013). Because novice and experienced teachers' practices and attitudes toward a subject can be expected to be different, this study considers teaching experience as a factor in CSIs' attitudes toward statistics and statistics teaching.

METHODS

An online survey was sent to 2832 faculty in mathematics or statistics departments at colleges or community colleges in the states of California, Texas, Pennsylvania, and Florida. While 154 responded, the analysis is based on the responses of the 92 participants who answered more than two-thirds of the 43 survey items.

We drew on the three surveys mentioned –EAEE, SAS, and ATS– as well as the specific practice recommendations in the GAISE college report (Aliaga et al., 2005) to construct the survey items. The survey consists of 43 items divided into two parts: Part I contains 15 items (Table 2) and Part II, 28 items (Table 3). Part I addresses instructors' attitudes toward statistics (as a discipline and a tool to solve problems) and is further subdivided, with five items for each of the three pedagogical components–affective, cognitive, and behavioral.

Table 2. Part I (15 items) of the survey on attitudes toward statistics

Pedagogical Components	Affective (A)	Cognitive (C)	Behavioral (B)
Items	1, 2, 3, 4, 5	6, 7, 8, 9, 10	11, 12, 13, 14, 15

Part II is designed to address statistics teaching practices, and is also subdivided by the three pedagogical components: affective (14 items); cognitive (7); and behavioral (7). We consider the cognitive and behavioral components only in the classroom context. However, in order to better understand teachers' feelings about statistics, we consider the affective component in a general context as well as in the classroom context, and we compare these two contexts for this component.

Table 3. Part II (28 items) of the survey on attitudes toward teaching practices

Pedagogical components	Affective		Cognitive	Behavioral
Context	General (A1)	Class (A2)	Class (C2)	Class (B2)
Items	16–22	23–29	30–36	37–43

The online survey consisted of two modules: (1) biodata and (2) the 43 items. The 43 items had a Likert-scale format. To increase the effect of responses, we used a 9-point scale (instead of the usual 5-point scale) from -4 ("disagree") to 4 ("agree"). To avoid apparent acquiescence, 27 items were phrased positively and 16 were phrased negatively; the latter were reversed for the analysis. We then converted the range of each item from the -4–4 scale to a 0–9 scale. The survey items were initially formed by the first author (a statistics educationist) and were then evaluated and modified by two of the other authors (a statistician and a mathematics educationist). Table 4 shows four example items (17, 24, 31, and 38), all of which address "real data."

Table 4. Example items

Item	Statements
17	I become thrilled when I see news or read articles that deal with real data.
24	In teaching statistics, I like using real data rather than hypothetical data.
31	There is little pedagogical value of using real data as opposed to hypothetical data for general data analysis in elementary statistics courses.
38	In my statistics classes, for general data analysis, I tend (or at least try) to use real data (archival, classroom-generated, available in media, or simulated).

Three sets of analyses were conducted. In each set, the participants' responses to different sections of the survey were compared across the participants by grouping them in three ways based on their biodata: by gender (male, female); by academic background (AB: statistics, mathematics, other); and by statistics teaching experience (STE: less than 2 years; 2–4 years; 4–8 years; 8 years or more). The first analysis compared the participants' responses to Part I of the survey to examine their attitudes to the three pedagogical components (A, C, and B). The second and third analyses used their responses to Part II: the second analysis compared their responses to only the survey items on the affective pedagogical component across the two contexts (A1: general; A2: classroom), and the third analysis compared the participants' responses to Part II items regarding all three pedagogical components in the teaching context only. In all three sets of analyses, the gender groups were compared using one-sided *t*-tests; the academic background groups' differences were analyzed using ANOVAs; and the differences between the different statistics teaching experience groups were analyzed using Welch ANOVAs. For every ANOVA and Welch ANOVA, the assumption of equal variance was checked to determine whether to use pooled or unpooled *t*-tests. Also, post hoc multiple comparisons were performed to identify the specific AB/STE groups that were the sources of differences in these analyses.

RESULTS

All significance tests were conducted at the commonly used significance levels of α = .05, .01 and .001. However, due to the limitations involved in making dichotomous conclusions based on *p*-values (Gelman, 2013), we also considered α = .10 to identify any potential factors. Throughout the data analysis, we use the acronyms: AB for academic background and STE for Statistics Teaching Experience.

Comparisons of the three pedagogical components of attitude

Table 5 summarizes the comparisons of the three pedagogical components of attitude grouped in terms of the three aspects of biodata considered in this study.

Table 5. Comparisons of the pedagogical components of attitude

Measure	A (n=91)		C (n=92)		B (n=92)	
	Mean	SD	Mean	SD	Mean	SD
	38.37	7.36	35.48	4.84	34.53	7.46
Gender (n=92)	p-value = 0.27		p-value = 0.34		p-value = 0.32	
Male (n=55; 59.8%)	38.78	7.2	35.31	5	34.84	7.71
Female (n=37; 40.2%)	37.78	7.66	35.73	4.66	34.08	7.14
AB (n=92)	p-value = 0.02		p-value = 0.17		p-value < 0.001	
Stats (n=12; 13%)	42.25**	3.41	36.33	3.96	41.08$^{\text{Ref}}$	3.42
Math (n=74; 80.4%)	37.62$^{\text{Ref}}$	7.78	35.62	4.82	33.65***	7.62
Other (n=6; 6.5%)	40.2	4.92	32	6.03	32.33**	4.27
STE (n=92)	p-value < 0.001		p-value = 0.04		p-value < 0.001	
≤2 (n=25; 7.2%)	33.21$^{\text{Ref}}$	8.46	33.64	5.26	29.32$^{\text{Ref}}$	7.69
2<&≤4 (n=31; 33.7%)	39.52**	6.37	35.94	4.33	34.9*	6.9
4<&≤8 (n=23; 25%)	40.35**	5.87	36.09	4.5	36.48**	6.07
8 <(n=13; 4.1%)	41.69**	5.38	36.85	5.32	40.23***	4.21

* p < .05; ** p < .01; *** p < .001. Subscript "Ref" marks the reference group in each column (i.e., to which the other groups were compared).

All comparisons were conducted using a one-sided *t*-test for gender, a weighted ANOVA for AB, and a Welch ANOVA for STE. Each individual's score could range from 5 to 45 (each component has 5 items, each of which ranges from 1 to 9). No gender difference is shown by the *p*-values, which are all greater than .05. The AB results are significant for two of the three pedagogical components: affective and behavioral. All three *p*-values for STE indicate significant differences across statistics teaching experience groups at $\alpha = .05$. Further, as STE increases, the scores improve dramatically for behavioral and moderately for affective and cognitive components.

Comparisons of affective attitude across general and classroom contexts

Table 6 summarizes the comparisons of the two contexts of the affective component.

Table 6: Comparisons of affective attitudes in the two contexts (general and teaching)

Measure	A1 (n=91)		A2 (n=89)	
	Mean	SD	Mean	SD
	50.12	8.62	49.06	8.2
Gender	p-value = 0.28		p-value = 0.07	
Male	50.55	8.47	48.02	8.92
Female	49.47	8.92	50.58	6.83
AB	p-value = 0.04		p-value = 0.88	
Stats	55.75 ^{Ref}	6.05	49.75	9.8
Math	49.49*	8.82	49.06	8.15
Other	46.5	6.25	47.67	6.09
STE	p-value < 0.001		p-value <0.001	
≤2	45.21 ^{Ref}	10.01	42.73 ^{Ref}	7.55
2<&≤4	50.29	8.41	49.87**	8.16
4<&≤8	51.35	6.2	52.7***	4.94
8<	56.62***	4.57	51.38**	8.67

* p < .05; ** p < .01; *** p < .001. Subscript "Ref" marks the reference group in each column (i.e., to which the other groups were compared).

Each individual's score could range from 7 to 63 (each component has 7 items, each of which ranges from 1 to 9). The *p*-value of .07 for A2 on gender indicates that the gender difference is significant (the females' A2 score is significantly higher than that of the males) at α = .10. Although the difference is not significant for A1, the results indicate that while male teachers' affect is slightly higher than female teachers' in the general context, it is the opposite in the classroom teaching context. In addition, while there is an AB difference on A1 (*p* = 0.04; significant at α = .05), there is no difference for A2 (*p* = 0.88). The AB difference on A1 shows that CSIs with a statistics background have more positive affect (feelings) about statistics (in the general context) than CSIs with a math background. Further, STE is a significant factor in teachers' affect in both the general context and the classroom context. In particular, the STE differences on A2 (49.87, 52.7, and 51.38) are more dramatic than those on A1 (50.29, 51.35, and 56.62).

Comparisons of the three pedagogical components in the teaching context

Table 7 compares the three pedagogical components in the teaching context.

Table 7. Comparisons of the three pedagogical components in the teaching context

Measure	A2 (n=89)		C2 (n=88)		B2 (n=78)	
	Mean	SD	Mean	SD	Mean	SD
	49.06	8.2	49.3	7.12	48.26	8.83
Gender	p-value = 0.07		p-value = 0.34		p-value = 0.12	
Male	48.02	8.92	49	7.34	47.31	8.91
Female	50.58	6.83	49.74	6.86	49.77	8.72
AB	p-value = 0.88		p-value = 0.54		p-value = 0.36	
Stats	49.75	9.8	51.17	5.47	51.82	9.8
Math	49.06	8.15	49.13	7.45	47.67	8.73
Other	47.67	6.09	47.5	6.09	47.75	8.14
STE	p-value < 0.001		p-value < 0.001		p-value < 0.001	
≤2	42.73[Ref]	7.55	46.13[Ref]	5.71	42.12[Ref]	8.34
2<&≤4	49.87**	8.16	48.1	7.95	47.81	9.43
4<&≤8	52.7***	4.94	52.3**	5.74	50.09*	5.8
8 <	51.38**	8.67	52.23	7.11	54.08**	8.39

* p < .05; ** p < .01; *** p < .001. Subscript "Ref" marks the reference group in each column (i.e., to which the other groups were compared).

Again, each individual's score could range from 7 to 63. At α= .10, female teachers had significantly more positive feelings toward the teaching recommendations (p = 0.7), and showed a greater, although not significantly greater, tendency to align their behavior with the recommendations (p = .12). In addition, the group differences in AB are not significant at α= .10. This result contrasts with the significant AB differences on A, B, and C (Table 5). Also, as in the other comparisons, the analysis found significant differences across STE groups for each of the three components (A2, C2, and B2). In particular, the STE group differences on A2 (49.87, 52.7, and 51.38) are more dramatic than those on C2 (48.1, 52.3, and 52.23) or B2 (47.81, 50.09, and 54.08).

DISCUSSION

Although relationships among the three pedagogical components of attitude were not apparent, the findings show some patterns. For example, there were more negative responses regarding the behavioral component than the other two components (Tables 5 and 7), which show a similar pattern. The findings further suggest that statistics teaching experience (STE) and academic background (AB) are, respectively, strong and moderate predictors of CSIs' attitudes toward statistics and statistics teaching practices. In particular, AB serves as a better predictor for attitudes in the general context ($p = 0.04$) than in the teaching context ($p = 0.88$) (Table 6). Such a large contrast indicates that although CSIs have different feelings toward statistics depending on their academic background, when it comes to teaching practices their feelings do not differ. This suggests the possibility of a disconnect between attitudes toward the discipline of statistics and attitudes toward the teaching recommendations, which is further supported by the results by gender. Gender seems to be a good predictor of attitudes toward the teaching recommendations in the classroom context ($p = 0.07$ (A2), 0.34 (C2) and 0.12 (B2); Table 7), but not of attitudes toward statistics (A, C and B) or attitudes toward the recommendations in the general context (A1). In particular, the strong contrast indicated by the reversed scores for A1 and A2 (Table 6) and the *p*-value of .07 for gender on A2 supports the disconnect.

The findings of this study have implications regarding the culture of statistics learning at the college level. Researchers made efforts in the 1990s to reform statistics course curricula. This reform movement emphasized the use of real data, more data analysis with less computation, and more use of software (Moore, 1997). These features are largely incorporated in the GAISE college report's teaching recommendations, but if the recommendations are to be successfully implemented, it is critical to understand how CSIs perceive them. This study's exploration of CSIs' attitudes toward statistics and the teaching recommendations is a first step in this direction. We hope that the study lays a foundation for future studies to explore the origins and factors of teachers' statistics attitudes, the dynamics of how CSIs' attitudes affect their teaching and pedagogical decision-making, and how the challenges in teaching statistics relate to CSIs' attitudes toward statistics.

This study has limitations. Although the sample size ($n = 78$~92) enabled us to make statistical inferences, a larger scale study would be necessary to arrive at firmer conclusions. Further, the data collection was limited to four US states and depended on voluntary responses, and the response rate was low (3.25%), constraining the extent to which the findings can be generalized.

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WHEN GEOMETRY MEETS THE LANGUAGE ARTS: QUESTIONING THE DISCIPLINARY BOUNDARIES OF A SCHOOL CURRICULUM

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Many mathematics educators consider mathematics a set of strict ideas that students have to 'learn,' and that mathematics teaching has little to do with humanities, thus often resulting to students who dislike mathematics (Kalavasis et al., 2006). In this paper we discuss our experience of developing an integrative and interdisciplinary project for teaching (geometry) mathematics through language education contents and performing arts. Several genres and modalities were used in order to create a curricular experience motivating students to become actively involved in their learning, to approach mathematical notions experimentlly and to renegotiate their perceptions regarding the nature of mathematics.

INTRODUCTION

The stereotypical 'traditional-style' mathematics class, particularly in the secondary school level, as described thoroughly by Susan Gerofsky (2015: 202, 203), is:

> 'A plain, grey, unadorned room, with students sitting still and silent in desks arranged in a grid pattern of rows, perhaps taking notes quietly, as the teacher stands at a chalkboard or overhead projector delivering an extended lecture, where there is little opportunity for them to engage in discussion, to voice basic questions, to express surprise or wonder, doubt or fear. Philosophical and contextual questions, discussion, physical animation, connections of mathematics with human nature and the more-than-human world, the use of objects to model mathematical structures are sterny discouraged'.

The ineffectiveness of a class like the one described above initiated the research community to explore and integrate all the available resources and techniques that could challenge the reality of this kind of mathematics classroom. Regarding, particularly, Geometry –a difficult school subject for the students– alternative approaches in its teaching have been explored (Clements & Battista, 1992). The use of new technologies (Laborde et al, 2006; Jones, 2011; Oldknow, 2008), the study of the applications of geometry in various sectors (Fletcher, 1971), the use of the History of Geometry applications with appropriate material from historical sources, (Gulikers & Blom, 2001), or from Ethnomathematics (Gerdes, 1988; Stathopoulou, Kotarinou, Appelbaum, 2015), as well as the use of the arts (Kotarinou, Stathopoulou & Chronaki, 2014), have created new educational circumstances, actively involving students in the process of teaching / learning.

In this article, our aim is to discuss how a group of adolescent students engaged in the cross-curriculum project '*The geometrical shapes through stories, poems and images*', implemented in a class of an upper secondary school in Athens (Greece) aiming to create a hybrid/ expanded learning space, where new tools and new Discourses were applied. This pedagogical endeavour challenged the boundaries of the diverse school subjects, encouranged students to approach mathematical notions experientaly and through a multilingual corpus of literary texts, and offered them the potential to renegotiate their perceptions regarding mathematics nature and particularly Eucledean Geometry.

HYBRID SPACE, CONCEPTUAL APPROACHES AND PROPOSALS FOR EDUCATIONAL PRACTICE

According to Homi Bhabha (1994), a theorist of postcolonial studies, borders or the boundary area between two fields is often an overlap area or hybridization, ie, a "third space" that includes an unpredictable and changing combination of the features each of them carries. The theory of hybridity argues that people add meaning to their world through the integration and interaction of multiple available social and cognitive resources and examines whether the status of "intermediate" can play at the same time restrictive and productive role in the development of the individual's identity.

The construction of the hybrid space emphasizes the in-between space that gathers knowledge and Discourses by individuals and various environments in which people participate today and which Discourses can be contradictory and competitive to each other.

Moje and her colleagues (2004), based on the theory of hybridity, focus on the dynamic learning that can emerge from the transformation of formal educational practices' to a "third space". The "third space", according to the proposers, is made through the active integration, in schools, of the various funds of knowledge and Discourses that are associated with the outside students' experiences. The ways i.e. of knowing, doing, talking, interacting, valuing, reading, writing, and representing oneself, produced and reproduced in several social and cultural communities in which they paricipate (Gee, 1996). A number of studies have examined the creation of a third space in the learning of Science at school with the merging of the Discource and scientific knowledge of this area with the students' outside school experiences about the natural world (Barton & Tan, 2009; Barton & Tan, 2008; Moje et al, 2004). Similar studies are examining the function of the third space to improve the teaching / learning of mathematics (Razfar, 2012; Flessner, 2009; Thornton, 2006; Cribbs, & Linder, 2013). In each case the relevant research reveal the learning benefits when teachers undertake the responsibility to bridge the boundaries between the two “worlds”, that of the lives of students outside the school and that in the classroom, describing the students’ involvement in the learning process as substantial and lasting.

In the educational context, “the third” space could be also reconceptualized as the integration of the varied disciplinary Discourses in the school curriculum and the creation of a fruitful dialogue between their own discursive practices in order to promote the acquisition of new knowledge (Wallace, 2004).

In our paper we argue that the integration of literary, visual and performing texts and practices in teaching Geometry expands the tools and the teaching processes that are used in school practices and creates a hybrid (third) space, where a richer repertoire for students’ participation possibilities is enabled. In such teaching environment –intertextual and cross-disciplinary– conditions and circumstances are created for the students to express their various identities and to experience the learning process through a different educational management, as evidenced by the implementation of the project below.

THE STUDY: DESIGN AND IMPLEMENTATION

The project was carried out in an inner city school in Athens, Greece. The school is one of the three schools of Arts in Greece where talented children in arts attend the official curriculum, enriched with extra courses on painting, theater and dance. The actual implementation of the project lasted over a period of two months, during the school year 2014–2015, with two classes of 10th grade students and the participation of six teachers.

In this project we explored the following main research questions: How can the reading of a literary work and visual and performing arts in the Mathematics classroom create a (hybrid) third space in which students can renegotiate the dichotomy of the Discourses of Science and Humanities? How might students’ experience of the expanded mathematical space transform their conceptualizations of mathematics and motivate their participation?

Data selection: The use of ethnographic research techniques (i.e. participant observation and interviewing) helped us to gather empirical evidence concerning students’ experiences; the majority of students’ activities were also videotaped and analysed. Semi-structured group interviews aimed to explore how students themselves perceived and processed their experience of participating in the project conceptually and physically. Our data collection was completed with a questionnaire with openended questions, given to the students at the end of the project implementation.

The Project in practice:

The interdisciplinary teachings were designed with the main objective for the students to approach the geometrical shapes as a means of artistic expression, but also for them to elaborate and respond to geometrical shapes in the context of artistic texts and to express themselves artistically in response to stories and poems etc. Our objectives regarding Mathematics were for the students to consolidate their knowledge of geometric shapes that had already been taught, to learn new shapes, many of which they would not have the opportunity to learn in their school life, while challenging their stereotypical image in mathematics.

In the next paragraph the different activities are presented, mainly focusing on the teaching issues regarding Mathematics.

- **Greek and mathematics class in dialogue (4 teaching periods)[1]:**

Students came in contact with the rendering of the Greek modern poet and mathematician George Vafopoulos (1990) through the poem "The large cone." They interpreted the parallelism by the poet of human life to a spiral that twists a cone and the description of the various human types with geometrical shapes as the square (a man with a square logic, Entrenched in boundaries), the zigzag line (a man meanders), the plane (a man without depth, a shallow man). The poem was the pretext, for the spiral of Archimedes and the logarithmic spiral to be defined and students were asked to design a logarithmic spiral based on the Fibonacci sequence. Then, having watched the short film 'Nature by numbers, a short movie' by Cristobal Vila (Etéreastudios), students approached shapes and concepts such as the golden rectangle and the golden section and looked at groups for linking these concepts with nature.

Moreover, through the poem the students were given the opportunity to define the conical surface, the cone and the conic sections and seek these figures in the architectural projects of Rem Kulhaas, Calatrava and Le Corbusier.The students themselves were asked to write their own texts or to create their own works of art (Fig. 1) based on the following questions: a) What shape would you describe yourself with and why? b) What shape do you experience your life in?

1. We inspired these activities from Stephanos Balis' book 'Mathematics and Poetry from Archimedes to Elytis'.

Last but not least, the short story 'The consolation of Geometry' from the book 'The silence of Giraffe' by Carlo Frabetti (2002), was read in literature classes, where students noted the important role geometric shapes and Geometry, in general, had in important mathematicians such as Archimedes, and Gauss and wrote their own point of view on what Geometry means to them.

Figure 1: My world as a cube

- **English and mathematics class in dialogue (5 teaching periods):** The students in the context of English language class studied and watched in cartoons, the story «The dot and the line: A romance in Lower Mathematics», starring a straight line and a circle, by Norton Juster (1963), defining the already known geometrical shapes -triangle, rectangle, rhombus, square- in English language, while they came into contact with new -ellipse, polyhedron, parallelepiped. In the end, students were asked to use their body as a tool to represent the above mathematical concepts inspired by the book, through dance (Fig.2) or through 'Drama in Education' techniques (hot chair, TV show and a press conference), a performance attended by the entire class.

Figure 2: Dancing the story 'the dot and the line'

• **French and mathematics class in dialogue (3 teaching periods)[2]:** In French Language class, the students expanding the content of the subject, studied poems from the collection «Euclidiennes» by Eugène Guillevic (1967) focusing on geometric concepts; at the same time, they had the opportunity to compare and contrast terms of everyday life with the corresponding mathematical register. They also matched eight selected poems of Guillevic; with pictures of monuments and areas of Paris, which point to geometric shapes, "played" with poems –personifying poems-shapes– and created their own expressive-creative recitations. Finally, they presented them in students of other classes.

• **German and mathematics class in dialogue (7 teaching periods):** In the interdisciplinary teachings, extracts from the literary book "DerZahlenteufel" of Hans Magnus Erzensberger (1997) were studied. Two students held a parallel narrative in Greek and German language from the introductory chapter, while two other presented math problems from the 10th chapter, in both languages. The previous activities were repeated to all their classmates along with an expressive reading with dramatized images of extracts of chapter 12.

• **Art History and mathematics class in dialogue (2 teaching periods)**: Our aim was to verify the relationship of applied disciplines such as industrial design, decoration and graphic arts with mathematics. Through the reference to the dynamics of non-performing art and through slides projection, students were given opportunities to understand how the three main sgeometrical shapes –square, triangle, circle- are reflected to works of sculpture, architecture and painting. A typical example is the paintings of Kadinsky and Klee, which helped students to understand the important role the geometric shapes play in their work and art in general. Furthermore, the students had the opportunity to study 20th century art movements, movements associated with abstract art, constructivism, de Styjl or Bauhaus.

• **Painting and mathematics class in dialogue (1 teaching period):** The Poincaré model of Hyperbolic Geometry was recontextualised in the work of Escher 'Circle Limit III' (1959). Prior to the introduction into Hyperbolic Geometry a discussion on the proof efforts of 5th Euclid's postulate took place. Students were also presented with the equivalent to the 5th postulate statements. Through the Java applet 'NonEuclid' by Castellanos, Austin & Darnell, the Poincaré disc and basic shapes, concepts and postulates of Yperbolic geometry were defined and presented, in comparison to the corresponding concepts of Euclidean geometry, while the students tried to locate some of these concepts in Escher's work (Fig.3).

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2. We inspired these activities from Stephanos Balis' book 'Mathematics and Poetry from Archimedes to Elytis'.

Figure 3: Trying to locate basic shapes from Poincaré disc

DISCUSSING THE RESULTS

Challenging students' perception about teaching and learning processes

In our project, tools and resources of different knowledge domains were gathered to transform traditional classroom practices. Whithin these expanded pedagogical contexts the students felt much more motivated to be involved emotionally, cognitively and physically in the classroom. The "conversation" of different genres and modalities seems to have provided them the potential to take a more active role in the teaching and learning processes. The students felt that they could expess their own ideas about the subject while the varied forms of peer collaboration developed during this teaching experiment appear to be a positive experience for them and in some cases they are associated with a better school performance, *'We could express ourselves freely and exchange ideas with our peers', 'We had the opportunity to reason and say our views'', 'I also liked that we worked in groups and did a good work'.*

> *'I enjoyed the whole process because it was something different than just doing exercises and generally an ordinary course'*
>
> *'As groups, it was nice because we split in the way we don't do otherwise'.*

Moreover, it seems that in this context the students on the one hand had the opportunity to deepen into mathematical concepts and on the other to understand the connection of mathematics to everyday life. *'It offered me greater understanding of the issues that concern me in mathematics','I won an extension of our curriculum in mathematics, with understanding and recognition of new theories', 'I saw the importance of mathematics in everyday life, outside the classroom and the limits that the official curriculum imposes on us'.*

Challenging students' perceptions about Geometry

Overall, students' experience of the expanded mathematical space had an impact on both their perceptions of Geometry (and mathematics in general) and on the developing of a stronger interest on the subject. The following quotes of students are inticative:

> 'What impressed me more is that while we think that the geometry we do in school is the geometry that we have in the world that we live in, it is actually not. It is a geometry that exists on paper and in reality things are very different than we think',
> 'I saw geometry and algebra that do not excite me, in a different eye',
> 'I like to draw shapes, but all of these rules ... When I saw through all that we did and the Yperbolic geometry, I was excited',
> 'I totally understand the examples of great mathematicians who found solace in Geometry. For me it is the main way to rationalize the world'.
> 'Geometry for me, is music. It is built with basic notes the lines, on the music sheet that sometimes is a flat surface, sometimes three-dimensional and with the proper consistency music pieces are created, the geometric shapes. This is for me, Geometry'.

Challenging formal curriculum

In contrast to the common curricular approaches, in this project teachers and students seem to appreciate this different curricular experience where the boundaries of the different objects were challenged and different discourses came together. Moje et al. (2004) speak about hybrid space where different funds of knowledge and Discourses coalesce to destabilize and expand the boundaries of official school *Discourse.*

> 'I liked that we understood how two different disciplines were related and that each discipline is not alone but it interacts with the other',
> 'What I liked was that this motivated me to try to connect Geometry with Literature',
> 'For me the most interesting was Hyperbolic geometry. Combined with this we saw some works of Escher that while it does not initially seem, they are based on a geometry different from the one we have'.

In a post-project set of reflections a student comments on the overall crosscurricular experience:

> "I would like to say an impression. The subjects we do, English, French, Mathematics everything, it is like a line, because they follow a particular curriculum. With what we did, because it is interactive in some way, we tried to leave the line and make circles and zigzags or anything. And that's why, from the one line that the lesson is: we managed to do something different. "

SOME CONCLUSIONARY REMARKS

The combination of varied mediation tools such as literary books, visual and performing arts as well as of varied disciplinary discourses may be understood in terms of the contextual, affective, and attitudinal approaches to a curriculum where mathematics may be humanized (Stathtopoulou, Appelbaum, Kotarinou, unpublished). The mathematics teacher mentioned continually arising tensions regarding debatable issues such as mathematics as an absolutistic subject out of our reality/ as a part of our lives, mathematics as culture free/ as connected to culture; using only the mind for mathematics/ using the body; listening/doing; doing mathematics/ playing with mathematics. These experiences enable opportunities for incorporating unconventional and informal practices that generate a third space where the different Discourses establish a dialogue, where discussion, movement, gesture, feelings, the use of material manipulatives and references to human personalities and the world beyond the classroom are strongly encouraged. This expanded space, so different from the traditional class allows students to approach mathematical concepts experientially, in interactivity with their diverse identities, and to renegotiate their perception of the nature of mathematics and especially of Euclidean geometry.

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THE IDEOLOGY OF RELEVANCE IN SCHOOL MATHEMATICS

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While (or because) the relevance of mathematics is a pedagogical, political and economic issue for both the practicing teacher and the researcher in mathematics education, we lack a sober discussion of the relevance of mathematics in academia. The normative discourse prevailing in mathematics education research is shown to be ideological and ignorant of critical objections. The main focus, however, lies on the perception of the relevance of mathematics by school students. The analysis of interview data shows that, in the discourse of the students, relevance is reduced to learning, skills and mundane applications. This one-dimensional restriction of the discourse does not only result in a lack of options for a more nuanced identification with mathematics, it is also efficient in obscuring other functions of school mathematics.

THEORIES OF RELEVANCE

The assumption that mathematics education in school is relevant for the life of the learners is not only crucial for the legitimisation of mathematics as a school subject, it also constitutes a basis on which power and resources are given to mathematics educators, be it in schools or research institution. Therefore, it eventually is a condition of possibility for this publication, as my research is financed by a German institution in the light of this assumption. Nevertheless, a critical contestation of this assumption should lie in the interest of anyone who considers research a self-critical endeavour.

In academic discourses, relevance is usually addressed through the discussion of educational goals of the mathematical subject, and different countries cultivated different and yet similar discourses about the relevance of the educational goals of mathematics. In the German case, to which I will confine myself here to provide an example, two contributions from the 1990s have shaped the discourse and also official curricula until today. Both contributions depart from the concept of ‘general education’ *(Allgemeinbildung)* which, in the German educational tradition, means an intellectual, ethical and aesthetical *Höherbildung*, that is an ‘elevated formation’, of the individual through experiences with and knowledge of cultural products of universal importance. The line of argument then states that general education is necessary for everyone to live a self-determined life, and that education in mathematics is a part of that general education. In the first contribution, Heinrich Winter (1995) sees the relevance of mathematics for general education in allowing students to make the experiences (a) to perceive phenomena in the world around us in a mathematical manner, (b) to understand mathematics as a deductively ordered world, and (c) to obtain problem solving competences which are fruitful beyond mathematics. In the second contribution, Hans W. Heymann (1996/2010) discusses the unique contribution of mathematics to the goals of general education which he sees in (a) the preparation for later life and in the promotion of (b) cultural competence, (c) an understanding of the world, (d) critical thinking, (e) the willingness to assume responsibility, (f) communication and co-operation, and (g) in the enhancement of the students' self-esteem.

These normative discourses assume a utopian classroom in which mathematics education can fulfil its idealistic goals. In contrast to that, contemporary school education in mathematics is reported to show activities and provide experiences that serve other social functions. Elsewhere, I have identified (a) the supervision of children, (b) their mathematical qualification, (c) their integration into and the legitimisation of the current social order, (d) their selection for further opportunities in education and occupation, and (e) the projection of societal hopes and fears onto mathematics education as such functions (Kollosche, 2016). This critical discourse provides explanations which are closer to the everyday experiences in the mathematics classroom but stand in conflict with the liberal educational theory represented, for example, by the concept of general education. Within this field, Paul Dowling's (1998) analysis of mathematics textbooks leads to a description of myths, which are established in the mathematics classroom and closely relate to the issue of relevance. He describes the belief that "mathematics can refer to something other than itself" as the "myth of reference", which tends to have totalitarian ambitions, claiming mathematics to refer to each and everything (p. 6). Under the "myth of participation" he understands the belief that mathematics is necessary for "optimizing the mundane activities of its students" (p. 8). Dowling, then, does not only argue that these believes are unjustified, but argues that they play a paramount role in the formation of the mathematical subject, the installation of mathematics as a tool of power and the reproduction of social differences. Relevance, here, is at least partly an ideological illusion.

As a consequence, the relevance of school mathematics can be formulated from different and yet unreconciled perspectives. These perspectives can be first of all considered theoretical discourses. However, these discourses have an impact on the way in which mathematics as a discipline is perceived by school students. Answers to the question "Why is mathematics relevant for me or us?" do not only motivate learning and provide it with an orientation, they also help to make sense of the experiences in the mathematics classroom altogether and to form a relationship to the discipline of mathematics, which might last a whole life-time. Eventually, the answers are political in that they transport a specific narrative which ascribes social importance to mathematics. In order to analyse the formation and the functions of such discourses among school students, this contribution will develop an understanding of the discourses on the relevance of school mathematics as an ideological narrative, before specific student statements recorded in an interview study are interpreted.

RELEVANCE AS IDEOLOGY

Developments in ideology critique (Žižek, 1994a) do not understand ideology as false consciousness which has to be destructed via the confrontation with the material reality of our existence in the Marxian sense. Instead, Slavoj Žižek (1994b) has generalised the concept through a psychoanalytic expansion. In his conception, "the real" is understood as the world we live in in its chaotic complexity which exceeds the capacity of thorough understanding. "Reality", then, is our cultural representation of that world; but as reality reduces an unintelligible complexity to mental and cultural representations, it necessarily leaves phenomena unexplained. The unexplained becomes problematic as what Žižek, following Jacques Lacan, calls a "symptom": an anomaly which cannot be explained with our representations, which destroys the tissue of our reality and endangers our whole conception of reality. Ideology, here, represents a narrative as part of our reality, which succeeds both in hiding the symptom and in linking to our desires and fears. For example, does the liberal parole for *liberté*, égalité and *fraternité* not correspond with every child's dream to break free from the protection of her parents and to unite with her equals, and thus –in the form of utopian ideals– give liberalism an esteem which helps covering the traumatic experiences of patronising and injustice in our contemporary liberal societies?

Interestingly, Žižek's (1994b) conception presents ideology as a necessity of any discourse. Ideology critique, then, does not only consist of the unfolding of the covering nature of a certain narrative. This step might not even be necessary, as often people already know that their actions and thinking is following an illusion. Nevertheless, Žižek points out, these people are acting "as if" this illusion was real. The question of ideology critique therefore is, how the illusion works and why it is attractive. Following Lacan, Žižek eventually argues that ideology is fuelled by the unconscious, by the promise of an unarticulated desire, by the promise of what Lacan calls *jouissance*.

In the case of the philosophical legitimisation of mathematics education discussed in the beginning, the construction of mathematics education through its educational goals is of fundamental importance for the survival of mathematics education as a school discipline and as a research field. Its integrity, however, is endangered by the experiences from the mathematics classroom, especially by the lack of the educational offers envisaged for students by Winter (1995) and Heymann (1996/2010) and by humiliating situations that cannot be justified within the scopes of a liberal theory of education (Kollosche, 2015). The symptomatic character of these experiences of contradiction is evidenced in their absence in the discourse. Just as the symptom is an unintelligible utterance of the real, experiences that contradict any liberal educational ideals are the unintelligible of the classroom experience. Their social relevance cannot be negotiated in any consistent discourse; their pervasiveness can only be met by limiting these experiences of contradiction to a deficient and yet-to-be-developed reality. The goals which justify the educational enterprise in mathematics are positioned in a utopian space which will never be realised. But why would a mathematics educator want to follow this ideological illusion, where is the *jouissance* in that? Assuming that the belief in the importance of mathematics and the possibility of education as *Höherbildung* is what motivated her to –for whatever reason– become a mathematics educator in the first place, and that the preservation of this motivation allows her a further identification with her economically privileged role in society, already a superficial consideration provides a reasonable explanation: The mathematics educator will find any discourse that installs mathematics as the tool for the desired *Höherbildung* attractive and any antagonistic experience threatening. The belief in the relevance of mathematics, therefore, has an ideological function. In the case of the student, however, the motivation might lie differently. How does this difference relate to the relevance ascribed to mathematics?

THE STUDY AND ANALYTIC METHODS

While students often do not enjoy their engagement with mathematics, different studies have shown that a vast majority of students do consider the school subject of mathematics "important" (Kislenko et al., 2007; Kollosche, 2017). Thereby, it has not yet been studied *how* students ascribe relevance to mathematics. However, these acts constitute instances of central importance for the role of mathematics in learning and in the political subjectivation of the learner. Here, student data will be used to reconstruct themes in the construction or destruction of the relevance of mathematics, to understand their ideological nature and to outline their socio-political implications. 23 students from grade 8 to 10 in regular public schools in and around Berlin were interviewed in school rooms during the school day by master students attending a research seminar at the Universität Potsdam in 2016. While two students went to the same class, all the other students attended different classes in different schools in the German states of Brandenburg and Berlin. The semi-structured interviews focussed widely on the students' relationships to mathematics, and featured the question "Do you believe that your mathematics lessons are or will be important for your life?"

In a thematic analysis (Braun & Clarke, 2006), I identified all statements which referred to the relevance of mathematics, grouped them into themes and interpreted them separately. Shaped by a Foucauldian paradigm (Foucault, 2011), I understand the students' answers as acts of constructing and legitimising a discourse around the subjectivity, that is the development and performance of techniques of the self, with which the students meet the requirements of the mathematics classroom. Following Žižek (1994a) but denying myself a deeper juxtaposition of both theories within the limits of this contribution, I will add Žižek's ideas for a psychoanalytical ideology critique to the Foucauldian discourse analysis in order to be better prepared to explain the detailed mechanisms of this power-knowledge. The leading question of the analysis therefore is: "How do students articulate their subjectivity in the field of discourses on the relevance of mathematics, and how can this articulation be understood from the perspective of an interplay between power and discourse?"

THE VOID OF RELEVANCE

A vast majority of 20 out of the 23 interviewees regard mathematics as personally relevant and associate this relevance to the mastery of mathematical skills. When asked why he stated that mathematics would be "important", Christian (all names changed, all answers translated into English by D. K.) follows a discourse which is very common under the interviewed students:

> Christian: What is it important for? Well, generally for later life. You often get in touch with numbers. When you have a profession, you have to be able to calculate. For example, so that they cannot cheat you at the checkout, then you must know a bit of maths. [...] In other professions you also have to do mental maths, like a bus driver – they have to do mental maths as the checkout only tells the price. And the police, in my opinion, they must know what they get, how expensive the fine is and what the change will be.

On a superficial layer of analysis, it is astonishing that Christian associates the relevance of mathematics with learning skills *in elementary mathematics*, such as calculating sums of money, which he left behind in the classroom in primary school. Seemingly aware of this restriction, he adds that "in many professions maths plays an important role", but "not to the extent of what we are doing". Concerning some contents he "wonder[s] what you need that for in maths, but what must be, must be." This restriction of the relevance of mathematics to elementary skills after the assertion of a general relevance of mathematics appears frequently in the interviews of the students. When it comes to restricting the everyday usefulness of the contents of school mathematics basically to those of primary school, this position is shared by Heymann's (1996/2010) legitimisation of mathematics education. However, it is interesting to note that while the students have made both experiences of relevance (in the past when they covered elementary skills in class) and experiences of irrelevance (in the present when confronted with advanced contents), students nearly consistently stick to the narrative of the general relevance of mathematics for life. In the light of these conflicting experiences, the students could just as well state that mathematics is irrelevant and position the example of elementary mathematics as an exception to that rule.

In Lacan's terminology (Žižek, 1994b), the assumed relevance of mathematics may be considered a master-signifier, a symbol that does not refer to any signified but serves as the basis of an ideological narratives and may be filled with various and often changing meanings, be it mathematics' contribution to general education or its relevance for the individual future. Christian's difficulties and ambiguities in explaining the relevance of mathematics show that, in his discourse, the idea of the relevance of mathematics is empty and does not point at any graspable experience or different discourse. This common speechlessness is best demonstrated by Kai:

> Interviewer: Do you believe that your mathematics lessons are or will be important for you?
>
> Kai: Yes, sure.
>
> Interviewer: How?
>
> Kai: Everyday life of course. Also, depending on what job you're doing, but architect, for example, you need very much. And well..., no idea.

Sverker Lundin and Ditte Storck Christensen (2017) refer to Johan Huizinga to understand mathematics education as a play whose "holy seriousness" results in an identification with the game's objectives. Indeed, allying with the discourse which presents mathematics as generally relevant eases the participation in the mathematics education enterprise which students cannot escape anyway. Thus, the empty promise of the relevance of mathematics serves as the ideological patch which on the one hand covers the symptoms of irrelevance and allow the students to enjoy mathematics in the hope of a bright but opaque future.

Admittedly, this future sometimes takes more graspable forms, occasionally in the form of professional aspirations or present forerunners. But when Olivia, who considers mathematics “indeed relatively important”, or Tim, who thinks that mathematics “will be quite helpful later”, say that they will need mathematics, because they want to became a farmer or a construction engineer respectively, their claim of the relevance of mathematics has somewhat materialised but has not changed its nature: The students explanation stops at that connection between relevance and profession, the relevance still lies in an inapproachable distance and does not connect to anything the students are doing in the classroom.

Two students report of singular experiences of the relevance of mathematics. Daniel uses mathematics when he is “doing constructions” (of what kind, we do not know) with his father and states that “then it is beneficial to know maths well”. Bianca tells:

> Bianca: Even if people don’t like to hear it: Unfortunately, maths is really important for your later life. Because, just recently, we had this sine function and I thought I would never need it again. Then we were at home and my mum came with a slip and said: “You must calculate that.” And there, I saw precisely that I need it for that. […] Well, you had the diagonal of a screen and there’s that right angle and the other two angles were indicated on a webpage, but not how wide or high this screen was […]. Fortunately, I could calculate that somehow.

Apart from this episode, Bianca’s interview resembles that of Christian. In this episode, the role of mathematics is ambiguous. From the perspective of problem solving, the prediction of width and height of a screen, which only the proportions and the length of the diagonal are known of, is a remarkable intellectual achievement. From a pragmatic perspective, Bianca could as well have found a webpage which provided her with the answer to her mother’s question. And of course, Bianca’s singular experience only connects one mathematical content with her life, but not all. However, the centrality of this episode in Bianca’s account suggests that it prove enough for her to assume the validity of a far more general assumption, namely the relevance of the mathematics she learns at school. It is as if this episode is the unique moment where the empty signifier of relevance and her school practices connect, and this moment fills her fantasy of a meaningful learning with life.

THE MATERIALITY OF RELEVANCE

On a deeper layer of analysis, it is astonishing that Christian associates the relevance of mathematics with learning *skills*. Indeed, *all* the students connect the relevance of mathematics solely with the mastery of mathematical techniques which can be beneficially applied in present of future life. This is even the case with Anna, the only student who expresses an unrestricted denial of the relevance of mathematics:

Interviewer: D'you generally believe that mathematics isn't necessary for your further life?

Anna: No, actually not, as I said, I got my mobile, I can always put everything into it if I have to calculate something or so. No, apart from that, I don't need it.

Interviewer: But, for example, what about situations like this one: You go shopping and on a pullover you have a tag that there is a 30% discount on the original price. Then you would like to know the new price of the pullover if you want to buy it.

Anna: Then, I would go to the shop clerk and ask her for the new price.

Even Anna, who does not admit any relevance of mathematics in spite of her interviewer's intervention, sticks to a discourse that associates the relevance or irrelevance of mathematics with the application of mathematical techniques. This materiality of mathematics education, that is the restriction of its educational scope to the learning and application of techniques, contradicts all normative discourses on the relevance of mathematics education. Interestingly, none of the educational goals expressed by Winter (1995) and –with the preparation for later life– only one of Heymann's (1996/2010) seven goals are associated with mathematics education in the discourses of the students. This means that neither the ideology of relevance that is cultivated in the classroom regards any of these goals as a part of the mathematical experience, nor do students make experiences in these dimensions which have the strength to find expression in their interview statements. Apart from the fact that the absence of any formational goals of mathematics education in the dominant classroom discourses sheds a bad light on the practical effectiveness of the theoretical legitimisation of mathematics education, that absence also limits the explanations with which students can make sense of their school experiences with mathematics and construct their discourses of relevance:

Interviewer: There are also many students who do not like maths at all. What do you believe is the cause of this?

Emma: [...] That you simply do not feel like it, because during exercises you simply think that you won't ever need that again. Because as long as you can calculate a bit, for shopping or so, then this is usually enough. Because none of us wants to study anything with maths later. And that's why it sometimes appears so senseless. And you do not understand why you should do it now, although you know that you actually won't need it ever again.

However, it would be naïve to believe that the mere introduction of alternative goals into the classroom discourse of mathematics education would lead to significant changes in the perceptions and motivation of the students. Instead, the implementation of these goals demands for a completely different style of teaching mathematics, a style which allows the students to experiences and reflect on the educational goals envisaged. Hitherto, such a teaching of mathematics is not only a yet unrealised project, it is also dubious if this project can be realised at all. Any attempts to implement new forms of teaching would demand enormous efforts by the teachers, and the reports of the students indicate that teachers usually avoid these.

The narrative that mathematics education is about learning skills that are needed in present or future life thus reduces the relevance of mathematics education to the materiality of learning. This narrative is obviously supported by Dowling's (1998) myth of reference, assuming that mathematics will be generally important in any further occupation, and by his myth of participation, assuming that mathematics will be needed in future life. But the 'myth of importance' described here goes beyond the mere insertion of an image of mathematics as a universal and indispensable tool of power. It does not only provide the ideological basis on which the learning of mathematics can be legitimised individually, it also erects the boundaries of the relevance discourse. By cultivating the discourse that the relevance of school mathematics lies in the preparation with mathematical skills needed in a distant and opaque future, the answer to the question for relevance is relocated in a space where it cannot be reached critically. With this indisputable position, it gains a symbolic power, which other, more immediate goals of mathematics education cannot have. It is therefore effective in excluding more immediate goals from the discourse and from classroom practice. Put simply, it is much easier to prophesy that a specific mathematical content will be needed in a distant future than to prove how it promotes cultural competence, an understanding of the world or even critical thinking in the here and now. This is how the materiality of relevance is productive in shaping the discourse.

HIDDEN RELEVANCE

On the last layer of analysis, it is astonishing that Christian associates the relevance of mathematics with *learning*. As discussed elsewhere (Kollosche, 2016), qualification is only one of the social functions in mathematics education that are discussed in research, and it is empirically unclear in how far mathematics education after primary school contributes to the mathematical qualification of learners at all. Of all the other functions identified in mathematics education, only selection is mentioned by the students, and only three out of 23 interviewees raise this topic. When asked what mathematics is needed for, Emma answers:

> Emma: […] And for school you also quite need it, because otherwise you do not achieve your marks and then you fail. So yes, you actually do need it.

Emma, who also believes in the relevance of elementary mathematics, here adds another dimension to the discourse of relevance. Irrelevant of whether the specific knowledge and skills acquired in the mathematics classroom are of any practical use at all, she addresses the relevance of mathematical achievement for a successful school career. While Emma only mentions the role of mathematics in assessment, Vanessa focusses on the ultimate function of such assessment when she states that "you have better job opportunities if you are good in maths". Patrick points in the same direction when he states that mathematics provides you with the cultural capital, in the sense of Bourdieu (1986), necessary to occupy elite positions in society:

> Interviewer: Wait a second! Why would you include it [mathematics] into your timetable at all?
>
> Patrick: Because I think you often need maths for your later career, if you want to become something, well, something big or so. Want to lead something, you would also need maths.

Notwithstanding these statements, the vast majority of interviewees did not at all mention selection in regard to the relevance of mathematic. The fact that the mathematics mark has a function as an indicator of economic success (Maaz, 2006) is either not obvious to the students or avoided as a topic to include in the interview discourse on the relevance of mathematics. Alexandre Pais (2014) argues that the devaluation of low achievers in the selection process is Lacan's traumatic real, which appears as an unintelligible friction of the educator's reality and has to be covered by the ideology that mathematics provides emancipatory qualification for all students. Thus, the widely ignorance of the relevance of selection through mathematics may serve as an example of how the classroom discourse limits the issue of relevance to socio-politically unproblematic realm of qualification for the future.

FINAL DISCUSSION

The normative discourse and the students' discourse on the relevance of mathematics are unable to address the social functions of mathematics education identified in research. However, this failure of the discourses is juxtaposed to their productivity of limiting the perceived relevance of mathematics to a utopian future. In the case of the normative discourse, this future is the ideal mathematics classroom which provides a truly emancipation education; in the case of the students' discourse, it is the future in which they will profit immensely by the mathematical skills acquired in school. In both cases, the participants are given what Žižek (1989) calls a "forced choice": In order to enjoy your profession as a teacher or your mandatory learning as a student, you cannot surrender yourself to critical thoughts about the relevance of mathematics but have to gratefully accept the narrative of the relevance in the future. The students' reports reflect this ambiguity towards the relevance of mathematics, but also document how this ambiguity is in all cases concealed by the experientially empty claim of a 'general importance' of mathematics. In the light of this inner disunity of the learner, it may be a relief to student that she is forced to attend school mathematics –an obligation without which school mathematics might cease to exist:

> Interviewer: Would there be maths in your self-made timetable?
> Vanessa: Probably yes, because you simply need it for your further life. I believe I would not do maths by myself, and so I would force myself to do it a little.

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UNDERSTANDING RELATIONS OF POWER IN THE MATHEMATICS CLASSROOM: EXPLORATIONS IN POSITIONING THEORY

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This paper examines the use of positioning theory in mathematics education and its potential for analyzing the relations between learning and student identity within the cultural and historical contexts of intellectual and social power relations. We offer a metasynthesis of the research in relation to Saxe's (1991) Emergent Goals Framework. We highlight three analytic themes: (1) microgenetic interactions that enable or constrain access to particular forms of identification and mathematical engagement, which are fundamentally inter-related; (2) ontogenetic processes that enable or constrain access to becoming mathematically-identified learners; and (3) sociogenetic processes that organize local discursive interactions around broader (gendered, racialized, and other social) relations of power.

INTRODUCTION

The United States is in an unprecedented era of gross economic and social inequality that is greater than any time in recent history (Hacker & Pierson, 2010). These disparities are evident in patterns of racial and economic resegregation in neighborhoods and schools and in variances in youth's access to high quality mathematics education. While discussions about such inequalities frequently note the increasing concentrations of power and resources among fewer and fewer institutions, groups, and individuals, they do not shed light on how these disparities likewise shape how mathematics is experienced and imagined in mathematics classrooms. Nor do they offer deep insights into how these inequalities inform non-dominant youth's mathematical learning, identities, and life pathways.

At present, few conceptual or analytic tools allow us to examine how macro discourses and relations of power influence micro-level interactions in local contexts like mathematics classrooms. While some tools are useful for understanding how discourses shape students' own processes of identification and the co-constructed nature of students' disciplinary, academic, and racial identities (e.g. Nasir & Shah, 2011; Varelas, Martin, & Kane; Martin, 2015), there is still a need for conceptual and analytic tools that examine how these processes and related patterns of inequality are instantiated, perpetuated, or transgressed through interaction at the classroom level. Likewise, local, classroom level examinations of power often make limited, if any, connections to issues of power and inequality beyond the context of those local sites. There is thus a need for coordinated frameworks that attend to the connections between macro-level discourses and micro level classroom processes.

In this paper we examine the use of positioning theory as a potentially promising approach. Positioning theory has become increasingly utilized by mathematics classroom researchers interested in the link between identity and learning. This link –between the processes of learning and becoming– is potentially fruitful in understanding the role of broader discourses at play in the mathematics classroom because of the direct connection between social (including racialized and gendered, etc) and individual identities (Langer-Osuna & Nasir, 2016). Positioning theory asserts the following: (a) human communication is generally mediated by socially constructed and historically situated *storylines*, which give meaning to our words and actions; (b) through communication (verbal and non-verbal), individuals locate themselves and others -that is, take on *positions* - often within existing storylines; and (c) individuals locate themselves within particular storylines through *acts of positioning*. According to positioning theory, not everyone has equal access to the same rights and obligations to perform particular acts (Harre and van Langenhove, 1999; Herbel-Eisenmann, et al, 2015). As this narrative access is also a way of framing access to power, positioning theory affords the study of the social organization of power as it is instantiated through interaction within the classroom.

Within the mathematics classroom, the questions arise: what students are afforded the right to learn mathematics robustly, actively, and with understanding? What students are obligated to learn mathematics in less productive ways? How might these issues of access be understood in relation to culturally and historically situated ways of organizing relations of power? In this paper, we explore the potential of positioning theory to serve as an analytic tool to examine how macro discourses and relations of power affect mathematics classroom interactions. To do so, we offer a meta-synthesis of literature that highlight analytic possibilities and potential ways to move forward in addressing issues of power as a field. We first describe our review methods before discussing how our analysis involves using Saxe's (1991) Emergent Goals Framework to both organize studies in mathematics education that use positioning theory and as a way to address some of the lingering conceptual questions regarding this theory. Utilizing this analytic approach, our metasynthesis ultimately highlights the ways in which relational power is fundamentally at play in the link between learning and identity processes at the classroom level and serves to moderate both access to trajectories of mathematical thinking *and* identification with mathematics as a discipline.

ANALYTIC APPROACH

Mathematics classroom researchers utilizing positioning theory have augmented it with frameworks that enable a consideration of timescales (Anderson, 2009; Herbel-Eisenmann, et al, 2015) in order to clarify the unit of analysis as well as the connections between moments of positioning and the storylines invoked (Herbel-Eisenmann, et al, 2015). Indeed, one way to understand the connections between learning, identity, and power, is to identify the units of analysis and determine how they become coordinated. We argue that the features of Saxe's (1991) Emergent Goals Framework afford this type of analysis, and are particularly helpful for examining the form and function of mathematics classroom discourse practices.

The Emergent Goals Framework considers the relations between discursive and other representational forms and their mathematical functions during social interactions within mathematics-linked social practices (Saxe, 1991, 1999, 2003). Saxe argues that words and other mediated actions take on mathematical functions within social practices across three levels: (a) microgenetic or moment-to-moment interactions; (b) ontogenetic or over time; and (c) sociogenetic or collective transformations at the level of the social practice itself, situated in historical time. Langer-Osuna (2007, 2014) has expanded the Emergent Goals Framework to consider how words and actions take on both mathematical and positional functions within mathematics classroom discourse practices across these three levels. Here, we analyze the literature utilizing positioning theory and locate the studies' units of analysis as micro, onto, or sociogenetic levels to consider whether positional functions focused on its relation to identity or mathematics learning. In doing so, we can map this body of work in a coordinated way, illuminating the ways in which power plays out in the moment, over time, and in relation to broader historical discourses.

METHODS FOR SELECTING STUDIES

Distinct from a comprehensive review of the literature, a metasynthesis is "an analysis and interpretation of the findings from selected studies" (Berry, 2016). We employ this approach in order to deepen our understanding of the current use of positioning theory in mathematics education and its potential to illuminate the broader relations of power that organize classroom processes. In particular, we use a cross-case analysis (Miles & Huberman, 1994) wherein we systematically coded, refined, and cross-referenced descriptive metathemes that arose across a sample of articles.

Guiding our metasynthesis was the question: how is positioning theory used to illuminate connections between identity, learning and relations of power in the mathematics classroom? Specifically, (a) what unit of analysis was utilized in the study; (b) what insights were generated about the teaching and learning processes of mathematics classrooms; and (c) how was power implicated in the analysis?

We conducted a comprehensive search of all peer-reviewed mathematics education research journal articles that cite positioning theory. First, we searched for peer-reviewed journal articles using key terms "positioning theory" and "mathematics" in ERIC database, and then additionally searched specifically within key mathematics education journals. From this initial set, we read through abstracts and skimmed papers to focus only on articles that: (a) were empirical studies rather than reviews or conceptual pieces; (b) drew upon positioning theory as articulated by Harre and colleagues; (c) focused primarily on mathematics learners (as opposed to teachers or others); and (d) were situated in mathematics classrooms. Altogether, this search resulted in 26 peer-reviewed journal articles. We coded the 26 articles in relation to: (a) the unit of analysis (micro, onto, or sociogenesis); (b) the focus of the analysis (learning or identity); and (c) how was power implicated in the analysis. We then looked for themes across articles, which we describe below.

RESULTS

Drawing on Saxe's levels of analysis, we highlight three analytic themes for capturing relations of learning, identity, and power: power in mathematics classroom interactions. Through these themes students negotiate: (1) microgenetic interactions that enable or constrain access to particular forms of identification and mathematical engagement, which are fundamentally inter-related with power as access to particular forms of mathematical engagement in the moment; (2) ontogenetic processes that enable or constrain access to becoming experiencing themselves as mathematically-identified powerful learners through identity development; and (3) sociogenetic processes that organize local discursive interactions around power as access to ways of being through broader (gendered, racialized, and other social) relations and discourses of power.

Microgenetic analyses: Power as regulating access to particular forms of mathematical engagement and identities in-the-moment

The majority of papers focused on microgenetic interactions that enabled or constrained access to particular forms of mathematical engagement, lending itself to a direct connection between acts of positioning and ways of engaging in mathematical work leading to particular learning opportunities (Bell & Pape, 2012; DeJarnette & González, 2015; Enyedy, et al, 2008; Esmonde, 2009; Herbel-Eisenmann & Wagner, 2010; Langer-Osuna & Avalos, 2015; Mesa & Chang, 2010; Tait-McCutcheon & Loveridge, 2016; Turner, Dominguez, Maldonado & Empson, 2013). This focus highlights the relational work of classroom mathematics learning and considers how individuals communicate and interact with one another in ways that enable or constrain the forms of engagement that are possible (Tait-McCutcheon & Loveridge, 2016). For example, Enyedy, et al (2008) noted that teacher revoicing of students' mathematical contributions tended to position students as legitimate authors of mathematical ideas.

A smaller subset of papers with a microgenetic unit of analysis focused directly on the identity-related functions of acts of positioning during mathematics classroom interactions (Bishop, 2012; Gholson & Martin, 2014; Wood, 2013). For instance, Bishop (2012) examines the peer-to-peer interactions of a pair of female 7th grader students in order to understand how they position one another as more or less capable math students and ultimately enact different mathematics identities within the same local context of their mathematics class. Similarly, Wood (2013) focused on both the mathematical engagement-related and identity-related microgenetic functions of classroom interactions together. The analysis found that during group work positioning affected available forms of engagement that could either undermine or support mathematics learning. Broadly speaking, in microgenetic analyses power can be understood in terms of particular forms of mathematical engagement that afford students' access to specific mathematical identities in-the-moment, with processes of engagement and identification being fundamentally intertwined.

Ontogenetic analyses: Power as regulating access to trajectories of specific mathematics-linked identities or ways of doing mathematics

A second analytic theme that emerged across papers explored how the power of particular positional acts over time regulated access to students' experiencing themselves as particular kinds of learners and doers of mathematics (Gresalfi, 2009; Kotsopoulos, 2014; Langer-Osuna, 2014, 2016). These papers focused on how a confluence of positional acts over time led students to develop specific trajectories of mathematical engagement and mathematics-linked identities, highlighting the link between identity and engagement over time into relatively stable structures.

Examining how relatively stable mathematics-linked identities are constructed over time, Kotsopoulos (2014) found that a student's own ability to reflexively position himself was overpowered by the interactive positionings of his teacher and peers. Despite the students' self-positioning, in his classroom context the focal student was frequently silenced and was unable to gain access to a positive learning identity. Other studies focused on the link between positional acts over time and relatively stable orientations toward ways of doing mathematics.

Consider Gresalfi's (2009) analysis that examines the ways in which particular dispositions toward doing mathematics stabilized over time as repeated microgenetic interactions positioned students in relation to a relatively wide range of mathematical competencies. Finally, Langer-Osuna (2016) analyzed how positional acts over time could even determine the solution path constructed among students during collaborative mathematics problem-solving, linking closely positional acts and mathematics learning. In studies that are focused on the ontogenetic level relations of power can be understood in terms of access to powerful identities or ways of doing mathematics that ultimately shape one's longer term trajectories of mathematical participation, identification, and potentially learning.

Sociogenetic analyses: Power as access to mathematically-valued storylines

A third theme that emerged across papers was the power of available narratives -such as storylines about ability, mathematics, race, gender, and language- to mediate access to particular forms of engagement in mathematical learning activities (Esmonde & Langer-Osuna, 2013; Gholson & Martin, 2014; Langer-Osuna, 2011, 2015; Takeuchi, 2016; Turner, et al, 2013). Some of these studies have focused more pointedly on race or gender, while other work has focused on the intersection of race and gender (Gholson & Martin, 2014; Langer-Osuna, 2011). For example, while Langer-Osuna (2011) found that gender mediated how peers responded to group leaders' directives in a largely African American classroom, Esmonde and Langer-Osuna (2013) found that in a racially diverse math classroom gendered and racialized storylines around friendship and romance mediated students' forms of engagement in a small group discussion. Gholson and Martin (2014) additionally reveal that "conceptions of smartness" are often mediated by racialized and gendered ideas from in and outside the classroom. These ideas seep into the social networks within mathematics classrooms and inform students' interactions with one another in ways that shape African American female students' learning opportunities and consequently their development of academic, mathematical, and racial identities.

Still other work has focused on language ideologies (Takeuchi, 2016; Turner, et al, 2013). Turner, Dominguez, Maldonado, & Empson (2013) found that local storylines about language mediate interactions among linguistically diverse learners in an afterschool mathematics learning setting. The teacher framed their learning context as one where bilingual participants would support communication across monolingual English and emergent English-Spanish bilingual students, enabling access to more central engagement in the mathematical discussion for all students. Irrespective of the type of storyline (racial, gendered, linguistic, etc.) being referenced, across these studies we are categorizing as sociogenetic, power can be understood in relation to access to particular narratives and practice-linked identities that align with historically and culturally valued assumptions linked to mathematics.

IMPLICATIONS

We see potential in the use of positioning theory to capture relations of power that affect opportunities for engagement, learning, and identification in the mathematics classroom. Our metasynthesis demonstrates how relational power is implicated across all three timescale: micro, onto, and sociogenetic. Moreover, in considering this body of work in relation to both the identity-related and mathematical functions of positional discursive acts at these three levels, we are able to better analyze the ways in which mathematical participation, learning, and identification are fundamentally linked and organized around broader discourses of power, such as ideologies of race, gender, and language among other social discourses.

We argue that analytical approaches that take into account timescales (moment-by-moment or longer term), context (local or classroom level vs. a broader or societal level), and the role of discourse when studying classroom interactions have the potential to unearth the power relations at play within mathematics classrooms that might limit students' opportunities to engage, identify with, and learn mathematics. Such coordinated approaches can allow researchers to better articulate what level of power relations are being examined and to make greater connections to how particular social discourses or storylines are implicated in classroom interactions in ways that shape access to more or less inclusive and productive forms of engagement and identification in mathematics learning activities.

Similar ideas have been considered by researchers utilizing theories related to but distinct from positioning theory in key ways (e.g., Heyd-Metzuyanim, 2013, 2015; Heyd-Metzuyanim & Sfard, 2012). For example, Heyd-Metzuyanim & Sfard, (2012) utilized a communications framework to examine how identifications were co-constructed in classrooms and not in the control of any individual person, affecting, sometimes adversely, whose ideas are attended to by peers. However, we argue that the expansion of positioning theory using a coordinated approach to examining timescale, context, and discourse, allows for an even fuller account of how learning, identity, and relations of power unfold in real time in ways that simultaneously index socio-historical ideas and norms and possibly create new ones. Coordinated approaches such as that offered here, have the potential to enrich the research methods and analyses of mathematics education scholars as well as serve to inform future classroom interventions and the development of interactionally and relationally minded-mathematics teachers.

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ON THE ENTANGLEMENT OF MATHEMATICS REMEDIATION, GATEKEEPING, AND THE COOLING-OUT PHENOMENON IN EDUCATION

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In this paper, I re-introduce and discuss the cooling out phenomenon in education (COPE), a concept that originates in sociology and has re-emerged across several academic disciplines, including education. This time, the purpose is to understand how COPE functions amid mathematics-curricular transitions at which learners are systematically sorted. The contexts for the reported studies are public universities in the United States, and the empirical focus is on undergraduates' experiences within remedial mathematics courses. Drawing on two recent studies in which I analyzed series of interviews with students in these contexts, I discuss the ways that COPE unfolded, who and what processes were involved in COPE in these cases, and implications regarding COPE for mathematics education research more broadly.

INTRODUCTION

> We real cool. We
> Left school.
> (Brooks, 1959, p. 373)

The "cooling out" phenomenon in education (COPE) has emerged, receded, and re-emerged as a contentious topic in the research literature (U.S.-based) on access to postsecondary education and students' persistence (Clark, 1960; also see Alba and Lavin, 1981; Anderson, Alfonso, and Sun, 2006; Bahr, 2008; Baird, 1971; Clark, 1980; Nielsen, 2015). As it has been applied in education contexts, cooling out refers to situations in which individuals' curricular aspirations are in some way deterred, invalidated, or denied while, at the same time, the person is strategically encouraged to view the outcome as less unfavorable than it might have seemed initially. Put differently, a person who is cooled out is methodically persuaded to alter their expectations based on the influence of some other entity –another person(s), an institution or institutional mechanism (e.g., program), or some other kind of agent. As a result, the cooled-out person is primed to be re-sorted socially from one identity category to another, with the latter being considered a lesser category to be. Although the more immediate hazard of COPE may be loss of access toward fulfilling one's role in a specific setting or their trajectory within a specified domain, the deeper-seated, psychosocial threat is not simply embarrassment or constrained access in the short term but more so a loss of identity associated with the domain.

The cooling-out phenomenon appeared originally as the subject of interactional sociologist Erving Goffman's (1952) classic essay in *Psychiatry*[1], in which he cogently unpacked the events and personas of confidence scenarios (i.e., cons) as a context for the more general methodology by which individuals adapt to failure. According to Bahr (2008), the original conception of cooling out entails,

> ...a process whereby an individual who has been the victim of a con game is eased out of the recently held identity of 'sure winner' by the *cooler* (the agent of *cooling out*) into an alternative identity *other than* 'victim' (p. 705, original emphases).

Goffman extended the concept as a metaphor on social interactions, suggesting that cooling out may occur in any situation whereby "individuals suffer involuntary loss or failure that reflects unfavorably on either their capacity to perform a particular role or their claim to a particular self or identity" (Snow, Robinson, and McCall, 1991, p. 425). It is that particular depiction –i.e., reflections on capacity to perform or otherwise to identify in particular ways– that underlies the current exploration of COPE.

Cooling out has been employed across a range of academic fields, populations, and situations: for instance, to characterize the methods by which poor persons are disregarded in child and family psychiatry clinics (Adams & McDonald, 2010); in whale-watching tourism (Nutch, 2007); to unpack the most recent international economic recession (Glynos, Klimecki, and Willmott, 2012); toward understanding the "relationship between gender, cooling-out, and the public order" (Snow et al. 1991, p. 425); job termination (Miller & Robinson, 1994); to describe a then-emergent relation between special education and "inner-city" schoolchildren (Johnson, 1969); among mothers of children with Down's syndrome (Thomas, 2014); or to unpack socializing practices in the law profession (Foster, 1981). Clark's (1960) oft-cited introduction of the concept to higher education, however, established the basis for questioning whether there is a special relationship between cooling out and students' experiences in developmental or remedial education programs. This particular question has sparked many studies and references to the "cooling-out function" of remediation, leading unproductively to "a small but constant stream of criticism of the community college" (Vaughan, 1980, p. vii).

1. In fact, "On Cooling the Mark Out" was only the second publication (pre-dissertation) of Goffman's illustrious scholarly record (see http://www.tau.ac.il/~algazi/mat/goffman.htm). Surprisingly little attention has been afforded the cooling-out concept –an "important yet neglected metaphor" (Thomas, 2014, p. 283).

PURPOSE, CONTEXT, AND GUIDING QUESTIONS

In this paper, I revisit the notion of COPE as it applies generally to educational access and mathematics remediation, but this time the specific aim is to shed some light on the relation between COPE and mathematics-learning experiences –i.e., how COPE unfolds experientially and *in situ* for mathematics learners– toward a nuanced and phenomenological understanding of what COPE looks like when it happens, how it is manifest in the context of mathematics-learning experience specifically (also see Larnell, 2016), and toward understanding how others' roles are involved as COPE unfolds in situ. Furthermore, the study context is both essential and central. Here, I explore COPE as it emerges and plays out amid learners' experiences in non-credit-bearing remedial[2] (NCBR) mathematics courses at four-year universities. As the research literature has substantiated across several decades, COPE happens, and it is particularly known to happen in relation to this kind of course (although it has been studied more often in two-year institutions). The goal of this article is to move toward understanding *how* COPE happens for learners in this kind of setting.

As I will discuss throughout the following sections, COPE as described here diverges from its original conception in a pivotal way. Both Goffman and Clark described cooling out originally as a designed structural mechanism or an otherwise intentional process. I argue and present empirical data that support an alternative notion that COPE can be manifested in less coordinated but nonetheless formidable ways. Toward unpacking this phenomenon further, the central questions of the article are:

(1) How does COPE emerge and unfold experientially for learners in NCBR mathematics courses?

(2) Who are the agents of COPE in these instances?

NCBR mathematics courses

During the past several decades, there has been a considerable increase in enrollments in NCBR mathematics courses in both two- and four-year colleges and universities within the U.S. (Attewell et al., 2007; Davis and Palmer, 2010). NCBR mathematics courses are offered in four-year universities to provide an opportunity to beginning or transferring college students who have been deemed underprepared to learn the mathematical content that will allow them to pursue postsecondary studies successfully.

2. NCBR courses are commonly known, widespread, but understudied in two-year and –to an even lesser extent– four-year universities in the United States. The term is used to emphasize the fact that these courses are often required but do not count toward graduation.

As such, these courses represent a controversial gatekeeping experience that is unparalleled among students who are entering U.S. colleges. Of these growing enrollments, mathematics is the most common subject (Parsad, Lewis, & Greene, 2003). On the one hand, NCBR mathematics courses are indispensable tools for equalizing access to postsecondary institutions. On the other hand, the credits for these courses are non-additive; so while students may be required to enroll in (and pay for) these courses before proceeding to credit-bearing courses, NCBR courses do not count directly toward a credential. And too often, student placements in these courses produce a familiar result: a repetitive cycle of enrollment in NCBR courses before a student is allowed to move forward. For too many students who are unable to withstand this pernicious process, the cycle produces a more invidious result: students often abandon their advanced-math-reliant concentrations or even their college careers.

INTUITING COPE: FROM CON TACTIC TO ANALYTICAL TOOL

To motivate the use of cooling out as an analytical tool, let us first consider its original conceptualization. According to Goffman (1952),

> Two basic ways in which a person can lose a role have been considered; [they] can be promoted out of it or abdicate from it. There is, of course, a third basic ending to the status story. A person may be involuntarily deprived of his position or involvement and made in return something that is considered a lesser thing to be. It is mainly in this third ending to a person's role that occasions arise for cooling him out. It is here that one deals in the full sense with the problem of persons' losing their roles (P. 455).

In this originating article, Goffman explored this third cooling-out ending by examining the workings of the confidence game, or con, which features certain roles –e.g., operator or grifter, mark, muscle, and shill(s)– and the interactions among the persons who take up such archetypal roles.

To illustrate briefly this allegorical scenario, imagine a classic "shell" game in which a crafty entrepreneur (or "operator") sets up a playing surface on which there are three overturned cups and a pea-sized ball. Three-card monte (or "find the lady") is the quintessential two-bit example of this kind of game. The basic play proceeds as follows: A mark is recruited to play the game –or is "put up." After the mark pays the operator's fee and the play is initiated, the operator shuffles the cups in a way that introduces the perception of uncertainty regarding the location of the pea-sized ball. The mark is then directed to locate the ball, usually with a group of onlookers standing by. The mark's hope is to earn double the money paid; for the operator, of course, the objective is to keep the money by deceiving the mark (while maintaining the appearance that all is left to fair chance).

Although it would appear so, the assemblage of onlookers is not simply incidental; there are often co-operators (e.g., shill, muscle) within the crowd. The shill's task is to lead and subtly manage the crowd, watch for law officers or other potential interruptions, recruit and encourage prospective marks to play the game, and to subsequently sway the mark's behavior –but not, however, to tell the mark what to do or how to choose. By subtly supporting (perhaps even cheering) the mark's play, the shill endears herself to the mark and becomes a kind of ally. In this scenario, the mark can never know, however, that the shill is actually the operator's accomplice. It is only through this tentative, faux alliance between the shill and the mark that the occasion arises for cooling out the mark. On the chance that the mark wins the game, the operator loses and is obliged to pay the mark. Customarily the operator does pay, but in other cases (e.g., a relatively big win), the muscle steps in and interferes while the operator packs the game and escapes (i.e., abruptly blows the mark off). This latter scenario is undesirable, of course, because it curtails the operator's opportunities to score money from other potential marks. Alternatively, a savvy co-operating shill will sway the mark to double down, given the mark's obvious luck (what possibly could go wrong?). With the shill's help, the operator has laid the rope, and the mark is hooked into continued play. In the event that the mark then loses (or is touched), the shill's alliance now becomes the operator's secret tool. Desperate to reconcile the loss, the mark may plead with the operator to play again or to obtain a refund. Before this can happen, however, the shill steps in and offers consolation while attempting to persuade the mark to accept the loss and move on. The shill becomes a cooler. The shill-as-cooler role is key to the continuity of the game; an upset and volatile mark can draw unwanted attention, incite violence, or convince other prospective marks to avoid the operator's unfair hustle. Cooling out the mark allows the game to go on.

Goffman stressed the metaphorical utility of the cooling-out phenomenon, arguing then as Snow and colleagues (1991) have argued more recently for "its extension to any domain of social life where individuals suffer involuntary loss or failure that reflects unfavorably on either their capacity to perform a particular role or their claim to a particular self or identify" (p. 425). Following this extension, in cooling-out episodes the 'mark' may not be the explicit target of a con game but more so a person who has suffered in the short term some embarrassment, failure, or loss of face and, in the longer term, possibly a loss of identifying within the domain of which they otherwise may seek to be a part. In that way, cooling out can be used as a kind of analytical frame:

> The analogy becomes a framework to explore how a 'mark,' the victim or potential victim of planned exploitation, comes to accept their loss and resolve taken-for-granted expectations (i.e., how they are cooled out). [Goffman] acknowledges whilst those participating in the confidence game

> are found in only a few social settings, the concept of cooling the mark out becomes an analogy for how individuals contend with "adaptations to loss; with defenses, strategies, consolations, mitigations, compensations, and the like" (Goffman 1952, p. 462, as cited in Thomas, 2014, p. 283-284).

RESEARCH METHODOLOGY AND DATA METHODS

This phenomenological project was guided by an overarching interest in shedding light on how COPE happens by drawing from participants' academic and mathematics backgrounds, their university and course-based experiences, their accounts of resources and constraints, and their evaluations of their experiences (see Larnell, 2016, for a related, more specific theoretical framework). The aim was to unpack the experiential nature of COPE by exploring questions about who participates and interrogating how interactions unfold –rather than to generate a general principle(s) or theory regarding the phenomenon's nature. In this way, the general approach taken here blends and pivots between aspects of hermeneutical and transcendental phenomenology –whereas the former focuses on interpreting lived experience, and the latter emphasizes a more birds-eye description that does not adhere so closely to the real-life, everyday world of interaction and experience.

To contextualize COPE and address the questions posed earlier, I re-examined data from two recent studies, one of which has been detailed elsewhere (Larnell, 2016). Both studies were conducted at public, research-intensive, four-year universities in the Midwestern United States. The studies combined multiple qualitative data collection methods: questionnaires regarding students' academic and math-specific backgrounds, observations of and field notes about students' interactions and teaching-learning experiences in NCBR math classrooms, and series of semi-structured interviews with students about their past and then-current NCBR mathematics-learning experiences. Although all of these sources inform the accounts presented here indirectly, the interviews provide the bulk of the data that inform the vignettes directly. More information about the specific data collection methods, theoretical frameworks, and the setting of the first study can be found in other texts (Larnell, 2016; Larnell, under review; Larnell, Boston, & Bragelman, 2014).

The selections of data included herein were identified as COPE episodes after several phases of open and theory-driven coding and through multiple readings of the available data corpus (cf. Larnell, 2016). COPE episodes are operationally defined and interpreted as those in which the narrator discusses an interaction(s) with a socialization factor/agent (e.g., a teacher, a peer, or institutional personnel) or an institutional policy, program, or practice that subsequently influenced how the narrator made sense of their experience in the NCBR mathematics course. Typically, these episodes were pieced together across multiple utterances or discursive turns based on mentions of the same socialization factor, and in some cases, the episodes were recounted across multiple interview sessions. References to various socializing agents were coded within the participants' narratives about their experiences, and the roles of these agents were interpreted and characterized (to the extent to which they were discussed) toward understanding the mathematics identities that students were constructing amid their told experiences.

FINDINGS

In a much lengthier and fuller version of this paper (currently under review), the findings are presented as three thematic vignettes that are each based on a single participant: Vanessa, Nicole, and Ruby. Across those months-spanning depictions, a kind of trajectory or "arc" emerges and uniquely evinces a phenomenological storyline for each of the participants. These arcs, I argue, align with the same essential stage-like features of cooling out as Goffman originally depicted. Moreover, each vignette depicts more richly and fully the roles associated with the participants' experiences with COPE, reported salience of those roles, and the impact of the various socialization agents who took up those roles amid each participants' experiences while they were students in an NCBR mathematics course.

In this abbreviated account, I present the findings as a composite storyline across the three participants, with constrained attention to their individual stories and therefore a much more narrow exposition on the phenomenon itself as evinced in parts of their overall experiences. Across the cases, the storyline is initiated by a common event before it splinters into distinctive experiential variations of COPE, with each variation involving different socialization agents, circumstances, and outcomes. The focus here is on two possibly essential phases of a COPE arc in this particular mathematics education context: students' initial placement in the NCBR mathematics course and, later, their more particular experiences receiving and negotiating messages that would encourage them to view their curricular location and their learning experiences therein as temporary, non-binding, and not as unfavorable as it may have seemed initially.

The "rope": The role of mathematics placement and institutional policies

At many U.S. public universities, new students are routinely required –often as one of their first experiences in their new academic environment– to complete a university-designated mathematics placement exam (students may also undergo a similar writing examination). The primary function of this kind of institutional policy mechanism is to sort students –a mathematics-specific gatekeeping function. All of the participants described inauspicious experiences with the examination; and surprisingly perhaps, Nicole's experience was not unusual among the participants I interviewed.

> Nicole: Um. Did I take the placement test seriously? No. My whole thing about coming to LMU[3]; I knew that I was not strong in math. Honestly, I went –the mindset that I had when I took the placement test was– I wanted to get a low score on it, so that I could start with the basic math here. So, then, I could build my foundation up here at LMU. So, if I got [into the NCBR mathematics course], then [the subsequent, credit-bearing algebra course] would build off of that, and so on and so forth. So, that's kinda what I wanted to do. I wanted to start off, you know, as low as possible and work my way up. No matter how long it took, I know that I would've been straight as a student when it came to math at LMU. Um; I don't even remember what score I got on the placement exam. I know it was low, because I'm (laughs) in [the NCBR mathematics course]. Did I take it seriously? Like I said: no. But that's just really the mindset that I went into when I took it.

This curious response to a testing situation may lead the reader to assume that the test-taker(s) was underprepared or otherwise deficient, but as I have reported elsewhere (Larnell, 2016), this response is neither unique, infrequent, nor necessarily an act of deficiency; it can be an act of agency. Across at least three other cases in the two studies, other students have reported to engage in what I have termed "satisficing" when met with this particular kind of entrance exam (p. 255): The students recognized that a poor score on the exam will result in their placement in an NCBR mathematics course, but for many reasons (one being the potential to avoid perceived opportunities to fail), the students opt to satisfy the requirements of the exam with minimal sufficiency. Beyond satisficing as an act, however, the role of the placement exam as an institutional mechanism that sets the stage for a mathematics-curricular experience that does not contribute directly to graduation credit.

Unexpected "shills": Two examples of cooling out the mathematics learner

NCBR mathematics courses often resemble –in terms of mathematics content and classroom environment– secondary school mathematics education, but the pedagogy in NCBR mathematics courses is typically direct instruction with a focus on procedural fluency involving fundamental number and algebra concepts: simplifying numeric and algebraic expressions (properties and procedures) and solving polynomial equations and inequalities. Ruby struggled throughout the course –especially with

3. LMU, for "Large Midwestern University," is a pseudonymous acronym. All other proper nouns are also pseudonyms.

the highly procedural algebra content– and near the semester midpoint, she began to consider alternative options to earn the necessary mathematics credits, informed by

> Ruby: Like really the only person that I do talk to about my academics is really my mom… In lots of ways, she knows me better than I know myself; it's that type of relationship. And then there are people at church who want to be resources. They're like, 'you'll do fine. Just stick it out. Math isn't that hard.' And they don't know me like that. But my mom, she was never that good at math, either. So, she understands. Because she took, uh, an equivalent course to [the university's NCBR mathematics course] when she was here. And she failed it. I couldn't believe it, as hard as she gets on me. But like, she failed, and she understands what I'm going through. So, I just choose to talk to her about it. And she was the one who said, 'when you get back, just be prepared to hit the community college or [the other local university], for when I come back next semester or next year. (Interview 3)

Ruby had a broad supportive network, mostly situated outside of the university environment. As she reports above and throughout her interviews, her most ardent and operative supporter was her mother, who was coincidentally also a student in the very same NCBR course when she attended the university years earlier. This legacy connection was central to Ruby's cooled out expectations and her gradual detachment from the course.

For another example of how COPE may involve socialization agents *within* the university environment, consider the following brief account about Vanessa's involvement with an institutional support program in which peer-to-peer mentoring was a central component:

> The 12-minute skit [which was itself named after the NCBR mathematics course] incorporated numerous issues and circumstances that involved transitioning to university life and reflected the [more experienced student-] actors' personal and anecdotally acquired experiences. The skit's loose plot was intended to reflect these classes and their typical routines, but it also showcased the student-centered perspective on what actually occurs. The skit portrayed students who were attempting to capture as much of the breakneck-paced lecture as they could, while for many of the actors this ended quickly as they instead opted to ignore the classroom activity and carry on with their social exchanges. Eventually, none of the actor-students were paying much attention to the actor-instructor, who was seemingly oblivious to the loss of audience. The skit ended abruptly as someone imitated a ringing bell (which was not a true element of the college classroom), and the actor-students dashed madly for the door, leaving their books behind (Larnell, et al., 2014, p. 54).

CONCLUSION

Mathematics as a curricular discipline has been utilized consistently and notoriously to cool learners' academic aspirations. Put differently, cooling out the mathematics learner has evolved as an essential individual-level part of a much broader project that positions mathematics as the premier academic gatekeeper. We have much to learn collectively, however, about how this gatekeeping functions at an individual-learner level. With further study, COPE may be a usefully analytical lens through which to unpack this various versions of mathematics-curricular gatekeeping –and to contribute to our understandings of mathematics learners' experiences more generally.

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MOVING UP OR DOWN THE LADDER: UNIVERSITY MATHEMATICS STUDENTS TALK ABOUT PROGRESS

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This paper explores how university mathematics students (re)produce discourses of progress in their talk about school and university, and the implications thereof for what they can do and be. A socio-political perspective and critical discourse analysis are used to analyse longitudinal interviews with students who are regarded as historically marginalized –on account of their race, class and language– in the context of an elite South African university. For these students, progress is about moving forwards or upwards, relative to others, towards quality education and the related social and material rewards for the individual and his/her family. When this narrative breaks down, students blame themselves, spend more time studying and become disconnected from material and human resources at the university and home.

INTRODUCTION

Recent mathematics education scholarship has explored how the notion of progress is given meaning in mathematics and mathematics education in a neoliberal era, and the implications for the production of the student as subject. This era can be characterized as a "re-structuring" of the relations between the social, economic, and political, and a "re-scaling" of relations between the individual, local, regional, national, and global (Fairclough, 2003, p. 4). Mathematics, it is argued, is viewed as powerful knowledge that drives individual, social, economic, and (democratic) political progress (Popkewitz, 2002; Skovsmose, 2009; Valero, 2008). Mathematics education –through which students are to access this knowledge– is thus viewed as personally empowering and as an unquestioned social good (Valero, 2008). The focus of mathematics education, therefore, is on developing efficient teaching and learning processes (Pais & Valero, 2011), with progress in this respect defined in terms of a measurable, linear temporality (Llewellyn, 2016). Such measures are accompanied by a neoliberal discourse that "offers the individual autonomy and the impression that we govern ourselves" (N. Rose, as citied in Llewellyn, 2016, p. 301), with failure pointing to a lack of individual student ability or effort. These discourses render opaque the related economic, race, gender, class, and language structures that shape mathematics education (Fairclough, 2003; Martin, 2013; Pais & Valero, 2011).

Mathematics education scholarship points to the prevalence of a linear, temporal notion of progress in mathematics education policy, research, curricula materials, and the talk of student teachers and students, with this notion working together with neoliberal discourses of growth, competition, individualism, performativity, quality, consumption, and choice (e.g., Doğan & Haser, 2014; Kirwen & Hall, 2015; Llewellyn, 2016; Llewellyn & Mendick, 2011; Smith, 2011). This, despite challenges to this notion of progress from the social sciences, as well as strong evidence that mathematics and mathematics education practices are not necessarily a societal and environmental good, but may produce inequality (Harding, 2009; Skovsmose, 2009).

In this paper, I use a socio-political perspective and critical discourse analysis of longitudinal student interviews to investigate (a) how mathematics students at an elite South African university (re)produce the notion of progress, (b) how this (re)production relates to other discourses, and (c) the implications of these discursive constructions for what these students can do and who they can be. While the structuring effects of dominant discourses are not easily visible (Apple, 1995), the context of this paper offers a source of "non progressive stories" (Llewellyn, 2016, p. 300); we see what happens when the narrative of progress as a "forward/ upward movement" (Llewellyn & Mendick, 2011, p. 52) breaks down. Firstly, the students in focus are those regarded as historically marginalized –on account of their race, class, and language– in the university context. On the basis of this classification, they were given access to the university via a four-year (rather than three-year) science degree programme. Measurements indicate that such students are likely to take longer than planned to graduate or might not graduate at all (CHE, 2013). Secondly, in post-apartheid South Africa the interaction between modernity, neoliberalism, and the socio-political history of the country produces a complex discursive space and a material reality of stark inequities in who makes "progress".

THEORETICAL FRAMING

While it is not possible for researchers to "step outside" (Llewellyn, 2016, p. 300) the dominant ways of thinking (Apple, 1995), mathematics education scholars have used poststructural perspectives (e.g., from Foucault), to critique dominant discourses. I use Fairclough's three-level perspective of the social and of language –referred to as a *socio-political practice perspective* here– to view the relationship between these wider discourses and what students say in research interviews.

From a socio-political perspective, mathematics and mathematics education are related networks of social, historical, and political practices. A practice is a relatively stable, recognisable combination of related material, psychological and discursive elements. The material and psychological elements of a practice are related to but not reducible to language use; discourse reproduces the activities, social relations, knowledge, values and so on, of the practice, but it is also a form of "action" (Fairclough, 2003, p. 26) that gives meaning to the social world. Thus, a discursive construct such as progress has material and emotional effects as it shapes what a student can do and feel. In addition, a socio-political practice mediates the meaning of abstract structural constructs such as race, class, and economic systems (Fairclough, 2003); a practice sets up particular subject positions for participants, depending on these wider social memberships. Yet these memberships do not determine a person's performance, as a participant can act agentically, that is, she or he can "do things, create things, change things" (p. 160).

METHODOLOGY

The data in this paper were produced in a larger longitudinal study of the experiences of historically marginalized students at an elite South African university[1]. I focus here on interviews with six students enrolled in an extended science degree programme, which required at least a first-year level mathematics course. Four of the six students graduated and two were academically excluded before graduating. The students with pseudonyms Colin, Joseph, Luthando, Philisani, and Thabo identified as "black African"[2] and male, and were learning university mathematics in English as an additional language. Josephine identified as "coloured" and female. All six students attended working class schools. For the study each student took part in an annual, individual semi-structured interview conducted by a trained interviewer. The verbal interaction in the three to five interviews with each student was transcribed.

From a socio-political perspective, a research interview is an instance of "recontextualized" (van Leeuwen, 2008, p. 3) practice. Firstly, the talk in an interview is "filtered" (Fairclough, 2003, p. 139) by the nature of the study and the resources of and power relations between participants. Secondly, in an interview what a participant says at a particular moment is "filtered" by her or his interpretation of past experiences (in Skovsmose's terms [2005], her or his *background*) and also her or his interpretation of future possibilities (her or his *foreground*).

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1. The study was funded by the Andrew W. Mellon Foundation.

2. The apartheid era racial classifications "black African" and "coloured" are still used to report educational performance in South Africa. Not only do these constructs still matter in terms of mathematics performance, but they also work together with class, language, and geographical location.

I used Fairclough's (2003) method of critical discourse analysis to analyse the content and form of the interview texts. This method involved working to-and-fro between the micro-level text and the macro-level discursive practices. At the micro-level I considered what meanings the lexical and grammatical features give to the text. At the macro-level I looked for traces of discourses of progress, quality, individualism, and so on in these meanings. In the next section, I briefly describe the macro-level discursive space of university mathematics education in South Africa.

UNIVERSITY MATHEMATICS EDUCATION IN SOUTH AFRICA

Skovsmose (2009) argues that "Modernity" –with its assumption of progress– and the special role assigned to mathematics in this view "marched along with colonization" (p. 325). In South Africa, the societal, economic, and educational divisions of colonialism became entrenched during apartheid (Badat, 2009), with mathematics being used to justify racial inequities (Khuzwayo, 2005). Emerging from apartheid in the early 1990s, post-Apartheid South Africa has sought to compete in a globalized, neoliberal world, while simultaneously redressing past injustices and developing an inclusive, democratic society in a context of poverty. Mathematics and mathematics education –at university in particular– are positioned as offering a "salvation narrative" (Popkewitz, 2002, p. 2) in this respect. These multiple goals have produced a complex, tension-filled discursive space (e.g., Barolsky, 2012; Posel, 2010). For example, we celebrate non-racialism, equal rights, and opportunity, but apply redress measures that appear discriminatory and have the potential to stigmatize. We encourage care for one another (given meaning in the concept of *ubuntu*), but also celebrate individual choice, hard work, and ambition in overcoming obstacles. The pursuit of the middle class dream –paraded as material accumulation– exists alongside extreme poverty.

These discourses take on particular meaning in South African universities. Soudien (2008) argues that the encounter between the features of the university –the value placed on individual identity development and independence, the nature of university-level knowledge, the elite nature of the university, and the possibilities for social mobility– and "middle class dreams" (p. 670) result in complex relations between the self, others, and the institution. In South African universities, he argues, this encounter has resulted in a certain type of racialized and classed experience.

After 22 years of democracy, there is stark evidence –most visible in civil protests and increasingly racialized discourses– that, for the majority of South Africans, progress is not equitable and not simply a matter of choosing to work hard and to dream big. While such protest has tended to be by the "poor" located on urban peripheries, university students have in the past two years entered this action, lamenting inequities in epistemic, social, and financial access to universities.

ANALYSIS: (RE)PRODUCTION OF NOTIONS OF PROGRESS

The students' transition to and through university required that they move from school, to the programme in which their first-year level courses were "extended" over one-and-a-half or two years, and then to the senior courses that constituted the rest of their degree programmes. Each student's talk about these practices is unique and has been discussed elsewhere (e.g., le Roux 2013), but the six voices are also "patterned in systematic ways by location, common values, comparable resources and shared experience" (Thomson, 2009, p. 8). It is these patterns –which point to the power of the structuring discourses– that are the focus of this paper.

Making progress as a top school mathematics student

Measurements –in the form of mathematics marks– were used by the students (and their peers) to identify them as different to others at school; "people knew me by my marks" (Luthando). For Thabo, this meant accessing limited resources at his school "first". The difference also lay in the students' identification of themselves as having no choice but to make good use of time to move upwards or forwards while their peers were static. When a teacher was absent most of the students in Josephine's class "would be waiting there and making noises," but she and her friends (who also got "good marks") "had to go to the office to find out what was happening". For Philisani, time was "running by":

> I was forced to be independent if I can say it because uhm... I had to think about my future and I had to think about what I want [...] uhm I couldn't just sit around and not do anything because the time was running by I had to figure out what I'm going to do and go out for what I wanted to do.

Getting good school mathematics marks school meant working "very hard" (Josephine). At times, this involved being static or going back; Luthando and Philisani identified themselves as having to "sit in" to listen to the teacher, and Philisani would "go back" to class during breaks to get help. Yet this action was about being independent, moving upwards and having "education" in the foreground: "you did what you basically could do in order to get education so it was more like standing up for yourself as alone" (Philisani).

The education in these students' foregrounds was at a university represented as the "best" (Luthando) and as a "world class African university" (Philisani). Philisani represented this university as opening "many opportunities" for students, "especially from disadvantaged" backgrounds. The university was also represented as opening opportunities for careers that would bring social and material rewards. Colin and Luthando identified certain family members as role models, on the basis of their material and social accumulation, that is, their cars, houses, and jobs. For Thabo, studying science offered a “faster way to go up the ladder” than arts disciplines. These were not just individual rewards, but rewards for their families who were “looking up to” (Colin) them and whom they could not “let down” (Colin). Thabo related the need to “care” for his family to his race; “we are black people, we have families we have to take care of and we need to go back home”.

Differences between school, the extended courses and senior courses

Views on the value of the extended courses varied across the six students and with time, yet there are patterns in how they represented these courses relative to school and to their senior courses. Most of the students spoke of their disappointment when hearing that they had been placed in these courses. Philisani represented this placement as “a little bit declining” and Luthando resented not having a choice in this regard. However, by reproducing the words of university staff to describe the extended courses as a “foundation”, the students located themselves in a university hierarchy and had making progress in their foregrounds:

> They’re wanting to build a foundation... so that I can... go in prepared to the science field... it started making a lot of sense [...] shows that they do understand my my from where I am standing and how and where do I need to improve... (Philisani)

Thabo represented the extended courses as designed “to lift you up a bit, to take you into the new level, the mainstream.” His choice of the word “mainstream” –commonly used in this context– identifies him as on a different path to other students. However, within a few months Thabo lamented that the pace in his extended mathematics course was too “slow”. Joseph represented the different pace as part of his transition from “running in a slow... slow... slow... lane” at school, to “moving on the fast lane” in his senior courses. Josephine consistently used the duration –four years versus three years– to distinguish her trajectory from that of other students. Joseph, Colin, and Philisani also identified themselves as needing to be progressively more “awake” as they proceeded from school (where Joseph was “asleep”), to extended courses and then to senior courses. Indeed, for Colin, his senior courses were “like an alarm saying that you are now in varsity.” There was no choice but to “wake up” or “grow”, the alternative of which was to “go down the drain” (Philisani).

Looking back at their first two years at university, all the students valued the extended courses as “comfortable” (Josephine) or “chilled”

(Colin) spaces where they formed “strong bonds” (Luthando) with students who shared their background. Many of the students likened the courses to being at home or with family, as Colin does in this extract; “it was the first family I met at [the name of the university], it is the first calm and relaxing environment, we are homies”. Yet some of these students also talked about their homes as places where they and their peers were static; Colin said that if he were to take a leave of absence from the university he would go “back home just to sit and not do anything”. Indeed, reflecting on the difficulty of the transition from extended to senior courses, Luthando (and Josephine) represented the former as not preparing them for when “there is not time to play” in the “totally different” senior courses. Not only was the pace different in senior courses, but the students were “left alone” (Philisani) without the support of their “family” (Colin) in the extended programme. Now they had to be independent; they had to “understand [...] by yourself” (Colin) and “manage my own self” (Philisani).

Signs that one is not moving upwards or forwards

At some point in the university careers of all these students’ measurements –in the form of marks for mathematics and/or other courses– signalled that they were not moving forwards or upwards. In addition, lecturers and tutors identified them in this way, that is, as “slowing down the discussion” (Joseph), and able to improve their marks if “I could get on track and on time with my work” (Colin).

Faced with evidence that they were not moving forwards or upwards, the students tended to blame themselves for not working hard enough: Colin attributed his “bad progress” in his fourth year of study to being “lazy”, and also not looking up towards his future: “I had lost that drive of aiming high”. Joseph identified himself as personally responsible for putting in more “practice” and “effort”. Indeed he said he could accept being excluded on academic grounds if he knew he had not been “lazy” but had “tried” and “put so much effort” into his studies.

The students’ responses to signs that they were not making progress drew on interpretations of the university as “all about time management” (Colin) and being “on your own” (Colin), on interpretations of their background as “disadvantaged” (Philisani) and on seeing progress in their foregrounds. When Josephine “failed” a senior course (not mathematics), she had two choices –having a “remark” and possibly adding marks or “redoing” the course. She chose the latter as she had “understanding the work” in her foreground and time on her side: “I only turned 21 this year, so if I would have graduated this year I would have been the same age as most mainstream students”. The students regularly rearranged their timetables by changing their sleep and study times. This rearrangement also involved repetition and studying for longer, that is, studying “more and mak[ing] sure I practice and do everything all over again, all over again" (Thabo).

For Colin, the problem of managing his time worked together with his identification as a "disadvantaged student" for whom the freedom and opportunity on offer at the university could be "overwhelming". He also identified himself as "separated" from "progress" on account of his learning mathematics in English and the "delay in thinking" as he "translated" between languages. Yet these students also suggested that, although their marks had dropped, these measurements did indeed represent progress for students with their background: "I think it is a huge achievement, like from where I come from fifty percent would mean a lot" (Thabo). Philisani measured his progress relative to others from his home community; some like him were "still surviving" (Philisani), while others had been excluded on the basis that "they don't wanna grow they don't wanna think about their future they just want to stick into that corner". As measurements of their performance pointed to gaps in their mathematical knowledge, some of the students emphasised their learning of other skills:

> ...that will help you towards the future like uhm being more... uhm manage managing your time... getting to know people and actually standing up and going doing things for yourself being independent. (Philisani)

The six students celebrated not yet being excluded from university, but as they spent more time working hard, all –with the exception of Josephine– became increasingly isolated from resources and others at the university and home. They stopped asking questions in class, on the basis that their questions were "low class" rather than "advanced" (Joseph) or that "if you don't know anything, you can't ask" (Thabo). When hard work, repetition, and more time did not translate into better marks, some students "gave up on the course" (Philisani). Luthando said he "didn't understand why I was doing maths," and only regained this foreground when he had to pass mathematics to transfer to an engineering degree.

The perceived need to spend more time studying meant that all six students made choices to spend less time on other activities such as part-time work, volunteer mentoring and tutoring of other students who shared their background, or religious activities. When facing personal difficulties, Luthando and Colin cited lack of time as the reason for not seeking counselling on the university campus. For some, more study time meant separating "my personal life from my studies" (Luthando) and less care for their families who faced difficulties. For Philisani, "pretending that I don't have a support system" was also about being someone who did not have the ability to "make it" as expected by his family:

> If everyone says all these great things about... you and how you can make it ... if you really don't believe that... and if you don't really don't have something within yourself...

DISCUSSION AND CONCLUSIONS

Scholars have pointed to the dominance of a narrative of temporal, linear progress, and related discourses of individualism, quality, consumption, and so on, in mathematics education (e.g. Doğan & Haser, 2014; Llewellyn, 2006; Smith 2011). Apple (1995) notes that dominant structures such as these make it difficult to answer questions about who benefits from mathematics education. In this paper, I have used a socio-political practice perspective and critical discourse analysis of longitudinal interviews to bring into view how "systemic constraints become lived as individual dilemmas" (Walshaw 2013, p.102) for six students at an elite South African university. In this context, there is stark evidence that the narrative of progress as a "forward/upward movement" (Llewellyn & Mendick, 2011, p. 52) breaks down for mathematics students –and indeed for many other South Africans– who have traditionally been marginalized on the basis of their race, class, and language.

For these six students, progress at school and at university was about moving forwards or upwards in a hierarchy. Placement in this hierarchy or "belonging" (Smith, 2011, p. 295) was relative; it was based on others –often those who shared one's background– being static or moving backwards. Progress at university was also about being awake rather than asleep, and being independent rather than being in a homely space. These students had no choice but to utilize the opportunities to progress towards a quality education and the related material and social rewards. This choice was not just about making individual progress, but about caring for one's family.

Moving up "the ladder," in Thabo's words, required working hard, being independent, and managing one's time. This progress narrative led to particular student responses when it broke down. They blamed themselves as "the only responsible person for their performance with the given opportunity" (A. Lewellyn & H. Mendick, as cited in Doğan & Haser, 2014, p. 1015). They spent more time studying. Most of the students worked independently rather than making use of university resources. Most of the students reduced their extra-mural activities and links to home. Taken together, performance measures –either passing but accepting one's lower marks on the basis of one's background, or failing and being excluded– meant feeling that one was lacking "something within [one]self" (Philisani).

The patterns in how the six mathematics students (re)produce a linear, temporal discourse of progress and related discourses points to the power of these discourses to shape what students can do and be. The critical perspective used in this paper brings into view how well-intentioned strategies aimed at redress of past inequities in the South African context may be in tension with these dominant narratives. This tension has consequences, not only for whether students achieve their academic and material goals, but also for how they connect with others and feel about themselves. Secondly, this perspective leads us to consider what other discourses might be possible in mathematics education, for example, a discourse of care for one's family and community.

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THE REPRESSION OF THE SUBJECT? – QUILTING THREADS OF SUBJECTIVIZATION

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Over the last decade, a flourishing discussion on the emergence of learner's subjectivities in mathematics classrooms has emerged in the socio-political strand of mathematics education. A variety of theoretical frameworks has been employed depicting that the space provided for the subject to recognize herself in school mathematics is minimized. Subjectivization in school mathematics, it appears, entails a repression of the subject. In this discussion paper, I identify and connect three different areas from which the thesis of the "repression of the subject" evolves – threads. I reflect upon these three threads through a theoretical lens borrowed from the contemporary philosophers Slavoj Žižek and Robert Pfaller. By quilting the threads, I seek to carve out the joint form of the three differentiated threads.

INTRODUCTION

In her recent article on students' interest in mathematics, Paola Valero (2015) points out the need to analyze the forms of subjectivity that emerge when pupils are confronted with school mathematics. She argues for a cultural gap between the forms of subjectivity promoted by mathematics curricula and the forms of subjectivity that derive from the cultural practices students engage with in their private lives. If students desire to cope with school mathematics, the process of dealing with this cultural gap seems to demand a repression of the subject. Although it has become a common observation that the space provided for the subject to recognize herself in school mathematics is minimized (e.g. Brown, 2011; Valero, 2015), research that analyzes "how the educational practices in the mathematics [...] curriculum are important elements in the constitution of modern subjectivities" (Valero, 2015, p. 29) is still rare. This discussion paper intends to contribute to this unfolding discussion (Pais, 2016). I argue that the thesis of the repression of the subject arises from three connected threads: (1) mathematics as a form of knowledge itself, (2) the social dynamics of mathematization, and (3) the learning of mathematics in school. Reflecting upon these threads through a theoretical lens borrowed from the contemporary philosophers Slavoj Žižek (2000, 2008a, 2008b, 2013) and Robert Pfaller (2011), I seek to relocate the analysis of the repression of the subject by shifting the focus away from the content (who or what is the subject?) towards the form of subjectivization (how does it come into being?). Despite all differences concerning the content, I argue that the process of subjectivization itself has a joint form in threads two and three. Further, I suggest that the repression of the subject is in itself not a straightforward process but a dialectical movement fueled by the interplay of antagonistic forces.

THREAD I: MATHEMATICS AND THE SUBJECT

> "I don't think that mathematical knowledge is that certain. Someone has *discovered* or, rather, *invented* it at some point, even though it makes sense in general. People who do not know our world certainly have a different view on mathematics." (Student, 8^{th} grade, 2015)

> "Mathematical knowledge is eternal. Certainly, new laws are *discovered* or *constructed* but no one can change the old laws. The current laws are already old and constructed by mathematicians in the past." (Other student, 8^{th} grade, 2015)

These students' utterances[1] from my 8th grade mathematics class sensitize for a field of tension between two different epistemological trends in the philosophy of mathematics, namely –a realist and an anti-realist strand. Within the tradition of Western philosophy, mathematics was embedded in an epistemology that conceptualized it as an abstract, superhuman, eternal and objective form of knowledge (Ernest, 2009). Here, mathematical objects exist a priori as ideal objects (independently from humankind) and can, therefore, be merely discovered by humans a posteriori. In contrast, trends under the umbrella term of constructivism developed an anti-realist philosophy of mathematics which depicts mathematics as a fundamentally social, cultural and historical phenomenon (Lakatos, 1976). Here, mathematical objects do not exist a priori but are, instead, contingent upon practices of constructing them.

The opposed epistemological positions build up a dichotomous frame: mathematics is either discovered or constructed. However, as illustrated by the aforementioned utterances, the clear distinction of the two positions collapses in human practice. Both students conceive mathematics as discovered and invented/constructed at the same time. According to Žižek (2000), we can conceptually overcome this dichotomy if we inscribe the friction between discovery and construction into reality itself. Žižek proceeds in two steps: Firstly, we have to accept the Kantian insight that any subject cannot but look at reality from her finite temporal standpoint. Secondly, such subjective localization must not be considered as the "epistemological limitation of our capacity to grasp reality" (p. 158) but, instead, as the "positive ontological condition for reality itself" (ibid.).

1. The students' utterances serve as illustrative vignettes and not as empirical data. Their function is not to provide "evidence" but to add meaning to intensely abstract hypotheses on a concrete layer.

Hence, irrespective of whether ideal objects exist independently of the human subject, the human cannot but build subjectivity into the objective entities she is supposed to discover. Any discovery thus entails a trace of construction. Consequently, the fallibility of mathematics is bound to the fallibility of the subject. This radically puts into question the idea of mathematics as a superhuman form of knowledge. The modern ideology of certainty (Skovsmose, 2005), which conceptualizes mathematics as abstract, eternal and objective has, however, survived until contemporary times (Ernest, 2009). Although, on the on hand, the subject is the necessary condition for the existence of any type of mathematics, on the other hand, she is repressed from it at the same time.

THREAD 2: DE|MATHEMATIZATION AND THE SUBJECT

> How would you complete this sentence: This is The Age of …? […] I would fill in the dots by saying that this is the computer age, or more sharply, the age of mathematizations." (Davis, 2009, p. 19)

In the 21st century, the computational algorithmic processing of personal data has started to penetrate nearly every sphere of social life through the use of mobile phones, payback-, EC- and credit cards, social networks, social media, and so on, and so on. Mathematics is effective in this development in two (related) respects: Firstly, mathematics has constituted the technological development of the computer as the crucial prerequisite (Davis, 2009). Secondly (and consequently), the computer contains limitations that are intrinsic to any formal mathematical description as such. In general, phenomena can be processed by a computer exactly then, when they are reduced to their quantifiable aspects: "Love", for example, becomes a matching of two persons qua psycho-metrics; "health" becomes the result of the combination of singular physiological characteristics, etc. In this way, human practice –and with it the subject itself– becomes reduced to those aspects that can be quantified in an apparently objective way, and the regulation of even larger social areas –hitherto contingent on human interaction– becomes replaced by mathematically formalized complements.

Supported by the ideology of certainty, mathematics is instrumentalized in the process as the privileged means for an objective description of reality, and mathematical models constitute themselves as an inherent necessity (Jablonka, 1996), although they are actually fabricating the reality that they pretend to simply describe: When, for example, a stock exchange algorithm, based on elicited preferences, "offers" a trader suggestions to sell or keep her shares, this algorithm is by no means a formalization of an objective description of real human behavior. Instead, it is a formalizing construction that re-inscribes the neoliberal logic of a subject (a subject that only pursues her own interests) into the practice of stock-trading. Since stock trading has transformed into a high-frequency endeavor, the trader cannot but act in accordance to the preferences provided by the algorithm. In addition to that, algorithms operate below the user-interface invisible for practitioners. Hence, it turns out that the mask of description conceals an underlying prescription and is further veiling the human influences in programming, selecting and implementing the algorithm. In this way, algorithms substitute the contingency of subjective experience in concrete life world contexts through a reified ontological order that disavows its very own "political foundation" (Žižek, 2000, p. 158).

Algorithmization appears to reinforce a phenomenon that, already in the late eighties, critical mathematics educators have conceptualized as "demathematization" which means that as soon as mathematical models are materialized in differential forms of technology, mathematics seems to disappear from the (visible) surface of social practices (Keitel, Kotzmann & Skovsmose, 1993). Thus, every mathematization always entails a demathematization. These two poles are dialectically bound together: Although they seem to oppose each other, demathematization can actually be conceived of as the inner negativity of mathematization; “an entity is negated, passes over into its opposite, as a result of the development of its own potential” (Žižek, 2008a, p. 180). Far from being two opposing poles, both processes simultaneously condition and reinforce each other (Jablonka & Gellert, 2007; Straehler-Pohl, 2016). Within the dialectic of mathematization and demathematization –in short, “de|mathematization” (Straehler-Pohl, 2016)– the prescriptive regulation of social practices can be implemented even more efficiently, because it mostly remains implicit or invisible for the affected human beings. In this way, the technological materializations of mathematical models function as black boxes that affect, change and substitute social practices far beyond their original intentions (Keitel et al., 1993).

The social dynamics of de|mathematization thus results in redoubling the repression of the subject: On the one hand, it elevates a formal language of description to an objective truth, disavowing its political foundation. On the other hand, it systematically replaces subjective accesses to reality by its formalized complements qua inscribing technologies – as materialized mathematical models – in the (invisible) regulation of social practices.

The Substance of the Self as a Quantum

In the 21st century, the colonization of social practices with prescriptive mathematizations and entailing demathematizations is especially effective in those areas, where social practices are optimized by quantifying the behavior of the individual and retroactively re-inscribing these “individualized” measurements into the organization of the very same social practices. The most obvious examples for such new form of social and individual regulation can be found in social media (e.g. dating platforms, social networks) and, especially, in the so-called self-tracking movement. The quantification of the self has the following effect: the subject is confronted with a representation of herself that appears more real than the lived experience of subjectivity itself, as the numbers appear to express her identity less ambiguously and more clearly (or "objectively") than life does. It is precisely this “ontologization” of the self to its supposedly objective characteristics that further urges the subjects to repress “their” subjectivity in favor of the objective self of quantification. This alleged objectivity, however, is nevertheless to be found within the subject itself, by presupposing "a movement that points towards a [supposed] solid inner core" (Illouz, 2007, p. 122).

In order to find the ontological self the subject is confronted with a two-folded process: On the one hand, the subject is pressured to supply the global big data players (e.g. Google, Facebook etc.) with personal data collected via the use of credit cards, search engines, social networks, and the like. On the other hand, the big data players promise to provide the technology that is able to “invent oneself” based on the collected data by means of an analysis, whose “grammar” yet escapes the subject's reach. The concomitant ideology can be translated into the following formula: “We can expose you to your substantial inner core! But to help us help you find your inner core, it is up to you to invent yourself by producing the necessary data.” While the subject is pushed to suppose an ontological self and search for it, she is also simultaneously pushed to manufacture it.

The fact that this two-folded logic is more than merely a thought experiment can be supported by referring to the Quantified Self movement. Quantified Self is a way of living based on the idea that the permanent quantification of all physiological values by means of technology (e.g. pulse, blood pressure, calorie consumption etc.) is the best way of getting to know oneself and optimizing one’s life. Thus, the self-proclaimed motto of the movement is “Self-knowledge through numbers” and "Self-optimization through numbers" at the same time. Assuming a substance of the self as a quantum intensifies the late modern never-ending and self-perpetuating vortex of self-observation and self-production.

Quantification as the Death of the Self?

Here, we are confronted with an ideological interpellation that masks itself as a helping mechanism to unfold the inaccessible "real" self by gathering its dynamics via a threefold disavowal: (1) the constitutive void of subjectivity is presented as a solid substance, (2) prescription is sold as description and (3) self-exploitation is portrayed as self-fulfillment.

The concept of “Self-knowledge through numbers” is based on a philosophical standpoint that ontologizes the self to an objective entity, which simply unfolds its inner potential. On the contrary, Žižek (2000) conceptualizes the self as a virtual entity: there is no such thing as a substantial core of subjectivity behind the mask of our various symbolic identifications (e.g. I am a mathematics education researcher, I am a Skateboarder etc.), but pure void. This void, however, is not the death of the subject, but quite the opposite: it is precisely the core of subjectivity (ibid.) –the void is the place, where the subject is actually "alive". Hence, the subject is decentered in her very constitution and the void is a necessary condition for any form of subjectivization as such (ibid.). Conceptualized as void, the "solid inner core" (Illouz 2007, see above) of the self can be neither measured nor manufactured. Instead, every (re-) formation of the self is contingent on a process of narration that retroactively sets its own conditions (Žižek, 2000).

Conceiving the self as contingent on retroactive acts of narration implies a radical break with a linear logic of historicity. This linear logic can be clarified by referring to the French language that differentiates between two forms of future: Avenir and futur. Futur simply describes the form of future that is “the continuation of the present, as the full actualization of tendencies which are already present” (Žižek, 2013, p. 264). It is the form that stands precisely for the linear logic of historicity. In contrast, avenir is “what is to come (à venir), not just what will be” (ibid.), an unpredictable future that retroactively sets its own conditions.

When the big data players prescribe our future behavior, interests and preferences by quantifying the present tense, the prescription is always limited to futur. This prescription forecloses the chance of a self à venir – a self yet to (be)come; a self that is more than the full actualization of its present appearances; a self that is more than the sum of its parts; or, simply put, a self that lives (in the sense of never being fully pre-determined) and hence cannot fully be formalized in mathematical terms. As Žižek (2013) puts it, the ultimate challenge of every emancipatory project is “to break the hold the catastrophic ‘future’ [as futur] has over us, and thereby to open up the space for something New ‘to come’” (p. 264). Thus, the promise to optimize and manufacture one’s self entails the death of the self à venir.

THREAD 3: LEARNING MATHEMATICS AND THE SUBJECT

The paradox of learning mathematics

> “In order to understand the logical, you have to think absolutely illogically” (student, 12th grade, 2015).

> “How can he [the human being] be the subject of a language that for

> thousands of years has been formed without him, a language whose organization escapes him, whose meaning sleeps an almost invincible sleep [...] and within which he is obliged, from the very outset, to lodge his speech and thought [...]?" (Foucault, 2002, p. 352)

The first utterance took place in a conversation between two students dealing with the issue of succeeding in school mathematics, whilst the second quotation is taken from the famous *Order of Things* by Michel Foucault. Both citations deal with a fundamental experience of alienation. In the first case, this experience refers to "the logical" of school mathematics; in the second case it evolves from the anthropological fact that any human being is born into a pre-existing language. Drawing on Wittgenstein's (2008) notion of *language games*, mathematics can be considered to be a formalized language game. When confronted with mathematics in school, the learner is thrown into this formalized language game, which rules are necessarily alien to her in the beginning. The dynamics of alienation constitutes itself in the question of how a human being can actually *become* the subject of this formalized language game "that for thousands of years has been formed without him" (see above). We can read the student's utterance as a response to Foucault's question (once we refer it to mathematics): "In order to understand the logical [to become the subject of the formalized language game of school mathematics], you have to think absolutely illogically [neglect your subjective experience of being-in-the-world]." Such interpretation implies a fundamental paradox experienced by many learners of mathematics: a yawning gap between students' subjective, lived experiences in relation to the supposed objective logic that regulates the language game of (school) mathematics. It seems that learning mathematics urges the subject to disavow herself. In such a way, the process of learning mathematics appears to repeat the repression of the subject that we have already observed in the first two threads.

Constructivism as a Return to the Subject?

In the meantime, the rise of constructivism towards a common-sense epistemology has largely promoted the emphasis on the subjective components of any learning process (Thompson, 2014). Today, it is hardly ever contested that the learner does not internalize knowledge but constructs it instead; learning processes are characterized by freedom and autonomy, relegating the teacher to a role of a facilitator of individualized learning processes (Radford, 2012). In theory, this shift inverts the repression of the subject by celebrating subjectivity and, therefore, retrieving what has formerly been repressed. However, it does not necessarily challenge the repression that is necessary for a legitimate acquisition on the side of the object of learning - mathematics. The student's paradox thus manifests a deeper problem: How shall it be possible for the learner to autonomously construct her subjective mathematical knowledge if what legitimately (objectively) is considered as truth has already been decided upon in advance? This field of tension between "emancipation" and "truth" has traditionally been understood as a psychological problem in mathematics education, with the consequence that its resolution has been conceptualized as a task for the individual. The learner herself is supposed to develop and enhance her cognitive construction processes: “The goal of intellectual education is to master the truth by oneself“ (Piaget, 1973, p. 106). In this way, the repression of the subject is not retrieved, but actually suspended by an act of replacement: While students are not perceived as objects of a process of a transmission of objective truth anymore, they are now required to become a subject; the legitimate subject is, however, not a subject à venir, but a subject that is already predetermined. Successful subjectivization relies on the subject's willingness to neglect her subjectivity when it comes to the matter of "truths".

The Necessity of Alienation

This momentum of alienation, I claim, cannot be resolved by a form of reconciliation at a higher level. Drawing on Žižek’s interpretation of Hegelian dialectics, I argue that this momentum of alienation is the irreducible and constitutive element of any subjectivization as such, or in short: no subjectivization without de-subjectivization. This means that any quest for a learning process, which would allow for a return to a “non-alienated” form of subjectivity, finds its fate in a dead-end from the very beginning. The student is subjected to the symbolic order of mathematics, which means that he or she will, by necessity, at first perceive mathematics as order of particular restrictions and rules, which is impersonal and external to him or her. Therefore, he or she can only subjectivize this order by means of an accompanying self-transformation, which then itself generates a surplus that allows further transformations or learning. There simply is no such thing as an “innocent” process of subjectivization: “what this misses is the way the subject emerges through the ‘self-alienation’ of the substance, not of itself” (Žižek, 2013, p. 261). Preconditioning an “innocent” self means to fall back on the illusion of a self that somehow emerges from its inner potential stored in a substantial ontological core of subjectivity (ibid.). The subject cannot, but

> “insist on the irreducible vicious cycle of subjectivity: ‘the wound is healed only by the spear which smote it’, that is, the subject ‘is’ the very gap filled in by the gesture of subjectivization [...] – the subject is both at the same time, the ontological gap as well as the gesture of subjectivization which, by means of a short circuit between the Universal and the Particular, heals the wound of this gap” (p.158 f.)

Hence, the only form of reconciliation is to be found in a shift of the subjective perspective, away from the idea of the alienation as an obstacle for subjectivization, towards a full identification with the *self-alienating substance*. Consequently, the irreducible ontological gap between “the logical” of mathematics as the object of learning and the subjective ‘logic’ of being-in-the-world (which is moreover already inscribed in mathematics as a form of knowledge itself, as shown in Thread I) has to be simply accepted as such. It is precisely this gap, the fact that we are never fully immersed in the Other (mathematics, culture, political economy, etc.) that “makes education necessary […] because it ultimately fails” (Pais, 2016, p. 13).

QUILTING THE THREADS: MOVING FROM THE CONTENT TO THE FORM

In order to move from the content of subjectivization to its form, I draw on the Austrian philosopher Robert Pfaller (2011). He denounces a new form of ideological interpellation (form of subjectivization) that he sees as constitutive for the postmodern perception of reality. While traditionally, interpellation was effective via the call for identification with a symbolic mandate (e.g. be a soldier, be a soviet, be a mathematician, etc.), postmodern ideology has replaced this interpellation with the imperative: „Be Yourself!“ (p. 30). Whilst this imperative grants the subject the right to freedom, self-realization, private enjoyments, etc., it rejects any offer of a symbolic mandate as invasive. However, as argued above, the subject is inevitably “decentered”, hence, the imperative to „Be Yourself!“ turns out to be a deadlock. The threads of subjectivization in thread 2 and 3 operate in a similar way: In the second thread, the ideological pathway of individualization is utilized by the big data players. Algorithmic data analysis “rectifies” individual preferences by formalized prescriptions. The dynamics of de|mathematization forces the subject to assume an ontological self, while simultaneously hollowing any sense of subjectivity by formalization. Similarly, when students deal with mathematics in a learning process brought forward by constructivism, they are firstly told to develop mathematics somehow from their inner "core", as if it has already been there and just needs to be unfolded. What they are confronted with, however, is a formalized language which rules are necessarily alien to them. Similar to the imperative to "Be Yourself!”, thread 2 and 3, yield a throwing back of the subject to its supposed (yet non-existent) inner core. In this way, the repression of the subject is simultaneously neglected and reinforced: "the subject is ordered to embrace freely, as the result of his choice, what is anyway imposed on him" (Žižek, 2008b, p. 36).

As my rudimentary analysis suggests, postmodern ideology and the accompanied forms of subjectivization result in a dead end because they stabilize a "fantasy" (*Žižek*, 2008b) that not only conceals the necessity of alienation but, furthermore, reinforces its repressive effects. This leads us to call for the need of a profound analysis of the interdependence of the forms of subjectivization active in the mathematics classroom in particular as well as in our contemporary mathematized society in general.

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RECONFIGURING MATHEMATICAL SETTINGS AND REPRESENTATIONS THROUGH WHOLE-BODY COLLABORATION

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We examine consequences of whole-body, multi-party activity for mathematics learning, in the contexts of number sense and ratio and proportion. Drawing on micro-ethnographic techniques, we compare two cases of whole-body, collaborative movement in mathematics activity. Informed by contemporary theories related to social space and embodied cognition, we illustrate how whole-body collaboration might transform how students experience, make sense of, and make use of spaces of learning. The analysis enriches our understanding of the changing spatial landscapes for learning and doing mathematics as well as how re-instating bodies in mathematics education can open up new forms of collective mathematical sense-making and agency.

INTRODUCTION

This study examines the consequences of whole-body, multi-party activity for mathematics learning, both in and out of the classroom. In particular, we report on and compare two cases of implementing whole-body, collaborative movement activities designed to engage learners in the mathematics of number sense and ratio and proportion respectively. By investigating how learners made sense of these mathematical concepts through physical action and interaction, we illustrate how whole-body collaboration can transform mathematics activity and learning, and shift aspects of disciplinary agency, embedded in representational infrastructure, to students' collective activity. The paper also contributes to research that seeks to expand sociocultural lenses on mathematics learning to take into account bodies and place.

To investigate these issues, we bring together (a) scholarship that attends to spaces of learning as both productive of and produced by human activity and (b) contemporary theories of embodied mathematical cognition that view mathematical thinking and learning as inseparable from body-based action, interaction, and experience. Attending to the dialectical dynamic between embodied interaction and place-making, we consider how whole-body, collaborative designs can disrupt representational infrastructure in a way that repositions learners in relation to mathematical content. We bring this theoretical focus on place, embodiment, and representational infrastructure into dialogue with two cases of whole-body, multi-party mathematical activity. In the first case, middle-school children in the context of special school programming participate in a series of Walking Scale Number Line activities taking place in the school's gymnasium. In the second case, elementary students during their regular mathematics class participate in a ratio-and-proportion activity called Whole and Half. In both cases there are deliberate designs to disrupt more typical uses of space and bodies in relation to school mathematics learning.

In what follows we introduce the theoretical perspectives we are bringing together with respect to spatial production, embodied cognition, and representational infrastructure as they relate to learning mathematics. We then summarize our research methods, followed with our analysis of each of this study's two cases. Finally, we discuss these findings together, focusing on how whole-body, multi-party activity can influence spatial production and the relationship between learners and relevant mathematics.

SPACE, EMBODIMENT, AND REPRESENTATIONAL INFRASTRUCTURE

In this paper we take up the argument that the physical spaces of learning should not be treated as static boxes waiting to be filled with human activity, but instead as complex, historically constituted, dynamically experienced, and socially produced settings (Leander, Phillips, & Taylor, 2010). In their study about arithmetic and grocery shopping, Lave and her colleagues (Lave, Murtaugh, and de la Rocha, 1984; Lave, 1988) noted that spaces do have durable material arrangements with design histories situated in some social, economic, and political context of use. They called this the *arena*. The arena, however, is experienced differently by individuals engaged in activity. They called this experienced space the *setting*, and argued that setting and activity are dialectically constituted, in the sense that "the setting both is generated out of grocery-shopping activity and at the same time generates that activity" (Lave et al., p. 73). Ma & Munter (2014) built on this relationship to consider how individuals, interacting together in activity, socially produce spaces, positing, in parallel with the dialectical relation between individual activity and setting, an analogous relation between collective activity and socially produced space.

In examining the dialectical relationship between collective activity and socially produced space, we deliberately focus on embodied aspects of that activity, including physical action, interaction, and experience. This choice is motivated by contemporary work in embodied cognition, communication, and experience, particularly as it applies to mathematics education. In particular, in this study we take an "interactionist" (Stevens, 2012) and "nondualist" (Nemirovsky, Kelton, & Rhodehamel, 2013) view of mathematical embodiment, understanding doing and learning mathematics as the body's (varyingly overt or covert) activity in its environment. We focus on how mathematical cognition is distributed across actors, material artifacts, and dynamically unfolding bodily activity (Hutchins, 2010).

From this perspective, the dialectical relationship between activity and socially produced space is necessarily infused with corporeal action, interaction, and experience. The multiplicity of space, of coexisting trajectories and stories-so-far (Massey, 2005), is a multiplicity of embodied selves under production, together in activity. Learners' bodies traverse these spaces individually but also in relation to each other, making up part of the dynamic material landscape while simultaneously producing the activity. A motivating principle for this study, then, is that possibilities for mathematical meaning-making are generated and constrained by this dynamic co-production of bodies and space, and our aim is to examine how deliberately novel deployments of whole bodies can create new opportunities for mathematics learning.

The whole-body designs in our two cases result in disruptions to representational infrastructure, the tools that allow representations to be produced, recognized, organized, manipulated, and interpreted. Examples of representational infrastructure in typical school mathematics include algebraic notation and the Cartesian Plane. In both cases in this study, number lines and intervals are key aspects of representational infrastructure that undergo destabilization and creative transformation in the context of whole-body, multi-party activity. A defining feature of infrastructure is that, when it works, it is invisible, or transparent to users (Star & Ruhleder, 1996). However, for learners, representational infrastructure is both a tool for learning and an object of learning (Hall & Greeno, 2008), or "simultaneously transparent and opaque" (p. 58, Kaput, Noss, & Hoyles, 2001). Kaput and colleagues (e.g., Kaput, Noss, & Hoyles, 2001) have argued that much of school mathematics is built on notation developed for the use of an "intellectual elite," advocating for developing more accessible representational systems. Our case comparison follows this line of reasoning by investigating two settings where representational infrastructure has been disrupted, then at least partially reconstituted to include whole bodies in interaction.

We take representational infrastructure to be socially constructed, historically sedimented, and flexibly used in local practice (Hall, Stevens, and Torralba, 2002). Breakdowns in representational infrastructure may provide analytical leverage for making infrastructure and local practices visible. At the same time, disruptions to representational infrastructure may open up possibilities for learning (Hall & Jurow, 2015). Ma (2016) built on this work, considering how a designed disruption to representational infrastructure might support conceptual agency by allowing geometry students to develop their own tools and routines using everyday objects and their own bodies for constructing large scale geometric objects (e.g., quadrilaterals) and relations (e.g., congruency). The case comparison presented here further investigates how learners might take up disruptions to representational infrastructure that involve whole bodies in interaction, as well as the consequences for mathematics learning.

METHODS

We draw on video recordings of learners engaged in the whole-body, multi-party activities in the contexts of Walking Scale Number Lines and Whole and Half. In both cases, we employed techniques from micro-ethnography (e.g., Streeck & Mehus, 2005) and multimodal interaction analysis (Jordan & Henderson, 1995) to understand how learners participated in and made sense of these activities through detailed sequences of talk, physical action, and socio-material interaction. We began with individual case analyses, focused on describing the emerging representational infrastructures and attendant mathematical activity with respect to the study's spatial, embodied lens. Then, taking a case-comparison approach (Hall & Horn, 2012), findings were put into conversation to bring into relief relationships among bodies, settings, and developing representational infrastructure. The study will be presented in this way below, to familiarize readers with individual case analyses before discussing their comparison.

WALKING SCALE NUMBER LINES

Our first case follows a group of students into their school gymnasium for special programming designed and provided by a dance educator, Malke, and a math educator, Max. The two, in conversation with other math educators and researchers (including the first author) planned activities that would place students as points on a giant number line represented by tape stretched across the gym floor, what we eventually began to call a Walking Scale Number Line (WSNL). Five groups of students between grades 2-8 experienced the activities over the course of two days, and Malke and Max revised their design after each group. Here we focus on a group of seventh and eighth graders.

The eleven students in this group gathered on a blue number line taped across the diagonal of the gym with evenly spaced yellow hashes. They were asked to choose a "home" position by choosing a yellow hash mark and taping their name tags in front of it. Students began by moving five units to their right, then two units to their left. Malke asked them where they were in relation to where they started, then Max pointed out that one of the students, Thad, was in the "exact middle" of the line (and, by design, the middle of the gym, as indicated by the basketball line markings and a picture of the school's mascot on the floor). This led to a sequence of dilation tasks where students were asked to double, triple, quadruple, and quintuple their distance from Thad. Finally, students were asked to identify their "opposite" if they had a student opposite, or just to name it if there was no student at that spot. The whole group was then tasked with finding a strategy of getting everyone to their opposites simultaneously in some efficient and safe (i.e., no crashing bodies) manner.

The WSNL setting placed material arrangements of the gym and students' past experiences in the gym into interaction with "familiar" mathematical tools (students were all familiar with number lines on paper). The open space of the gym typically used for play, competition, and performance was transformed by tape and the designed WSNL activities. The floor of the gym, painted with lines for basketball and four square, was temporarily augmented with number lines of brightly colored tape running parallel to the long wall and one long diagonal blue line, the "paper" for students' representations and problem solving. The gym arena along with students' moving bodies took on new meanings in the context of WSNL. Students' bodies became meaningful aspects of representational infrastructure for themselves and each other, beyond individual quantities moving and operating along the number line. Quantitative relationships were understood and talked about as spatial relations between students' home positions and bodies. They tracked their walking and described their locations as "two thingies over. From where I started," or "I'm where [Thad] was."

The familiar representational infrastructure of number lines was newly materialized as large-scale walkable physical phenomena embedded in the gym floor, tacitly agreed-upon attributes (of number lines and of the gym) no longer so readily available. Left and right, negative and positive were experienced variably, depending on individuals' embodied orientations in relation to each other; even "to Thad's left" became problematic as soon as Thad turned around to face the other way. Static aspects of the space (the wall, the stage) became stand-ins for direction (left, right), taking on mathematical meanings in the service of performing and describing operations.

Students' unique and distributed perspectives from their positions along the line also became resources for reasoning. Toward the end of the workshop the group discussed strategies for moving to their opposites all together without bumping into each other. Maggie, nine units to the right of Thad, thought they could all walk along the line, and when they encountered another person they would hold hands, lean back, and swing each other around (Figure 1, top). Thad suggested that if he held onto

Morgan (two to his right) and Kian (two to his left) with either hand, he could just turn around and rotate them to their opposites (Figure 1, bottom). He then revised this to include the whole group: "Wait, we could all grab hands with each other, and then I spin around, and you spin around." Maggie and Thad solved the problem from their respective physical and mathematical perspectives in the material arrangements of the space –Maggie from nine units to the right of Thad, needing to get to nine units to the left, and Thad needing to stay put but have everyone on either side of him swap to the other.

Figure 1: Top row: Maggie (second from left) demonstrates holding Theresa's (far left) hands and swinging her around to switch places on the line. Bottom row: Thad (third from left) demonstrates holding Morgan and Kian's (on either side of him) hands and spinning around to place them in each other's home spots (their opposites).

In sum, in WSNL designers physically modified the arena (school gym) with tape to produce a large-scale version of the familiar number line. Together, students' bodies operated as quantities on the line, performing displacements, dilations, and an "opposite" routine. As students engaged in the tasks, and instructors responded to them, they developed new meanings for their bodies, the arena, and the spatial relations among them. Individual and distributed perspectives contributed to this meaning-making, and as the new representational infrastructure of the WSNL emerged, particular forms of mathematics became available. We next describe how representational infrastructure emerged in a different multi-party, whole body design.

WHOLE AND HALF

Our second case examines the incorporation of whole-body, multi-party activity into a 5th-grade mathematics classroom in 'North Lake' Intermediate school. The activities we examine here take place in the context of classroom preparation for an upcoming visit to a museum exhibition about ratio and proportion. On the day before the field trip, in anticipation of the exhibition's emphasis on physical movement and kinesthesis, 5th-grade mathematics teacher Ms. Collins assigned her students a suite of classroom tasks involving collaborative physical movement. We focus here on a pairs task called Whole and Half (W+H). To play W+H, one person creates an interval of space between two hands, or one hand and the floor. The second person must respond by placing a hand halfway between the ends of the interval. As Whole varies her hand placement, Half must keep up by moving her hand accordingly. Players can vary the game by alternating who plays Whole or Half or by experimenting with different proportions.

After introducing W+H, Ms. Collins launched the activity by directing students to "get out of your seats and start working," a directive that indexed how the activity of W+H entailed a marked reconfiguration of the routinely practiced space of the classroom. To meet the practical demands of whole-body collaboration, the students needed to de-center their mathematical activity away from its usual locus in the classroom's tightly packed rows of desks and toward atypical regions: spatial margins between the desks and the walls, a cluster of goldfish tanks used for science class, and the corridor in front of the desks, typically occupied by Ms. Collins, that houses the Smart Board.

Similar to WSNL, W+H recruited participants' bodies and body parts as meaningful components of a representational infrastructure in which (a) W's bimanual hand positioning embodied an interval-like whole, (b) H's single hand represented a half, and (c) the spatial relationship between W's and H's hands created a multi-party, body-based instantiation of a part-whole quantitative comparison. Expressing and holding constant a part-whole relationship became both a matter of intricate social and embodied coordination. Interactional breakdowns made particularly visible how participants were incorporating multiple bodies and the dynamic spatial relations among them into representational infrastructure. For example, just after Ms. Collins's directive to "get out of your seats and start working," Katie lingered at her desk making notes while her partner, Claire, skirted around her desk to the front of the room and, taking on the role of W, positioned her hands to materialize a diagonal whole in front of Katie's desk. But Katie, still writing in her notebook, left Claire hanging for about a quarter of a minute. Holding her hands still to keep the diagonal whole interval in place, Claire waited for Katie, growing increasingly impatient, re-iterating the activity's directive in physical terms ("stick your hand in between it"), and urging Katie to hurry up ("come on Katie"). This brief interactional breakdown was simultaneously a breakdown in representational infrastructure; without Katie's cooperation, Claire's whole lacked its comparative half and she could not complete the task.

Once Claire had elicited Katie's collaboration and the breakdown had been provisionally repaired, each student took a turn as W, producing sequences of bimanual intervals to which her partner, H, responded. Like many of the North Lake students, Claire and Katie discretized the activity, with W posing staccato progressions of intervals to H as a sequence of punctate tasks or challenges. Within this game-like appropriation of W+H, the students enacted progressions of representational innovations, leveraging bodily capacities and limitations as resources for authoring and revising an emergent representational infrastructure to heighten and diversify possible challenges. For instance, early in their engagement with the activity, after Claire had produced four different Wholes, Katie observed somewhat plaintively, "I'm not really having to move my hand very much." A few turns later, the students switched roles and Katie embodied a sequence of wholes that were progressively more challenging scenarios for Claire-as-Half. For example, taking advantage of Claire's finite reach, Katie positioned two interval wholes asymmetrically with respect to the median plane of her own body so as to be just out of reach for Claire (Figure 2a-b).

a. b. c. d.

Figure 2: a-b: Katie-as-Whole (right) creates a W sequence that challenges Claire's (left) reach. **c**: Claire-as-Whole (left) uses her right hand and the floor to create W, treating the floor as the other end of the interval. **d**: Katie-as-Whole (right) indicates the top of the Smart Board as one end of W, while treating the floor as the other end.

To produce these hard-to-reach wholes, Katie not only made use of hers and Claire's physical possibilities and limitations, but did so in a way that opportunistically leveraged the newly "free" space around her body. Access to this space was facilitated by the spatial disruption to representational infrastructure. Displaced from the confines of desks, newly mobile intervals could occupy and incorporate alternate corridors and materials of the classroom arena, as students transformed mundane features, such as the floor (Figure 2c) or the upper edge of the Smart Board (Figure 2d), into meaningful components of a mathematical representation.

To summarize, W+H recruited students' bodies as components of dynamically shifting interval representations of part-whole quantitative relationships. Playing Whole and Half disrupted the routine spatial practices of the classroom, as participants relocated mathematical activity to new classroom regions and flexibly incorporated eclectic material features of the classroom into representations of part-whole relations. Participants creatively leveraged new possibilities for – and constraints on – physical movement in relation to the environment in order to make innovations and elaborations on the emergent representational infrastructure.

SELECT COMPARATIVE THEMES

We now bring these two cases together by highlighting select themes that emerged from comparative analysis: (a) the dynamics of friction and augmentation in spatial disruptions to learning environments, (b) ecologies of mobility and durability in disrupted representational infrastructures, and (c) the consequences of whole-body collaboration for learners' mathematical agency. First, both WSNL and W+H involved re-purposing the arena in which they took place. Yet, while WSNL deliberately capitalized on the histories of participation associated with the gymnasium (whole-body movement and performance), W+H was taken up in salient contrast to histories of classroom practice, attendant embodied and spatial routines, and the material arrangements of the classroom that both indexed and enabled those routines. Thus, the WSNL case predominantly made visible the ways in which whole-body, collaborative design can intentionally highlight, leverage, and augment a setting for doing and learning mathematics. W+H, on the other hand, made salient how this kind of design can lead to meaningful contrasts –or induce friction– with the built environment of the mathematics classroom and associated sedimented histories of embodied spatial practice. Together, these cases illuminate how both spatial friction and augmentation may be present in educational designs that disrupt the space-activity dialectic.

Second, both cases illustrate how whole-body, collaborative designs reconfigure the environments to which they are introduced, with important consequences for the representational infrastructure of learning and doing mathematics. Because the dialectical relationship between embodied activity and setting can influence possibilities for representing mathematical concepts and processes, disruptions to the activity-space dialectic are simultaneously disruptions to representational tools and practices. Thus, in both cases, representational infrastructures were disrupted and reconfigured to incorporate whole bodies, body parts, and new regions, materials, and features of the arena. Yet, the resulting reconfigured infrastructures were comprised of remarkably different material ecologies. While in both cases students' bodies were deliberately recruited for the mathematical content of the activity, our analyses unpack how bodies played significantly different roles in the emergent representational infrastructures. In WSNL much of the infrastructure was determined by the tape on the gym floor and, as a result, had a relatively immobile and durable quality. Bodies as points along the line became the dynamic part of the infrastructure and the possibilities and constraints for making sense of numeric operations hinged on the negotiated interplay between the static, durable frame of the tape-augmented gymnasium floor and collective physical movement. In contrast, in W + H, interval boundaries were not durably congealed but, rather, were partially constituted by moving hands such that the emergent representational infrastructure was less fixed to any one particular aspect of the classroom arena. In other words, in W+H, where the bodies went determined where the mathematics was. As intervals became unfixed and re-tethered to moving bodies, performers of Whole quickly and flexibly re-oriented (e.g. turn diagonally), re-scaled (e.g. stretch or shrink), and translated (e.g. move to the right) interval boundaries. Leveraging a newly mobilized representational infrastructure, students spontaneously and opportunistically incorporated material elements of the arena into dynamically changing intervals, producing a setting in which unexpected regions of the classroom might suddenly become salient and saturated with mathematical significance. The distinct representational ecologies we find in WSNL and W+H highlight two possibilities for how multiple bodies might play a part in representational infrastructure: as mobile parts framed by a materially stable, designed space, on the one hand, or as constituting the entire representational tool, on the other.

Finally, these analyses illustrate how whole-body, multi-party activity can create different kinds of opportunities for conceptual agency, the nature and extent to which learners are positioned as genuine authors or creators of mathematical ideas. In both cases, the incorporation of learners' bodies into representational infrastructure physically positioned learners as mathematical objects and learners' physical movements as mathematically significant operations or events. Because of this, students' repertoires of bodily movement –their possibilities for, constraints on, and histories of physical action– became resources for mathematical invention. Even the subtlest of bodily movements (such as changing the orientation of a palm in W+H) could be taken up as mathematically meaningful and incorporated into the local development of representational infrastructure. And, as the infrastructure shifted and evolved, participants differentially selected from these embodied repertoires (e.g. standing up on tip-toes or linking arms with a partner and spinning around) to author and negotiate new representational forms. Thus, we suggest that both activities collapse –or at least trouble– distinctions we might make among disciplinary, conceptual, and material agency (e.g. Pickering, 1995) in these contexts.

CONCLUSION: RE-CENTERING SPACES, BODIES, AND MATHEMATICS

This study represents an attempt to foreground and interrelate spatial and embodied perspectives on mathematical thinking and learning. In particular, we pieced together a framework that views mathematical representational tools and practices as emergent from a dialectic between embodied activity and interaction, on the one hand, and the social production of space, on the other hand. Using this framework, we drew on micro-ethnographic and case-comparative techniques to investigate how whole-body, collaborative activity can create new meanings for physical movement and interaction while simultaneously transforming how learners experience, make sense of, and make use of the spaces in which these activities unfold. Broadly, this study aimed to contribute to an understanding of the changing social spaces –both in- and out-of-school– for learning and doing mathematics as well as the detailed consequences of re-instating embodied physicality for mathematical thinking, learning, and agency.

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AN EMPIRICAL STUDY INTO DIFFICULTIES FACED BY 'HINDI MEDIUM BOARD STUDENTS' IN INDIA AT UNDERGRADUATE MATHEMATICS AND ITS SOCIAL IMPLICATIONS

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The research was an attempt to bring out the difficulties which arise majorly due to the transition of language from Hindi to English in the first year of Undergraduate Mathematics in India. The transition is not only linguistic but also social, regional and psychological. The population subject to such difficulties are the Hindi Medium Board Students (HMBS). HMBS here refers to the students who take their final school exam in Hindi Medium. The objective was to find out the kinds of difficulties faced by HMBS when they pursue Mathematics (Major) degree course in University: the frequency of such difficulties and the kinds of facilities, learning environment and resources available to them and used by them. The findings are based on the surveys and interviews with teachers and students.

INTRODUCTION

India has historical significance in education. The ancient texts of Vedas have enlightened the world with its diverse knowledge sphere. Sanskrit was the core language of learning since ancient times (Ramaswamy, 1999). Indian mathematics has contributed a lot to the world of mathematics. The ancient Bākhshāli manuscripts, the great works of Indian mathematicians by Āryabhṭa, Māhāvirācāryā, Mādhava and several discoveries in Indian mathematics were succinctly coded in the form of metrical compositions in Sanskrit (Ramasubramanian, 2012).With the change in time, Sanskrit lost its charm and the number of texts produced in Sanskrit today is fairly low when compared to other languages in India. Sanskrit literature has fared no better (Pollock, 2001). Hindi, being one of the closer languages to Sanskrit is the most popular and widely used language today in India. Moreover, Hindi is the 5th most spoken language across the globe following Arabic, English, Spanish and Chinese (Summary by language size, 2016). In fact, Hindi is the most widely used language across India and it also serves as an important medium of instruction for teaching and learning in schools and colleges of India. In India, Hindi becomes the major language and, according to the census 2001 around 41% people have their mother tongue as Hindi and treat Hindi as their medium of communication (Jain, 2014). The language policy of the Ministry of Human Resource Development, Government of India is intended to encourage the citizens to use their mother tongue in certain domains through some gradual processes. The goal of the policy is also to help all languages to develop into fit vehicles of communication at their designated areas of use. The status as major, minor, or tribal languages should not matter during this development phase (MHRD, 2016). Around 22 major languages are recognized by the constitution of India under the *8th schedule* (Constitutional provisions relating to Eighth Schedule). The *National Policy on Education 1968* also placed emphasis on the use of three language formula for teaching and learning, where the third language could be the regional language apart from Hindi and English. It also placed emphasis on the use of Hindi as a link language (NPE,1968). In India, especially in northern and eastern states, Hindi is the medium of teaching and learning in schools at all levels i.e. from primary level to senior secondary level. The majority of these schools are government schools. Mathematics is also taught in Hindi up to class XII. From textbooks to medium of instruction, all mathematics is covered in Hindi. The final examination for class XII is organized by the state examination boards and a majority of the students take their mathematics paper in Hindi language.

However, for a vast majority of Indian children, the language of mathematics learnt in school is far removed from their everyday speech (NCERT, 2006). In India while entering the college and university system there is a sudden shift in the formal medium of instruction from Hindi to English, especially in the faculty of sciences. Higher academic research in the sciences and social sciences is often considered impossible to conduct in Hindi. This shift also generates an inferiority complex in those who cannot read or write English well (Deshpande, 2000).

The researcher came to realize about this language transition when he was teaching mathematics in a government school in Delhi. The medium of instruction was Hindi and the students were completely unaware of the mathematical terminologies in English. The HMBS pursuing undergraduate mathematics also discussed with the researcher their problems generated due to this transition. Most of the students studying mathematics in Hindi in government schools come from rural backgrounds with little knowledge of English. Their language skills are not good to communicate in English and are prone to miss many opportunities (B.S. Gomathi, 2014).

There is a great diversity of students in the University of Delhi where students come from all parts of India to study. Among such students, there are many who pursue mathematics at undergraduate level. There are large numbers of students enrolled in the University of Delhi who have completed their prior schooling in Hindi medium. Usually English as the medium of instruction is followed across the university especially in the faculty of science. Every student who pursues mathematics, especially as honours at undergraduate level, is a potential human resource in mathematics. However, considering the large number of students coming from Hindi medium board, the present study tries to focus on the HMBS in their first year of undergraduate mathematics.

IMPORTANCE

The study is relevant to the present scene of undergraduate studies in India. It also tries to present the current status of a socially and politically popular language, Hindi, in higher mathematics. In the Indian context, very little research is available in the area of undergraduate mathematics. Venkatraman, Sholapurka and Sarma (2012) talk about improving mathematics at tertiary level. As every student of mathematics is a potential human resource in mathematics, it is important to provide students with the best educational resources to meet his/her academic needs. Language shouldn't be an obstruction in learning and doing mathematics. Mathematics is a language in itself, but to understand this language of mathematics one has to go through another language. It is important to find out the difficulties faced by students when suddenly shifting from Hindi medium to English medium. Change of language should not create problems for HMBS to meet their full potential in learning and doing mathematics. It is also interesting to see how the social stratum created by language affects the confidence, motivation and self-esteem of HMBS, during the first year of undergraduate mathematics.

OBJECTIVES

a. To identify the areas of difficulties in mathematics in first year of undergraduate mathematics
b. To identify the availability of resources of mathematics used by HMBS
c. To explore the status of communication and interaction between teacher and HMBS
d. To suggest ways to facilitate learning for the HMBS and create support systems

RESEARCH DESIGN

The present study was conducted with first year students pursuing mathematics as honours, teachers and staff from the support department of the university. The reason why the first-year students were chosen for study was simply because first year students are the direct victim students of language transition. Since they come directly from school with experience of learning through Hindi medium, with no intervention program, they can discuss the challenges with a fresh perspective. A total of 67 HMBS were chosen through purposive sampling from various colleges of Delhi University. A detailed questionnaire consisting of 22 items was used to gather information from students. Four teachers and one official of the Directorate of the Hindi Medium Implementation[1] were also interviewed. The questionnaire for the HMBS was in Hindi language and the questions covered the following major areas:

e. Awareness and use of mathematical terms in English
f. Types of resources used and awareness related to resources by HMBS
g. Communication and interaction between teacher and HMBS
h. Difficult topics in undergraduate mathematics
i. Narratives describing the problems faced during the first year in undergraduate mathematics

RESULTS

The data collected through questionnaires and interviews was analysed qualitatively and quantitatively. The analysis highlighted multiple problems faced by students. On the basis of the survey and interview, the main highlights of results can be described as below.

In response to a question, "*Do you face difficulties in undergraduate mathematics due to change in the medium of teaching from Hindi to English*?" If yes, rate the frequency of your difficulty on a five-point rating scale from Always, Often, Sometimes, Rarely and never; responses of the students were as follows:

90.1% students replied yes, they do face the problem. This means almost everyone coming from Hindi medium background faces difficulties when he or she enters undergraduate mathematics. A majority of 50.8% students rated the frequency of such problem as 'sometimes'. Although the problem may be infrequent, it is significant that the majority of students face it.

Other important findings are discussed in different subheadings below:

1. The Directorate of the university publishes textbooks in Hindi

a. Awareness and Use of Mathematical Terms in English

77% students did not study any proper English medium textbook of mathematics during their senior secondary level (Class XII), i.e. before entering under graduation. 70.5% said that their textbooks of Class XII did comprise mathematical terminologies in English apart from Hindi also. 75.4% students face difficulties while writing or solving a problem of undergraduate mathematics in English. In undergraduate mathematics, 83.6% students were partially aware of the English terminologies used. These results reveal the lack of awareness about use of mathematical terms in English by students during their school level. Whilst there was presence of mathematical terms in English in their Class XII textbooks the majority of students did not study any textbook of mathematics in English and, as a result of this, students did not use or know how to use mathematical terms in English at Undergraduate level. The mere presence of words is not helpful and the communicative gap in transition from secondary to tertiary is also revealed. Also Bill Barton (2004) discusses that in order to learn the use of mathematical vocabulary; it is not enough to learn lists of words. The words must be learnt within particular mathematical contexts. Generally, students from non-English background have to jump directly from awareness of mathematical terms in English to completely using them without any prior exposure and training.

Table 1. Awareness and Use of Mathematical Terms in English

Awareness and Use of Mathematical Terms in English	Percentage of Students
Students didn't study any English medium textbook of mathematics during Class XII, i.e. before entering under graduation	77%
Students experienced the presence of mathematical terminologies in English apart from Hindi in mathematics textbooks of class XII	70.5%
Students faced difficulties while writing or solving a problem of undergraduate mathematics in English	75.4%
Students are partially aware of the mathematical terminologies used in English	83.6%

b. Types of resources used and awareness related to resources by HMBS

Regarding access to resources, 80.3% of students relied on the books and materials refereed by the university, while others relied on internet and books by other authors in undergraduate mathematics. The point to look upon was that the materials and books refereed by the University are all in English. Undergraduate mathematics resources, including the internet, have an abundance of English based texts, but hardly any are written in Hindi. 93.4% students were not even aware of any textbook of undergraduate mathematics in Hindi language. 91.2% students consulted dictionaries to know the meaning in Hindi of mathematical terminologies given in English. 96.7% students were not even aware of the Directorate of the Hindi Medium Implementation. The Directorate of Hindi Medium Implementation is a body of the University which facilitates students with materials and textbooks in Hindi language. There is lack of undergraduate mathematics textbooks in Hindi and even the students and teachers are unaware of it. The textbooks in disciplines of social sciences are easily available in Hindi while the textbooks of mathematics in Hindi are hard to find.

Table 2. Types of resources used and awareness related to resources by HMBS

Types of resources used and awareness related to materials by HMBS	Percentage of Students
Students relied on books referred by university	80.3%
Students were not aware of any textbook of undergraduate mathematics in Hindi	93.4%
Students consulted dictionaries to know meaning of undergraduate mathematical terms in English	91.2%
Students were not aware of the Directorate of Hindi medium Implementation	96.7%

c. Communication and Interaction between teacher and student

81.9% of students felt uncomfortable while communicating verbally in class. Only 39.3% of students talked to their teachers about the problems generated due to language transition. Teachers also said that most of the HMBS were introverted and shy in nature. Students' lack of confidence stopped them interacting with them. Such issues related to language transition was discussed or raised with teachers only when students initiated the topic themselves.

Table 3. Communication between teacher and HMBS

Communication and Interaction between teacher and HMBS	Percentage of Students
Students felt uncomfortable while communicating verbally in class	81.9%
Students talked to their teachers about these issues of language transition	39.3%

d. Difficult topics in Mathematics

The first year of undergraduate mathematics comprises of Calculus, Algebra, Analysis and Differential Equations. Students were asked about the most difficult topics among these from the point of view of language. 82% of students rated Analysis as the most difficult topic. The results reveal that topics in descending order of difficulty were Analysis, Algebra, Differential Equations and Calculus. Some students also expressed that they were unable to use language to write the statements generally used in analysis and algebra. Calculus and Differential Equations have problems often related to calculations, so students were able to understand and solve problems in these topics easily as compared to the use of expressions and terminologies in Analysis and Algebra.

e. Narratives of Students

Narratives of the students bring out the social and psychological issues generated due to transition. Usually, HMBS use the books referred by the university. The books recommended are mostly by foreign authors and the language is many times more difficult for HMBS to grasp. In order to understand in a simpler way, students also study books of local authors.

> “I feel embarrassed to ask anything related to mathematics in the classroom. When I had topped my class XII examination in mathematics, I was so happy to pursue mathematics in my future life. Now, everything has got suddenly changed. I did not expect that I would get troubled by the use of English in college level mathematics.” *(Comment by one of the HMBS)*

Such comments were also made by students who had come from one of the neighbouring states to study in Delhi. Similarly, students talked about the problems while adjusting to English vocabulary.

> “In our town, right from the primary level we have studied mathematics in Hindi. When we entered the undergraduate mathematics, terminologies appeared before us like aliens. To expect from us that we already are aware of these terms is unfair. Students from English background are able to score and indulge more in activities of mathematics more than us and we just keep managing ourselves with the vocabulary and the environment.” *(Problem shared by one of the HMBS from the neighbouring state)*

A few students also shared their experiences of leaving questions in the examination due to unfamiliarity with terminologies in English.

> "I already knew such terminologies in Hindi but because of inability to recognize the same term in English I left the questions unanswered." He further added, "However, I came to realize it later. I could have solved the question". (*Experienced shared by one of the HMBS*)

HMBS had already studied terms like 'reflexive', 'symmetric' and 'transitive' as 'स्वतुल्य [2]', 'सममिति[3]' and 'संक्रामित [4]'. They were already familiar with such terms in Hindi, but considered them new while facing them in English. They also regretted the loss of time as they came to realize the pre-known concept. A majority of students shared that they had also lost their confidence in mathematics and the classroom which they had earlier. A few even thought of leaving mathematics and opting for some other discipline. Some also talked about feeling embarrassed while presenting or answering the problems in English in front of the class. Most of the highest attaining students in mathematics were from English medium board during the first year undergraduate mathematics. Definitely English gives an advantage to score over the Hindi medium board students in undergraduate mathematics.

DISCUSSION

Hindi, being one of the widely-spoken languages across India and various societies, is not given due importance in the curriculum of undergraduate mathematics in India. The study also reveals the lack of interaction between teachers and students about this problem. There is very little demand for Hindi medium books and text materials in Hindi, but when students are asked if they needed such books, the majority of them openly talked about the need and availability of such Hindi medium books in the market. Educators have not raised this issue of transition seriously and hence the issue is left unaddressed. India is a country where the development in mathematics has historically been in Sanskrit but it has lost the significance of research work in its native languages. Efforts should be made to revive it. A major part of Indian society comprises of students from the Hindi and rural speaking belt who are quite good at mathematics. They experience low-confidence and feelings of embarrassment due to such transition. Venkatraman, Sholapurka and Sarma (2012) talk about creating a pool of students who would continue research in mathematics. Such students should not be discouraged to move out of the field of mathematics.

2. Hindi translation for Reflexive.
3. Hindi translation for Symmetric.
4. Hindi translation for Transitive.

REFLECTIONS AND SUGGESTIONS

The mother tongue is the most comfortable medium of communication and understanding. The mother tongue brings ease in understanding even the most difficult context whereas learning through any foreign language compounds the complexities in learning. This belief is more relevant in the Indian context as India possesses huge language diversity with 22 official languages and more than 300 dialects, for example different regional states have different languages. Education in India is the state responsibility with central government only in an advisory role. Most of the students get school education in the regional language of that particular state. So when a student moves to central university for higher education he/she suffers from high academic loss due to language transition leading to underachievement and sometimes withdrawal from the course. Since higher education is important to develop academic rigor among students, this kind of loss is detrimental for the student as well as for the nation.

Though the present study looked into the cases of Hindi medium students only, and that too with a limited sample, the findings point to the deep-rooted problem of ignoring learner-centred practices of teaching. Access to knowledge shall be the fundamental right of every student in a democratic country which gives official status to many of its traditional languages. When a child is allowed a school education in the mother tongue then how can he/she be denied the pleasure of learning in higher education?

India is blessed with a huge quantity of human resources which shall be transformed into a useful national resource by providing academic and skill development. It is therefore important to create a repository of learning resources not only in Hindi but also in all regional languages. Central and State universities should have frequent interactions to share learning resources. National knowledge Network shall be used to maximize the reach of resources. Popular course and research books shall be translated in Hindi as well as in all regional languages. Subject experts from different states shall be invited to write course relevant books in regional languages. There shall be a central core committee of academics who shall be responsible to balance disparity in academic resources due to language dominance. The committee shall also be responsible to maintain high standards in developing learner responsive learning materials. University teachers shall be trained in multi-lingual teaching in cases where teachers teach learners of more regional languages.

Language differences also exist within a variety of other countries. Such initiatives are important not only for India but for any nation who takes pride in its tradition and culture.

OPERATIONAL DEFINITIONS

HMBS: Hindi Medium Board Students: It refers to those students who have graduated from school by passing class XII exam (which is the final school leaving examination in India) in Hindi medium. It means that their language of writing in the examination paper is in Hindi. These students have studied mathematics in Hindi medium.

Undergraduate Mathematics: It refers to the mathematics course taught at graduation level or college level.

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WHAT IS DESIRED AND THE GOVERNING OF THE MATHEMATICS TEACHER

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In this paper a Foucault-inspired discourse analysis is deployed, with the aim of unpacking the traces of what is desired, and governmentality techniques put in operation for conducting the conduct of the mathematics teacher. The enunciation of the desired has effects of power in the fabrication of mathematics teacher' subjectivities. What is desired shape to diverse discourses and dispositive of control, where are regulated and standardized the mathematics teacher and teacher education, promoting, among other things, successful practices and models of (re)training that meet current social needs and demands.

INTRODUCTION

Discourses about social well–being, progress and development are circulated in diverse social level, in which are tracing a better future. OECD (1989) stated that the education is an important factor in the social and economic development of countries.

> Achieving greater equity in education is not only a social justice imperative, it is also a way to use resources more effectively, increase the supply of skills that fuel economic growth, and promote social cohesion. (OECD, 2016a, p. 4)

Moreover, the development of the mathematical and scientific knowledge has become relevant to achieve a successful social and economic development. OECD (2015b) asserted that a person with high mathematical competencies is more productive and able to face the challenges of the modern world, favoring the individual well–being and socio–economic progress, since, first, "an increase in cognitive skills increases the probability of a number of positive outcomes, such as completing tertiary education, finding a job and earning a good salary" (p. 34), and second, to think mathematically is a powerful mean to understand and control one's social and physical reality, through developing of certain tools and skills that help people to undertake diverse tasks and problems of everyday life, and of their contexts (OECD, 2014). Mathematics and sciences become technologies for fabrication efficient and productive citizens.

Discourses (re)produced by international agencies and research seem to be configured with the aim of promoting successful experiences, practices and performances (Gutierrez, 2013), embodying what is desired by society. The enunciation of what is desired by society is redefining social demands and needs, promoting rationalities. Thus, this paper problematizes discursive formations about what is desired and governmentality techniques put in operation for conducting the conduct of the mathematics teacher. Through a discourse analysis, inspired in Foucault's ideas, is unpacking dispositives of control and the mathematics teacher's subjectivities.

Discourses are put in operation from what is desired, configuring forces, dispositions, rationalities, and truths. The enunciations of what is desired by society govern the mathematics teacher, deploying diverse forms of control and a kind of subject.

UNPACKING WHAT IS DESIRED AND ITS DISCOURSES

International agencies and research in the field of the mathematics education are (re)producing continually discourses. The discourse is a group of statements (Foucault 1972, p. 117) –statements used with some regularity– that constitute a particular language, knowledge and regimes of power, through the configuration of valid form of enunciating and arguing. Discourses operate within governmentality techniques (Foucault, 2010) to the enunciate what is desired by society –desired ways of being, acting, thinking and producing meaning, among others–, and the to promote a rationality.

In this work, the desire is not understood as a manifestation of some lacks, rather as a continuous process of becoming, in this case, the becoming of the society and the mathematics teacher. In this fashion, it articulates forces, discourses and dispositions with the aim of configuring an image of future. What is desired is identified from circulating statements, in specific, statements that constantly are enunciated and used by international agencies and research on the mathematics teacher. Diverse forms of control are deployed for achieved what is desired by society, For example, international standardized testing, such as PISA and TIMSS, become a form of control, through competition and comparison system of reason.

> PISA, has become the world's premier yardstick for evaluating the quality, equity and efficiency of school systems [...] PISA allows governments and educators to identify effective policies that they can then adapt to their local contexts.(OECD, 2014, p. 3)

A Foucault-inspired discourse analysis (Arribas-Ayllon & Walkerdine, 2008) is deployed with the aim of contributing to an understanding of the effect of power on the making of the mathematics teacher, through the unpacking of discourses about what is desired, governmentality techniques and forms of control. The empirical materials consist of documents produced by international agencies, such as OECD and UNESCO, and the research about the mathematics teacher, in specific the research released within the last five years of three journals: Journal of Mathematics Teacher Education, ZDM and Educational Studies in Mathematics. With this analysis is seeking, on the one hand, to show the circulating discourses formulate from what is desired by society, and the role given to mathematical knowledge, its teaching and learning. On the other hand, to problematize the mathematics teacher as object of policy. And finally, to analyze how the mathematics teacher is governing and conducting.

THE MATHEMATICS TEACHER AND WHAT IS DESIRED

Socially are enunciated discourses, in which are circulating and promoting ideas about social well–being, progress, quality of lives and development. Discourses (re)produced by *the OECD Better Life Initiative* are a clear example of that, since this initiative "focuses on developing statistics that can capture aspects of life that matter to people and that, taken together, help to shape the quality of their lives" (OECD, 2013, p. 1), with the aims of measuring well–being and progress in diverse societies and promoting better policies for better lives. The formulations of these discourses have portrayed a kind of society, and the ways of being and acting of its persons.

"Today's socio-economic climate poses a number of challenges that requires individuals to manage complexity and diversity in their private, work and social lives" (OECD, 2015b, p. 42), where social and emotional skills are just as important as cognitive skills. It is recognizing that "[s]ome aspects of wellbeing (such as household income, wealth, jobs and life satisfaction) are generally better in OECD countries with the highest levels of GDP per capita" (OECD, 2015a, p. 5)

Education and social progress has been connected by OECD in diverse of is reports, in this fashion, in OECD (2015b) asserted

> During the past 30 years, important gains have been made in some indicators of social progress, especially in access to, and participation in, education [...] Education can contribute to raising motivated, engaged and responsible citizens by enhancing skills that matter. Cognitive ability such as literacy and problem-solving are crucial. (p. 27)

Also, it considers that to invert in education is one key policies "for addressing today's numerous socio-economic challenges, and for ensuring prosperous, healthy, engaged, responsible and happy citizens" (OECD, 2015b, p. 27).

> Education systems share the goal of equipping students, irrespective of their socio-economic status, with the skills necessary to achieve their full potential in social and economic (OECD, 2016a, p. 39)

Moreover, it is been repeatedly enunciated, on the one hand, the relevance of the teaching and learning of mathematics and sciences for society (see, OECD, 2010, 2014). The social and economic development have been closely related with the development of these kind of knowledge, configuring the insatiable need of improving its teaching and learning, by aiming at ensuring its higher quality, and with that achieve what is established as desired by society and producing a particular kind of person.

Lerman (2012) asserted that "[m]athematics as a field of knowledge production has a privileged position in the eyes of governments, business and parents" (p. 188), success in mathematics has a gate-keeping role. On the other hand, the relevance of teachers for society, the roles that they have in the establishment of quality education (see, Luschei & Chudgar, 2015; OECD, 2005, 2014), in a more just society, closing achievement gaps between advantaged and disadvantaged students (see, OECD, 2012), and in the social development and progress, the teachers become relevant since they are who "brings progress to society through the social administration of the child" (Popkewitz, 1998, p. 2).

From empirical material, it is possible to see an alignment of research on the mathematics teacher with discourses of international agencies, through the resonance of its discourses. It is possible to find resonances about effectiveness: effective teachers are important (OECD, 2005, 2012).

> Educators in some countries are engaged in intense debates regarding the best way to assess teacher effectiveness and the difficulties and potential risks involved in linking teachers' performance to their students' test scores. (OECD, 2016b, p. 146)

"The question of what constitutes good or effective mathematics teaching is at the heart of educational research" (Hemmi & Ryve, 2015, p. 501). Lee and Kim (2016) asserted "to believe that initial teacher training programs should include more specific investment in the effective use of classroom dialogue for learning" (p. 378). Concerning the learning of mathematics,

> "[t]he understanding of how children learn mathematical ideas effectively and how best to facilitate that learning must define the very core of mathematics education research." (Seah & Wong, 2012, p. 35)

Regarding the research and the mathematics teacher, Gellert, Hernández, and Chapman (2013) enunciate that research wants its findings to be applied to the professional development of teachers, modifying education practices, in the frame of social changes. The research becomes a form of control, with the research is seeking to have the tools, knowledge and effectives methods for guaranteeing the quality of the mathematics teacher –its effectiveness and competitiveness– with the aim of assuring the mathematical knowledge and skills of new generations. It configured a standardization of all aspects of the teacher that work as a quality parameters. The development of standards is based on a constant process of abstracting of the mathematics teacher aspects for its generalization. The mathematics teacher performances are responding to quality standards configured socially (Beswick, Callingham, & Watson, 2012). The standards are controlling the becoming mathematics teachers, through the regulation, normalization and disposition of the desired mathematics teacher.

The social demands are redefining what is considered as 'good' –right, successful and effective– practices, repertoire of techniques, knowledge and ways of being of mathematics teachers, shaping a desired features and qualities –standards– that mathematics teachers must have. The standardization of the mathematics teacher is put in operation through educational policies, efforts and initiatives; moreover, the standardization together with international comparative surveys is deploying logic of comparison and competition, where is setting a numerical language and teacher have become samples or data (Deleuze, 1992).

In navigating through the discourses that circulate about the mathematics teacher, it is possible to identify, on the one hand, that the mathematics teacher is opened and conditioned by the political (OECD, 2005), the cultural (see, Andrews & Xenofontos, 2015) or the social (see, Beswick et al., 2012). The mathematics teacher is an object of policy (OECD, 2005), planned and designed from what is desired and as means for the realization what is desired. On the other hand, that the mathematics teacher always is or has some deficit, mathematics teachers becomes a subject in debt, in debt with the system, students and themselves (Montecino & Valero, 2016). And finally, that are put in operation forms of control –a control short–term and of rapid rates of turnover, continuous and without limit (Deleuze, 1992)–, with trace the becoming of the mathematics teacher. Research on the mathematics teacher is controlling, influencing mathematics teachers, their professional development and practices (White, Jaworski, Agudelo-Valderrama, & Gooya, 2013).

DISPOSITIVES OF CONTROL, GOVERNING THE MATHEMATICS TEACHER

International agencies and research on mathematics teacher configure an expert knowledge, which puts in operation discourses, rationalities and truths. Through of discourses that (re)produce is traced what is desired by society. This expert knowledge operates as a technology for fabrication of desired subject.

What is desired by society traces demands and interests, producing a reality (Deleuze & Guattari, 1977) and promoting a kind of subjectivity, conducting the becoming of the mathematics teacher. "[D]esire is understood as a primary active force rather than as a reactive response to unfulfilled need" (Patton, 2000, p. 70), a force that configures a social production of reality –"social–production is purely and simply desiring–production itself under determinate conditions" (Deleuze & Guattari, 1977, p. 29).

The desire is a productive force. Just as Deleuze and Guattari assert that desire produces reality, Foucault argues that power is productive, "power produces; it produces reality [...] The individual and the knowledge that may be gained of him belong to this production" (Foucault, 1991, p. 194). Power is understood as force relations, which shape dispositives, discourses, subjectivities, truths and forms of control. But, "if we suppose that all social relations are power relations as well as desire-relations, then one and the same social institution may be considered either as an apparatus [dispositive] of power or as a complex circuit of desire" (Patton, 2000, p. 69). Discourses, institutions, expert knowledge and what is considered as true and valuable are configured from a particular interest and from what is desired.

What is desired is shaping to *dispositives* (Foucault, 1980) of control, a dispositive is a *multilinear ensemble* (Deleuze, 1992) of techniques, forces, dispositions and discourses put in operation for conducting the conduct of the mathematics teacher –conducting towards what is desired– and have a strategic function (Foucault, 1980). Dispositives have effects of power in mathematics teachers' subjectivities. The teacher is governed through the enunciation of what is desired, and the production of system of knowledge, discourses and rationalities.

That society desires a kind of mathematics teacher does not only mean that it desires a specific teachers, but it also desires the rationality that they will promote, the kind of student that they will be able to develop; it desires the impact in the social growth and development that teachers will have, and the conditions that they will promote. Thus, the mathematics teacher is related to many social aspects, where is configured a complex network of relations where the teacher has implications, being these complex relations of what is really desired by society.

AS A CONCLUSION

International agencies and research on mathematics teacher as an expert knowledge conducts the making of the teacher, producing knowledge, truths, discourses and forces from what is desired. Regarding the mathematics teacher, it desires a teacher capable of meeting the demands of new times, productive and competitive, and that promote a particular rationality. Dispositives of control are put in operation for configuring and to governing the mathematics teacher. The teacher is subjected to rationality and a regimen of truth, in which, what is desired by society promote practices and configure demands.

The becoming of the mathematics teacher is a product of what is desired, the desire is defining the ways in which it is valid and legitimate to think and research the mathematics teacher, in other words, it is regulating what is said regarding teachers. The mathematics teacher is becoming an agent within economic system, an agent that is controlling under logic of debt and favouring an economic and social model. As agent the teacher has to promote this model and its development through the mathematics. In other words, the mathematics teacher must shape a kind of student; students able to integrate themselves into the ways of acting and thinking of an economic and social model –that regulate the society– through mathematics.

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FROM POLICY TO PRACTICE: DISCOURSES OF MASTERY AND "ABILITY" IN ENGLAND

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Recent policy discourse in England has adopted "mastery of mathematics" as a desired aim and approach to teaching, yet this is understood in a variety of ways. One key component of the official discourse is the claim that mastery will enable ALL pupils to achieve and move through the curriculum together. This paper explores the ways in which this claim is recontextualised in the discourses of agencies involved in teacher professional development.

INTRODUCTION AND BACKGROUND

Recent changes to the curriculum and official policies in England attach strong value to the term "mastery" both as an objective for pupil learning and as an approach to pedagogy. This value is being carried through into practice by initiatives, some funded by government sources and some independent, including programmes of teacher professional development and the development of new textbooks, all of which are labelled with the term. There is, however, some lack of clarity about the nature of mastery and the pedagogy that might support it. This is evident in a number of recent publications providing critique (e.g. Wells, 2016) and pointing to common "misconceptions" (National Association of Mathematics Advisers, 2015).

Official rhetoric (emanating from government sources and echoed in the popular media) associates the introduction of mastery with learning from jurisdictions that have performed well in international tests. In particular, the government has funded a programme of teacher exchange visits with Shanghai and is supporting the development and dissemination of textbooks for primary schools based on those used in Singapore and is using ideas about the pedagogy in these countries in forming the recently revised National Curriculum and other policy initiatives. It thus appears that the UK government is attempting to make use of international test outcomes in order to inform policy making, although it is worth asking whether it is developing new policy ideas on the basis of analysis of the new knowledge produced by TIMSS and PISA or whether it is, like the policy discourses studied by Pons (2012), drawing on these studies as a rhetorical strategy to strengthen an existing positions in the policy debate.

The difficulties and dangers of adopting practices from other countries without paying due attention to cultural differences have been pointed out elsewhere (e.g. Clarke, 2002). It is also over-simplistic to draw conclusions about causal relationships between particular features of an education system and student performance. In this paper, however, my intention is not to provide another critique of UK government policy but to understand how ideas of mastery are transformed as they move between fields – between research, policy and practice.

In the context of an earlier policy reform in England, involving changed approaches to assessment, Morgan, Tsatsaroni and Lerman (2002) drew on Bernstein's (1990) notion of recontextualisation in order to understand how teachers' practice in implementing the reform was shaped by the various official and unofficial discourses about mathematics, pedagogy and assessment. Bernstein describes the movement of discourse from the field of production into the field of reproduction, the school, through a process of transformation within the recontextualising field. Mathematical and pedagogical knowledge and theory are originally formed in the field of production, the academy, but their presence in teachers' practice is mediated by curricula, assessment regimes, textbooks, policy, guidance and training –discourses formed by selecting and transforming elements of the original discourse and combining these with elements of other discourses in order to suit them for their new practical pedagogic purpose. The production of these recontextualised discourses is the work of a range of agents and agencies (governments, teacher educators, publishers, etc.) each with their own interests. The study developed by Morgan, Tsatsaroni and Lerman took teachers' discourse as its starting point and sought to trace its discursive elements back to their sources. In the present paper, the focus is on the ways in which the discourse of the "mastery" reform is constructed by various recontextualising agents and agencies. This will enable exploration of the discursive resources that may be available for teachers to draw on in order to "perform" mastery in the classroom.

As Lerman and Adler (2016) demonstrate, action in the field of recontextualisation is complex: they map the various sources drawn upon by policy-makers as they produce the official discourse of educational policy. However, the action of schools and teachers in the field of reproduction is not only directly regulated by government policy but is also shaped by the transformations of official discourse into resources and guidance for practice produced in the Pedagogic Recontextualising Field (PRF). The PRF itself comprises two sub-fields: the official (OPRF), which is directly regulated by the state, and the unofficial (UPRF), whose agents and agencies have some degree of autonomy.

As has been identified elsewhere (Wells, 2016), the various recontextualising sub-fields draw on discourses from several sources within the field of production of educational theory. The term “mastery” itself is often traced to Bloom; the concrete-pictorial-abstract pedagogy incorporated in the officially endorsed pedagogy in Singapore draws on Bruner; academic discussion of pedagogy in Shanghai and other Confucian-tradition jurisdictions makes use of the notion of variation, aligning itself to some extent with Marton’s variation theory, though developed independently and with some cultural differences (Sun, 2011). My concern is to map the ways in which elements of these discourses and of others have been selected, transformed and combined to produce messages for mathematics teachers in England to guide and regulate their practice. In this paper there is space only to consider a small subset of discursive elements as explained below – those elements concerned with “achievement for all” and notions of “ability”

METHODOLOGY

The data are the texts of public domain documents chosen to represent the official and unofficial discourses of mastery produced in each of the ORF, OPRF and UPRF. They are published by three agencies, each forming part of one of these fields:

1. the government *Department for Education* (DfE) – an agency of the Official Recontextualising Field (ORF)
   - 1a the statement of aims of the Mathematics National Curriculum for primary schools (extracted from Department for Education, 2013). While produced before the explicit policy turn to mastery, the official discourse claims that it is nevertheless a “mastery curriculum”.
   - 1b a press release reporting the announcement by the schools minister Nick Gibb of funding for schools to introduce mastery: South Asian method of teaching maths to be rolled out in schools
   - 1c the text of a speech by Nick Gibb in 2014: *Nick Gibb speaks to education publishers about quality textbooks*
2. the *National Centre for Excellence in Teaching Mathematics* (NCETM). The NCETM is directly funded by the DfE and is currently charged with coordinating professional development for teachers and dissemination of the official version of mastery. It may thus be seen to be part of the Official Pedagogic Recontextualising Field (OPRF), converting policy texts into texts that aim to shape pedagogy directly.
   - 2a The Essence of Maths Teaching for Mastery – a summary of key principles of mastery
   - 2b Meeting the needs of all without ability setting
   - 2c Using a high quality textbook to support teaching for mastery

   2b and 2c are “case studies”, each describing how a school is introducing key aspects of mastery
3. *Mathematics Mastery* (MM). MM describes itself as a professional development programme for teachers. It operates as part of Ark, an

educational charity responsible for a chain of academy schools in the UK. These schools are all expected to follow the curriculum and pedagogic approach designed by MM. Other schools, not part of the Ark academy chain, may also buy into the MM training and materials. Founded in 2009, MM predates the adoption of mastery into the official policy discourse. As will be seen in the analysis below, there are tensions, in some cases made explicit, between the discourse of MM and the discourses of the DfE and NCETM. MM is independent of the government and hence forms part of the Unofficial Pedagogic Recontextualising Field (UPRF). Because of the scale of its resources and its institutional position it has a widely recognized public presence.

3a *What is the Mathematics Mastery approach? – a summary of the key principles of MM*
3b *Mastery – facts, fictions, fashions and fads*
3c *Textbooks – A useful piece of the puzzle?*

Texts 3b and 3c are blogs posted by senior officers within MM, explicitly engaging with the official discourse of mastery.

Apart from 1a, the texts were accessed from the internet during July 2016. They were selected to enable comparison of the treatment of key themes across the fields.

These sources clearly do not encompass the full variation in current discourse about mastery. In particular, social media afford opportunities for individuals, including teachers themselves, to contribute to public communication about policy, without the support of the resources of an official or non-governmental agency. Such blogs, tweets, etc. are often oppositional, constructing and critiquing a particular version of mastery and often advocating an alternative pedagogy.[1] Such texts are also products of the UPRF and analysis would provide further insight into some of the possible forms of compliance and opposition, but this is beyond the scope of the current paper.

The approach to analysis focuses on how the texts function ideationally – what version of the world do they construe? In particular, what are the characteristics of mastery, of pupils and of teaching and what actions are involved in performing mastery in the classroom? An initial process of open coding was conducted, supported by nVivo. Clusters of codes were generated and refined, identifying utterances related to each of these questions as well as others that emerged during the coding process.

Given the limited space in this paper, I shall focus on just one cluster

1. Established examples include: http://www.inquirymaths.com/posts/inquiryandmastery and https://tothereal.wordpress.com/2015/04/29/why-this-post-is-wrong-and-dangerous-response-to-mastery-overload/ (both accessed 1/9/2016). There are also many less formal and more ephemeral examples.

of issues emerging from the data: the idea that pupils are different, achieve differently and/or should or should not be taught in different ways or even in different groups. This has been an on-going source of debate in the United Kingdom for many years. As recently as 2014, reports in the media suggested that the ruling Conservative Party was about to make is compulsory to teach all secondary school pupils in “ability groups” (Paton, 2014). Although the government distanced itself from this rumour, many secondary and primary schools nevertheless increased their use of such grouping. Since the official adoption of mastery, however, the idea that pupils with different levels of attainment should be taught separately and differently has been replaced by emphasis on achievement for all pupils. This is evident in the texts produced in all three fields:

> The maths mastery approach is marked by careful planning, ensuring no pupil’s understanding is left to chance. (ORF)

> Mastery is something that we want pupils to acquire, or rather to continue acquiring throughout their school life. All pupils. (OPRF)

> Our approach is designed to enhance understanding and enjoyment, as well as raise attainment for every child. (UPRF)

These three examples are all included within the “ability” cluster because they directly refer to achievement (understanding, mastery, enjoyment, attainment) for *all children*. Other sub-codes within this cluster referred to the identification of different groups of children classified by attainment or ability (e.g. “learners who struggled”, “the most able”) and to organization of teaching for children of different “abilities”, including: *acceleration*, *differentiation*, *keeping together*, *keeping up* and *setting*.

While statements about this topic occur in texts from all three agencies, in order to map the recontextualisation of mastery across the ORF, OPRF and UPRF it is necessary to look in more detail at the kinds of messages produced in each field. Having extracted all statements coded in the ability cluster, the next stage of analysis started at the level of the clause, identifying in each case who or what are the actors in what kind of process. This laid the groundwork for looking at semantic patterns reoccurring within each set of texts and across the texts as a whole. For this, I have adopted a version of Lemke’s thematic analysis (Lemke, 1983), simplified to consider only the transitivity system of actors, processes and circumstances. This form of analysis identifies common semantic structures through the cohesive devices present in the text. A common structure may be detected not only in the direct repetition of specific actor/process relations but also in lexical covariation, such as the presence of synonyms, and in grammatical transformations of similar relationships. (See Morgan (2016) for a detailed account of this method applied to official texts constructing “good practice in mathematics teaching”).

For example, across the texts of the ORF, the following statements exemplify a common semantic structure, ascribing agency to curriculum, teaching approaches, textbooks or other resources, all of which are said to “ensure” some form of achievement for all pupils:

> The national curriculum for mathematics aims to ensure that all pupils become fluent in the fundamentals of mathematics
>
> careful planning, ensuring no pupil’s understanding is left to chance
>
> textbooks are used [...] to ensure that all pupils [...] achieve

The semantic pattern evident in these statements is summarised in the theme:

> TEACHING/CURRICULUM/RESOURCE ENSURES ACHIEVEMENT FOR ALL

The analytic process seeks to identify such themes within the texts produced in each field and to examine relationships between the themes across the three recontextualising fields.

ANALYSIS

The number of statements in each text coded to “ability” or one or more of its sub-codes is summarised in Table 1. This gives an indication of the universality of reference to this construct but does not give a reliable indication of the degree of emphasis as the texts vary in length and in their main focus. For example, text 2b, with 17 statements coded as referring to ability, presents a case study of a school that had recently stopped grouping pupils by ability, while text 3c, with only two such statements, discusses the use of textbooks.

Table 1. Statements coded as "ability"

Field	Text	Number of statements coded to “ability”
ORF – DfE	1a	4
	1b	1
	1c	4
OPRF – NCETM	2a	5
	2b	17
	2c	5
UPRF – MM	3a	10
	3b	6
	3c	2

Semantic patterns of the ORF – DfE texts

Within the texts of the ORF the analysis identifies two main types of semantic pattern. The first type ascribes properties and actions to pupils:

PUPILS ACHIEVE	As seen in the extracts shown above, achievement may be taken to include fluency, understanding, "grasp of the fundamentals" and possibly other, unnamed, outcomes. This action is consistently qualified as for ALL pupils.
PUPILS MOVE THROUGH THE CURRICULUM	While this appears closely related to the notion of achievement, it has the additional metaphoric content of movement, which may vary in speed. This action is qualified in possibly contradictory ways. On the one hand ALL pupils "keep up". On the other hand we are told that only "the majority move at the same pace", while even those pupils who have already achieved understanding "should not accelerate". There is no explicit identification of any group of pupils who do not move at the same pace – presumably more slowly (hence not "keeping up").

The second type of pattern ascribes agency to teaching approaches or actions (though not to teachers), to official instruments (curriculum and examination regimes) or to textbooks or other teaching resources. As shown in the previous section, one such pattern is summarised in the common theme:

TEACHING/CURRICULUM/RESOURCES ENSURES ACHIEVEMENT FOR ALL

There is, however, also a complementary theme that picks out "the most able" pupils for special treatment:

TEACHING/CURRICULUM/RESOURCES CHALLENGE **SOME** PUPILS

Semantic Patterns of the OPRF – NCETM texts

In the texts produced by the OPRF, we again find the themes:

(ALL) PUPILS ACHIEVE

and

PUPILS MOVE THROUGH THE CURRICULUM (TOGETHER)

There is no suggestion in this case that some would move at a different pace but there is consistent emphasis on no pupil being left behind. As in the discourse of the ORF, agency is ascribed to teaching approaches, though not specifically to the curriculum or resources (and without the certainty of "ensuring" achievement):

TEACHING BENEFITS ALL PUPILS	Specific benefits are identified for HIGH ATTAINING PUPILS: they are themselves ascribed agency in relation to mathematics as they: DEMONSTRATE DEEPER UNDERSTANDING ENJOY LESSONS MORE than previously. The benefits for other groups of pupils are not identified separately.

Unlike in the ORF, teachers themselves are ascribed agency and this agency acts differently for different groups of pupils:

TEACHERS CHALLENGE HIGH ATTAINING PUPILS
TEACHERS SUPPORT LOW ATTAINING PUPILS

Whereas the policy discourse of the ORF picked out higher attaining pupils for special attention, it did not suggest that others might need support.

Another feature of the OPRF is the construction of contrast with the past, challenging perceived obstacles to the mastery approach.

> Maths teaching for mastery rejects the idea that a large proportion of people ‘just can’t do maths’.
> “We had traditionally taught ability sets in Years 5 and 6 for many years, believing that the gap was so vast by this point we couldn’t conceivably support and challenge children in mixed ability classes.”

This use of contrast is a feature of a persuasive rhetoric. Whereas the policy discourse has a more absolute modality, stating how things should be in a way that excludes any possible challenge, the NCETM texts recognise and reject possible objections to the changes it is tasked with implementing. Another rhetorical device employed to counter objections is the recruitment of teachers’ voices, as in the second extract above. The case studies in texts 2b and 2c both include quotations from teachers, claiming that implementing the mastery approach has overcome their own past objections and those of their colleagues.

Semantic patterns of the UPRF – MM

The discourse of the UPRF repeats the basic theme that:

PUPILS MOVE THROUGH THE CURRICULUM (TOGETHER)

It also echoes the ORF claim for the mastery teaching approach:

TEACHING ENSURES ACHIEVEMENT FOR ALL

The notion of CHALLENGE is also present, though here it differs from the discourse of the other two fields in that “students of different levels of attainment” are all to be challenged, not only those who are identified as high attaining or high ability:

TEACHING CHALLENGES **ALL** PUPILS

Indeed, these texts construct an oppositional discourse about ability, challenging the idea that pupils can be labelled by their ability:

We believe our 'abilities' are neither fixed nor innate, but can be developed through practice, support, dedication and hard work.

Whereas the ORF and OPRF both construct a model of mastery in which, while all achieve, those identified as higher attaining or higher ability achieve more (greater depth of understanding, enjoyment, meeting challenges), the UPRF explicitly extends these additional benefits to all pupils:

All learners benefit from deepening their conceptual understanding of mathematics, regardless of whether they've previously struggled or excelled.

Unlike the teachers' voices in the OPRF texts, those UPRF texts chosen for analysis here do not engage directly with the practicalities of classroom implementation. They do, however, refer to other texts, "our curriculum structure, our coaching and depth materials", that are claimed to "enable teachers to plan for those at different levels of attainment".

DISCUSSION

The analysis of texts from each of the three recontextualising fields identifies common themes, characterising "mastery" as involving achievement for all pupils, moving through the curriculum at the same pace. The notion of challenge also appears across all three.

As might be expected, the OPRF draws closely on the official policy discourse but transforms it in ways that reflect the NCETM role in transforming policy into forms that can be taken into the classroom. An important aspect of this is the ascription of agency to teachers themselves; this reflects the fact that a major role of the NCETM is to disseminate messages to teachers that will enable them to transform policy into practice. The different teacher actions construed in relation to higher and lower attaining pupils –"challenge" and "support"– draw on widespread pre-existing discourses of ability and work with the rhetoric of contrast with the past to persuade teachers that achievement for all is possible.

Interestingly, the message of challenge for high attaining pupils is especially strong in both the ORF and the OPRF. Text 2b provides a hint of the interests that may lie behind this, quoting a teacher in a primary school that had abandoned ability setting:

> "It was an unpopular move with a handful of parents at first; however, we have been careful to ensure the most confident children are always challenged and engaged in class and so any resistance was short-lived."

Vocal (and probably mainly middle class) parents not only pressurise individual schools but are also likely be seen by the government as an important constituency that needs to be persuaded that the new policy will benefit their children.

In contrast, the discourse of the section of the UPRF considered here explicitly displays its autonomy, construing Mathematics Mastery to involve challenge for all pupils and positioning itself in opposition to the notion of “ability” used in the official discourses. As noted earlier, the Mathematics Mastery organisation pre-dates the policy adoption of mastery and it appears to be struggling to defend a claim to “own” the term. Although it reaches out to all schools, MM has a power base in the Ark academy chain which imposes common policies and practices that may differ to some extent from those current in other types of school. Evaluation of the first year of an attempt to disseminate the MM approach to other secondary schools identified a perceived lack of support for lower attaining pupils as a problematic issue for teachers outside the Ark chain (Jerrim et al., 2015).

The question of how teachers may interpret and adapt these discourses of mastery in the context of their practice is yet to be investigated. In this paper I have only presented analysis of the presence of one construct, “ability”. There is a long history of use of this construct in England to differentiate educational provision for different groups between and within institutions and classrooms. It remains to be seen how the core message that ALL PUPILS ACHIEVE and the varying messages about challenge are recontextualised in the field of reproduction as teachers draw on new and existing discourses in order to form their classroom practices.

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PRIORITIZING VISUAL SPATIAL MATHEMATICAL APPROACHES IN FIRST NATION EARLY YEARS CLASSROOMS

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This paper describes the Math for Young Children project (M4YC), a professional development research project that works with teams of educators in an adapted lesson study approach to promote, design and assess visual, spatial approaches to mathematics learning in early years classrooms. Since 2011, our work has focussed on supporting communities historically underserved by the educational system. We present methods and results of our ongoing work in First Nations communities in the Treaty 3 territory of Northwestern Ontario.

INTRODUCTION

The research and professional development work we have been carrying out in Ontario, Canada in our Math for Young Children project (M4YC) investigates the potential of visual/spatial approaches to support early years mathematics learners, particularly those historically underserved by the education system. We situate our work in the growing literature on the importance of spatial reasoning and spatial kinaesthetic approaches to mathematics learning in the early years (e.g., Davis, 2015: Newcombe 2010; Sinclair & Bruce, 2015). Further, our work resonates with a proposal of Gates presented in a 2015 MES paper in which he speculates on the potential for reducing SES-related differences in early math learning by shifting the focus in mathematics teaching from its current emphasis on symbol, language and text, to a focus on the visual aspects of mathematics. Teaching approaches to school mathematics, he asserts, depend on, "language and textual communication to the near exclusion of other modes of communication –most notably the visual" (p. 517). Gates argues that a more equitable approach to mathematics instruction may be one that places greater emphasis on the visual aspects of teaching and learning mathematics.

In this paper, we discuss these issues in the context of our work in the Math for Young Children project. We begin with an overview of M4YC in its first two years; its rationale, goals and professional development approach as well as details of the visual/spatial lessons and activities that were co-designed as part of the process. The main focus of this paper, however, concerns our M4YC work in First Nation communities, and we present the methods and results of research we have been conducting to assess student learning and to learn more about the potential of this visual spatial approach. Finally, given the overall success of this project, we speculate on the implications of this work and consider how a rigorous spatial geometry curriculum can provide a culturally responsive context to support early years mathematical learning in First Nation communities.

THE MATH FOR YOUNG CHILDREN PROJECT

Since 2011, we have been collaborating with teams of educators, including early years teachers and their students, to design, implement and assess new spatial approaches to early mathematics learning, particularly in the area of geometry.

In the first few years of the M4YC project, we worked in low-resource urban school districts with schools ranking among the lowest in provincial math scores. During that time, we collaborated with more than 12 professional learning teams, each comprised of university researchers, school board mathematics coaches, Ministry personnel, and early years educators (released for seven days during the school year), and their more than 2,000 Kindergarten to Second Grade students. Our approach to this professional development involved an adapted form of Japanese Lesson Study (Lewis, Perry, Murata, 2006) with four key adaptations: a) teachers engaging with mathematics, b) teachers conducting clinical interviews with their students, c) teachers co-designing exploratory lessons, and d) teachers creating resources, lessons and activities for wider use in early years classrooms (Moss, et al., 2015).

Our over-arching goal for the M4YC project has been to re-think/re-envision what might be the most equitable, culturally responsive and effective starting point for early mathematics learners. Specifically, an essential part of our goal has been to work in collaboration with classroom teachers to shift the focus of early mathematics instruction away from the typical emphasis on number to one that is more focussed on children's intuitive mathematics, their emerging spatial knowledge, and their embodied and aesthetic experiences. In previous papers (e.g., Moss, Bruce & Bobis, 2015), we have written about our rationale and motivations for the M4YC project. Below we offer a brief review.

The starting point for our work has been the well-known literature on SES-related disparities in early math readiness. It is now well recognized that young children have strong intuitions about, and many informal understandings of, "everyday mathematics" (Ginsburg, Lee & Boyd, 2008) that are foundational to formal school mathematics. It has been reported by many (e.g. Case & Okamoto, 2005), that despite similar starting points, by the time children enter formal schooling, there are already striking SES-related disparities in readiness to engage in mathematical activity and this lack of readiness can have cascading effects on children's future mathematics learning. Recent findings on the predictive nature of early math competence for later *overall* academic success (e.g., Duncan et. al., 2007) speak even more strongly to the urgency of working towards improving the situation. Indeed, given this inequity, the U.S. National Research Council (NRC) has urged educators and policymakers to provide young children with, "extensive, high-quality early mathematics instruction that can serve as a sound foundation for later learning in mathematics and contribute to addressing long-term systemic inequities in educational outcomes" (Cross, Woods, & Schweingruber, 2009 p. 2). Despite such calls, current practices in many early years school settings (at least, in North America) are usually limited to a focus on number.

Why geometry and spatial reasoning

Our choice of geometry and spatial reasoning was based on many factors. Initially, we were looking for a curriculum-based math focus that would allow students to engage in visual/spatial ways of doing math. We were aware that geometry is an underserved area of study in early years classrooms (e.g., Sinclair & Bruce, 2015; Van den Heuvel-Panh uizen and Buijs, 2005) and, while geometry is inherently spatial, typically geometry instruction in early years is limited to sorting and labelling shapes (Clements and Sarama, 2011).

Research from cognitive sciences also contributed to the rationale for our spatial focus. Mix and Cheng (2012), among many others, have confirmed the close link between spatial reasoning and mathematics performance and have, for example, shown that the ability to visualize, to engage in perspective taking, and to rotate figures mentally, not only predicts overall mathematics abilities, but also success in other school subjects as well. Furthermore, while it has been believed that spatial reasoning is a fixed talent, there is now conclusive evidence that spatial reasoning is malleable and can be improved with practice and training in people of all ages (Uttal et al., 2013). And finally, our reason for focussing on geometry and spatial thinking relates to student motivation and interest, and is linked to what Sinclair (2001) refers to as, "the aesthetic dimension of student interest" (p. 25).

The collaborative creation of lessons, activities and teaching materials

Because there was a scarcity of teaching materials that focussed on spatial reasoning and geometry for young students, our work with our M4YC teams increasingly focussed on lesson design. Following the model of Japanese Lesson Study, all of the lessons and activities were co-created by the participants and put through a careful iterative process involving the implementation of these new lessons in many classrooms and settings, followed by critiquing and reflecting, revising, testing and re-testing. Although the content of the lessons and activities were often above typical grade level expectations, the design of the contexts, inviting guided playful pedagogy and thoughtfully designed scaffolds, made these lessons accessible and were enthusiastically taken up with our diverse K-2 students.

For example, in one lesson, very popular with students from all grade levels, entitled *The Magic Keys*, the students were charged with finding the 12 unique shapes, that can be composed with five squares and that comprise the full set of pentominoes. This is a considerable challenge that involves children grappling with concept of congruence, while also visualizing and conceptualizing the transformations of rotation and translation. In the second part of this lesson, once the 12 pentomino shapes have been discovered, the students were asked to visualize and then justify which of the 12 pentominoes, when folded, could make an open box – thus adding more visualizing, in this case from two- to three-dimensions.

Indeed, all of the lessons that were developed in the M4YC project involved significant challenges and sophisticated mathematics: *The 3D Cube Lesson*, for example, challenged students to find and construct the full set of unique shapes comprised of five cubes, requiring mental and physical transformations in three-dimensions; The *Symmetry Game* involved the students in recognizing and creating reflectional symmetry around vertical and horizontal axes; *The Tile Lesson* provided students with spatial approaches to grid structure for area measurement; and, The *Garden Patio Lesson* focuses children's attention on composing, decomposing, and transforming area. Each of these lessons, it should be emphasized, were designed to be taught in playful, inquiry-based ways combining play and specific curriculum goals (Fisher, et al, 2013).

In addition to the lessons, the teachers and researchers also designed a series of quick image activities that focussed on supporting and enhancing children's spatial thinking as well as reinforcing geometry concepts. Designed as brief (10-15 minutes) and easy-to-implement challenges, the quick challenge called for short bouts of intense visual-spatial attention from the students. These activities included drawing, building, copying, and visualization exercises. The main aim here was to develop the children's ability to engage in various features of spatial visualization, including the ability to generate, recall, maintain, and manipulate or transform visual-spatial information in mind and with the aid of manipulatives. For a full description of these lessons and quick image activities (Moss et al., 2016).

MATH FOR YOUNG CHILDREN NORTH WEST (2013 – PRESENT)

In our third year of the M4YC project, we were invited by a rural school district in Northwestern Ontario to work with teachers and students in schools serving a high percentage of Ojibwe students from neighbouring First Nation communities (Nigigoonsiminikaaning First Nation, Seine River First Nation and Naicatchewenin First Nation). The majority of students travelled by bus to attend these public schools. The school board was interested in being part of a research study that could provide a better understanding of the potential of this spatial geometry approach to early mathematics learning for their students. The school district had in place First Nation instructional leaders involved with curriculum development, as well as Educational Counsellors from each First Nation community whose role it was to act as a liaison between community and school. We welcomed these new partnerships and the opportunities to learn from First Nation communities.

Community Involvement

There were a number of ways that the research team and the various communities came together. Elders welcomed the research team to the communities with a traditional fish fry. In the words of an educational counsellor from one of the First Nation communities, "We wanted to make sure that the researchers knew the communities the children came from even before the math work was to begin." Other invitations were the "Fall Harvest" an annual gathering for local schools in which Elders and community leaders share important traditions and cultural practices enabling us to develop an awareness and understanding of Indigenous histories and perspectives. The Family Math Nights, which included a fish fry, local drummers and an Elder-led ceremonial opening, were co-planned with First Nation communities and included both culturally relevant and school-based math activities. As our time with the project continued, new connections were forged with the communities.

The PD Process and the Educators' Involvement

Our PD process in the Northwestern Ontario school district, while retaining much of our initial PD design, outlined above, involved several significant differences that reflected the particular context. However, an important difference was the composition of this new professional learning community we have come to refer to as the NW Team. As in our previous work in M4YC, we included teachers, principals and math coaches, along with the research team, but as mentioned above, also included were the First Nation early years consultant, the First Nation curriculum coordinators, the school board's Anishinaabe Language teacher and the education counsellors from the four First Nation communities in the district. This was the first time that this group of educators had been invited to a mathematics professional development process in the school district and this turned out to be a significant factor in building new connections between the school and its First Nations communities. Important features of the earlier PD model remained intact. These features included a focus on teacher-led clinical interviews (Ginsburg, 1977) and the design of exploratory lessons, both of which, as we had learned (Moss et al, 2015) encouraged teachers to listen more closely to their students, and to be better able to interpret and analyse their students mathematical reasoning –generally, to become better able to relate to their students. The PD process involved seven full day meetings with three visits by the M4YC team and two interim meetings on Skype.

Table 1: A brief overview of a record of events of the PD sessions.

Sessions	Record of Events
Visit 1: Day 1	Team engage in geometry and spatial reasoning challenges. Teachers are introduced to research on spatial reasoning and geometry, and conduct clinical interviews.
Visit 1: Day 2	Teachers and researchers co-teach selected lessons in classrooms from our full team observations. Teachers choose quick image activities to try with small groups.
Skype Day 3	Each teacher shares and presents examples of lessons field-tested in their classrooms. Team plans new lesson collaboratively, which is implemented in real-time via Skype. Team reflects and begins to design further lesson.
Visit 2: Day 4	Team works on more geometry and spatial reasoning challenges, teachers present further examples of lessons they have adapted and designed.
Visit 2: Day 5	Team introduces new lessons and new quick image activities, again co-teaching in a variety of classrooms.
Skype Day 6	Each teacher shares and presents examples of lessons they field-tested in their classrooms. Team plans new lesson collaboratively which is implemented in real-time via Skype. Team reflects on lesson and plans next steps.
Visit 3: Day 7	Team works collaboratively to document and communicate the professional learning process and to create a bank of new resources for other educators.

RESULTS: HOW DID THE M4YC PROJECT WORK IN THE NORTHWEST?

In this section, we first present qualitative results pointing to the effectiveness of the project and then present a brief overview of quantitative results of quasi-experimental studies we carried out to assess change in the children's mathematics learning over the course of the project. We then describe the way the DGC spread in the communities and the invitations the M4YC team has received to expand the work.

As researchers, we spent a substantial amount of time in each of the Kindergarten to Grade Two classrooms, either co-teaching lessons, or observing children working in small group activities and engaging in quick image activities. What became evident was that this spatial approach, the DGC, suited the children who were highly engaged and appeared to rise to the challenges the activities presented. The children proved capable of engaging in transformational geometry, in visualizing and conceptualizing congruence in three dimensions, mathematics not usually addressed in early years classrooms. Many of the teachers noted that those children whom they had viewed as "lower achieving" appeared to flourish with this new approach.

While it was widely recognized that the children enjoyed and seemed to thrive academically and emotionally with the DGC approach, there was, as well, a collective interest by the team, the school district and the communities to learn more about the effectiveness of the DGC on students' developing mathematical understandings. Each year, we tested the Kindergarten to Grade Two children on a range of measures at the beginning and the end of the school year, and we compared their progress to comparison groups of children (who in turn would become part of the experimental group in the following year and whose teachers would participate in the M4YC PD).

Our pre- and post-measures included tests of geometry and spatial thinking, numeration and arithmetic as well as KeyMath, a standardized curriculum-based math achievement test. The details of this research are beyond the scope of this paper, (see Hawes, et al., submitted), but briefly, each year the results were extremely impressive and followed a similar pattern of achievement. While we expected that the students who participated in the DGC would make good gains in both geometry and in measures of spatial reasoning, what was unexpected were the significant gains that these students made, in comparison with the control group, in areas of mathematics not emphasized in the project, including basic numeration, arithmetic, and problem solving. Furthermore, our testing also showed that the students' scores were well above the expected Canadian norms, a very strong achievement given that the schools placed at the lower end of the provincial standardized math scores (Hawes et al., under review).

Maybe more than any other indicator of success of the project was how the M4YC approach was incorporated and adapted in new ways by the participating educators in First Nation communities. First, the day cares in each of the First Nation communities incorporated M4YC activities into their programs, creating a first ever math focus in their early years day care programs. Second, in some communities, the DGC became the focus of after-school programs for school-age children. Third, Family Math

Nights have served as a model and annual event for other schools and communities in the district. (Please see Caswell et al 2013 for video link to Family Math nights.) Finally, the M4YC NW project, which began in 2013 in two schools, is now a collaborative endeavour in all eight elementary schools in the district. Maybe, most significant, however, in terms of reach, was the invitation in 2015 to become partners with a federal First Nation education authority, Seven Generations Education Institute's First Nation Student Success Program to work collaboratively with K-3 teachers and indigenous educational leaders in four First Nation federal schools.

IMPLICATIONS/SPECULATIONS

Given the success outlined above, we speculate on a number of contributing factors. In our initial urban work, we analysed how the M4YC process of PD supported teachers to gain a deeper content knowledge and broader conceptualization of geometry and spatial reasoning and, importantly, how the project empowered them as designers, even as researchers (Moss et at 2015). This has also been a feature of our work with the NW Team. As Kincheloe & Steinberg (2008) remind us about collaborative professional development and research in partnership with Indigenous communities:

> PD should produce new levels of insight amongst the participants; in particular, that PD should demand that educators at all academic levels become researchers.

We have also analysed why the DGC was appropriate for the young mathematics learners, particularly those who might otherwise have struggled with the subject. This list includes how a spatial approach: provides multiple entry points for the learners; proves very motivating; involves embodied experiences; and for many children their spatial reasoning and awareness develops in advance of their number sense.

Finally, the fundamental factor when we look across all of the work is that of establishing relationships. First, our PD model and "processes of partnering" (Bang, 2016) were structured to be collaborative and non-hierarchical and allowed for genuine relationship building between researchers, teachers, and community members. The PD model with the inclusion of clinical interviews and lesson design afforded teachers new ways of relating to and understanding their students.

Our work differs from that typically considered "culturally responsive." Our goal was not to mathematize cultural practices but rather to form a collective in which every member of the team could bring expertise to the table. A significant contribution from Jason Jones, the Anishinaabemowin (Ojibwe language) coordinator for the district, was his creation of a new word for math that encapsulated the PD process: *Gaa-maamawi-asigagindaasoyang*, meaning, "Gathering to learn and do mathematics together.” The inclusion of the Anishinaabemowin teachers in the math PD was a first for the district and offered a chance for non-Indigenous members of the team to become aware of the verb-based features of the language (Lunney Borden, 2009) and, more importantly, to show solidarity in the struggle to revitalize and reclaim Anishaabemowin that was taken away from First Nation peoples during the era of cultural genocide.

Elder Mike Kabatay of Seine River First Nation, on reviewing our PD process and viewing the research results, remarked, “You’ve reawakened something that is already in our children.” This statement reflects his understanding of a way of thinking that was repressed as part of the residential school era and that is now in the process of being “reawakened.” Elder Mike Kabatay’s statement also reflects how the focus on visual spatial mathematics created a space for children to engage with math in a way that they may not have previously experienced. Former National Chief of the Assembly of First Nations, Chief Shawn A-in-chut Atleo referred to our project in terms of reparation between community-school relationships, “This *is* reconciliation.”

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WHEN AN EDUCATIONAL IDEOLOGY TRAVELS: THE EXPERIENCE OF THE NEW MATH REFORM IN LUXEMBOURG

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Since 1958 a series of new reform programmes, known as "New Math reform" tried to fundamentally deconstruct the mathematics education of schools in the United States. This reform aimed to promote the "problem-solving" abilities in students and was a means to modernise not just the school math education but also the idea of why students should learn mathematics. Later, the reform project travelled to Europe through the support of the OECD and some other international or European organisation. This paper briefly reviews the process of the adaptation of this reform project in Luxembourg during the 1960s and 1970s. The aim is to look at how the ideological background about mathematics education, and in general education, mattered in the preceding of this school reform.

INTRODUCTION

This paper is part of a research project on the process of the adaptation of a particular school reform in Luxembourg during the 1960s and 1970s. The reform movement, which was called "the New Math reform" started in the United States with a particular ideological background and expectations, and travelled through the Western countries with the support of the Organisation for European Economic Co-operation (OEEC) and its successor, the Organisation for Economic Co-operation and Development (OECD). As a consequence of the Cold War, this reform caused a tremendous excitement, during the two decades of the 1960s and 1970s, in most Western countries. The fact that the OEEC/OECD organised these conferences shows the politico-economic aspect of this reform. The primary purpose of the research project is to reveal how the ideological background of the New Math reform was translated into the educational system of Luxembourg as a Western European country. To do so, the research studies the expectations of the school on different levels, school policy, and the relevant interactions in various contexts at the national and international levels regarding the New Math reform. This will provide an insight into the idiosyncrasy of Luxembourg's school system, and its cultural logic concerning the school policy in general and toward math education. I endeavour to give a sense of the issues about the connection between what was intended and what was practically implemented for the area of mathematics education in Luxembourg. Furthermore, I take the case of Luxembourg as an example to show how the cultural infrastructure of a context can affect the way of transferring an idea from one context to the other. Later, I will explain more about what I mean by ideology and its translation.

The paper will proceed in six steps. At the first step, the paper gives a short terminological explanation about the terms “ideology” and “translation” as they are used in this paper. It continues at the second steps by a brief literature review of the New Math reform and its ideological background in its native context, the United States. At the third place, I will introduce briefly about the methodology that I used for studying the case of Luxembourg. The fourth step will explore the case of Luxembourg. This part will review the aim and the process of modernisation of the school mathematics in Luxembourg. It starts by discussing briefly what the stakeholders in Luxembourg sought from the time they decided to reform. Then, the paper goes through a more detailed story of how the reform advanced for the secondary, middle and primary school separately. This historical overview gives an insight into the context of an educational system in Luxembourg and prepares the readers for the future discussions in the next section. In the fifth step, the paper will discuss the translation of the first ideology of the New Math reform into the Luxembourgian school system. Finally, the sixth section will conclude the findings of the paper.

THE MEANING OF IDEOLOGY IN THIS PAPER

As Eagleton (1991, p. 1) notes the term ideology has a whole range of useful meanings, not all are compatible with each other. Ideology in my work refers to a system of ideas, belief and ideals that form a basis for a theory, and shape a system or a language of reasoning to support that theory at the practical level. The focus of the paper is on this system or language of reasoning in the context of Luxembourg, and its comparison with the reasoning in the United States as the native country of the reform. I use the ‘system of reason’ similar to what Popkewitz (2009) describes as considering the rules and standards that order the practices of a curriculum. Accordingly, the ideology of the New Math reform stands for a system of ideas, beliefs, ideals that formed the basis of the reform movement that wanted to modernise the school mathematics curriculum, and gave the necessary tools to reason and justify this reform. Consequently, the translation of that ideology in a new context denotes how the initial ideas, beliefs and ideals of this reform movement adapted in the new context. Furthermore, how these ideas and ideals showed themselves in the process of the reform and the reasoning behind the reform.

THE INITIAL IDEOLOGY OF THE NEW MATH REFORM IN THE UNITED STATES

After the end of the Second World War, almost all western countries started reforming their educational system, specifically their maths and science education. The reason was rooted in their wartime experience and the new economic and security needs (Philips, 2014, pp. 32-38; OECD, 1961, p. 11). In 1952, a commission of mathematicians and mathematics educators was formed at the University of Chicago as entitled College Board's Commission on Mathematics. In 1955, this commission issued some recommendation for mathematics teachers to better prepare students for college (Loveless, 2001, p. 188). Their efforts aimed to promote a "discovery method" based on the Bourbaki's 'modern mathematics' (Phillips, 2014b, p. 53). The discovery method meant giving students "unconstructed tasks" to help them "learn to solve problem", and in this way to discover mathematical structures themselves (Barlage 1982, 24 cited in Loveless, 2001, 188). The work of this commission coincides with the launch of Sputnik satellite by the USSR in October 1957. As Phillips (2014b, p. 40) indicates Sputnik raised the profile of the American National Science Foundation (NSF) and its education initiatives. These education initiatives included some reform plans in mathematics and sciences education. In 1958, the School Mathematics Study Group (SMSG) was funded and financed by the National Science Foundation (NSF) (ibid). SMSG received the best finance from NSF to create the New Math reform, which in fact included several reforms. The SMSG's work and stands regarding mathematics was at least partially based on work by the College Board's Commission on Mathematics, and the work of this commission was considered as "reasonable starting point" (Phillips, 2014b, p. 49). The New Math movement tried to systematically replace Euclid's Elements, which was the main source of mathematics education, by the Bourbaki's 'Eléments de mathématique' (Sriraman, 2008, p. 24; Malaty, 1999, p. 231). The New Math reform was a means to modernise not just the school math education but also to modernise the idea of why students should learn mathematics (Phillips, 2014b, p. 99). By the great financial and political support from NSF, the SMSG was able to vastly and rapidly disseminate the idea of the New Math reform by producing model textbooks for publishers all over the United States (Loveless, 2001, p. 188). The New Math reform attracted politicians by the promise that the programme can promote the problem-solving ability in students, and therefore to prepare the 'modern" minds that have to employ "rational," "mathematical," and "structural" approaches (Phillips, 2014a, p. 470).

RESEARCH METHOD

My plan for achieving the aim and answering the questions is to study the documents relevant to the reform, and then historicise and contextualise the extracted information from them. I look at this reform of school mathematics within its context. This context includes the national and international educational, cultural and political discussions. I try to bring in as much as relevant information as possible. I consider the time and the social environment where that text was produced while reading these sources of information. I also look at how they link with other related sources or texts. Furthermore, a critical part of the contextualisation in my research is the authors of texts or actors of events. While analysing this information, I need to consider the social, educational, and in some cases the political background of the authors. Moreover, I examine the case of Luxembourg in the context of Europe, and as a member of the OEEC/OECD and other international organisations or unions. These ways of contextualisation will help me to understand and to interpret my sources form different national and international standpoints to examine how the adaptation of the New Math reform proceeded. Moreover, as Tröhler (2007, p. 17) notes the contextual approach helps to “avoid believing in ‘timeless truths’ to be found in different texts”.

To have a better insight, I use a three-levels model to analyse the data. David Labaree (2010) names four complex levels that a reform must undergo: the rhetoric level, the level of formal structure, teaching practice and student learning level (pp. 109-111). The primary focus of my research is on the first two levels as I adapt them according to the education culture in Luxembourg. I call the first level the level of policy makers, and the second is the level of teachers who had a more leading role in the school system of Luxembourg. My study includes the interactions between these two levels with each other, and with international organisations. Although not as a main focus of my study, I support my dissertation with a third level, which can be a combination of what Labaree names the “teaching level” and the “student level”. On this level, I am more interested in the interaction between this level and other levels.

THE PROCESS OF MODERNISING THE SCHOOL MATHEMATICS IN LUXEMBOURG

This section gives an overview of the course of the modernisation of the school mathematics in Luxembourg to make readers ready for the further discussion about the ideology and the reasoning behind the reform.

In 1958, the Ministry of National Education of Luxembourg introduced an initial school reform plan in the *Courrier de l’éducation Nationale* signed by Pierre Frieden, the Prime Minister and the Minister of National Education of the time (Frieden, 1958). This reform plan expected to make a link between all levels of schooling and provide a hegemonic school system that would also make links between education and citizens' lives. However, despite this wanted hegemony, the approach had to be different for different levels. Schreiber (2014) shows in her dissertation that despite this ambitious initial plan, the reform project was actually proceeded and voted separately in three separate levels: primary, middle and secondary levels (p. 282). Moreover, another departure was necessary based on school subjects. For the mathematics education, the National Commission of Mathematics Teachers started working on the reform of the school mathematics since 1961 (MEN1158_Item01, 1967). This Commission comprised the representatives of seven high schools in the state and about 60 mathematics teachers of secondary schools. The mission was to adapt the programmes for different classes and to introduce more appropriate textbooks and enrich traditional subjects with fundamental concepts borrowed from modern mathematics. Since 1962, almost half of the teachers attended the courses organised in Arlon by the Belgian Centre for Mathematics Pedagogy (CBPM).

SECONDARY SCHOOL

The reform for the secondary education occurred in 1968 together with a structural reform of the secondary education. In 1967, the Commission proposed introducing the different volumes of a French mathematics textbooks collection (e.g., Bréard) and adopting the corresponding French programme, from the lower classes (MEN1158_Item01, 1967). This suggestion met with some difficulties because of some differences between the structures of the school systems in the two countries. For instance, one of these reasons was the number of mathematics courses per week. Therefore, the commission was determined to postpone the introduction until the structural reform of secondary education in 1968 (ibid).

Before the structural reform of 1968, there were three tracks of education for the secondary level: classic education for boys, modern education for boys and education for 'young girls'[1]. The weekly hours for mathematics courses were different in each of these school tracks. The reform of 1968 had a capital favour of the mathematics education of the country by increasing the weekly hours of mathematics courses and stopping the differentiation between boys and girls regarding mathematics education (Klopp, 1989, p. 254; MEN1135_Item01, 1968). Also, within this reform, the modern mathematics was introduced into the secondary education by using textbooks from the Bréard collection (MEN1135_

1. Lycée des jeunes filles.

Item01, 1968), with some modification. The introduction of the modern mathematics through the Bréard textbooks created difficulties and unhappiness among teachers and students as can be seen in media, teachers' reports to the ministry or published in the teachers' journals for instance in (Dieschbourg, 1968; Klopp, 1989). Also, physics teachers complained because the new mathematics programme was not compatible with the physics programme. Particularly for the first year of the secondary school, students didn't have the necessary calculus knowledge for their physics lessons (MEN1136_Item02, 1972). Nonetheless, this programme could breathe new life into the body of the mathematics education of the country. This happened both by increasing the hours and giving equal access to mathematics to girls and boys. The New Math became an excuse for demanding more hours of mathematics from the Ministry (MEN1135_Item01, 1968; Schreiber, 2014, p. 356).

MIDDLE SCHOOL

The reform for the Middle Schools in Luxembourg was created in 1965. It was one of the alternative options after the six years of the primary school. The middle school was based on providing a general education aimed at preparing boys and girls for some jobs in the lower and middle careers in the administrative and the private sectors (Memorial_A_n°60, 1965). The curriculum makers for this level had the chance that in the process of developing the curriculum, they could take into account the results achieved in the modernization of mathematics courses. Also as the teaching language of mathematics education in the middle school was French, there was no problem with choosing a Belgian textbook. From the same time teachers were taught to use concepts of modern mathematics in the primary schools without mentioning notions and mathematical terminology. However, the complete adaptation of primary school mathematics was more complicated, as I will explain in the next part.

PRIMARY SCHOOL

For the first time, the New Math reform was introduced in 1970 in one primary school. That school was chosen as a pilot school to examine the New Math programme provided by the Centre Belge de Pédagogie de la Mathématique (CBPM). The pilot study, in Luxembourg, was led by Robert Dieschbourg, a Luxembourgian teacher (Dieschbourg, 1971). Dieschbourg was a key figure in the adaptation of the reform for the primary school due to his connection with the CBPM. During the 1970s, gradually more school joined to the program. During this time, the official textbooks of Luxembourgian primary school had no trace of modern mathematics. However, some manuscript manuals were provided to be used voluntarily by teachers in the classroom. For instance, a series of books called

Mathématique moderne, published in Belgium, were employed in the Institut Pédagogique of Walferdange (a city in Luxembourg). This series contained six volumes of books for six different levels of primary school (Simons, Mouraux, & Van Cutsem, 1977) but they were not distributed among the pupils.

The report by the OECD about Luxembourg, based on the information collected in 1959, states that the ministry selects the class textbooks on the recommendation of a committee appointed for this purpose, and the same books are used in all classes of the same category (OECD, 1961, pp. 183-184). This tradition was respected during the 1960s and 1970s (probably also after). Also, important exams, like the passage exams between different levels had to be based on the knowledge of the official textbooks and not based on what teachers might teach. This issue was also mentioned in a report about the progress and problems of the reform movement in Luxembourg (MEN1158_Item01, 1967). This is why I consider the appearance of the modern mathematics in the official textbook as the official introduction of the New Math reform in the Luxembourgian school system.

The Language was one of the issues that challenged the process of the reform for the primary school in two ways. As mentioned before, from the beginning, Luxembourg started the attempt for the reform through collaboration with French and Belgian mathematics communities (MEN1158_Item01, 1967). However, the teaching language of the primary education in Luxembourg is German, so the use of French or Belgian textbooks was not possible. The other language-related obstacle in the way of the New Math reform was the hours of language courses in the curriculum, which thus restricted the hours of mathematics education. While pupils in schools in Belgium or France had mathematics for five hours per week, in Luxembourg it was only three hours in 1967 (ibid). Therefore, it was not possible to adapt the exact programme of those countries. Also, it is not difficult to conclude that with only three hours of maths every week, it is not possible to go much further than simple calculation skills. As the official portal of the Luxembourgian government mentions on its web page, the trilingualism is an important part of the national identity in Luxembourg (Portal_GDL). Therefore, the solution for the improvement of mathematics education could not restrict the language education or the use of any languages.

Another issue that was mentioned as an obstacle in the way of adapting the reform was the lack of “Luxembourgian” textbooks, which made even the realisation of the pilot classes challenging (MEN1158_Item01, 1967). However, I believe that the meaning of “Luxembourgian textbooks” or “manuels Luxembourgeois” was not only related to the language. In this era, there was an inclination for producing the textbooks for primary education in Luxembourg by Luxembourgian authors with Luxembourgian examples and flavour. Before the 1960s, the textbooks for primary education were all German textbooks. This caused unhappiness among some people, and they were aiming to produce the textbooks and make a school of a “brand” of Luxembourg (Frieden, 1945, p. 35). The crucial importance of textbooks in Luxembourg is a vast matter and is the subject of a future paper that I am working on.

IDEOLOGY VS. IDEOLOGY

So far, the paper tried to make an outline of the process of the modernisation of school in Luxembourg. Based on the information given in the previous sections, this part discusses how the original ideology of the New Math reform was translated in Luxembourg. The initial ideology of introducing the modern mathematics in general and especially in the primary schools was the belief that students who learn in this way “would learn how to acquire reliable knowledge” (Phillips, 2014b, p. 19). The national goal and promise of the reform were to equip the future citizen with enough “intelligence” in the scientific battlefield against the USSR.

Luxembourgian stakeholder, however, were pursuing a different goal. In the context of the school system in Luxembourg, with its unique language policy, the initial ideology of the reform was almost faded. Although some Luxembourgian teachers also repeated the idea of the relation between the New Math reform and the improvement of brain functionality, this research did not find any evidence that this argument had any impact on the process of the reform. In fact, the main teacher who was fighting with this reasoning complained about his letter being ignored by the Ministry of Education (Dieschbourg, 1975). In the arguments of the supporters of the reform, the main reason for modernising mathematics education was to bring Luxembourgian schools up to the level of other European countries (d’Letzebuerger-Land, 1959). This reason worked well for the advancement of the reform for the secondary school. However, it was not a sufficient incentive for the primary level.

Luxembourgian students who planned to pursue their education in a university had to go to another country, as there was no university in the country. Therefore, it was crucial to provide them with the same level of education that the students of neighbouring countries received. This ideal for the modernisation was not as functional for the primary school as it was at the secondary school. Pupils were not supposed to go to a neighbouring country for continuing their education after the primary school. Not all pupils were expected to go to the secondary school after finishing the primary level. Even though the length of compulsory schooling was nine years, there were other options for the remaining three years, such as middle school or technical school. Therefore, besides the challenges like the language education, lack of Luxembourgian textbooks, and the structure of Luxembourgian school system, the arguments based on the neighbour-level-modernisation ideal were not forceful enough to overcome those challenges. When it came to the primary school, this ideal had to compromise with other ideals that were already running the school system and still had power. This was the reason that the modernisation process of the primary school mathematics lasted for such a long time.

Stakeholders of the educational system in Luxembourg were after a plan that serves the best, all pupils of the country, regardless of which job they will choose in the future (Michels, 1964, p. 75). Furthermore, Lenz, Rohstock, and Schreiber (2013) also show that Luxembourgian stakeholders intended to promote maths and sciences in their country but not in an "American way", and the New Math reform was identified as an American product. The ethical and moral education had a great importance in this country. Lenz et al. (2013) even argue the 'conservative and more cautious educators in Luxembourg believed the Sputnik debate would endanger their predominantly humanistic educational ideal'. The goals and promised outcomes of the New Math reform, as mentioned before could not actively motivate the educational system in Luxembourg. The reason rooted in the definition of the ideal future citizen, for at least, "conservatives" educators who were actually dominant in that era. This definition was not totally following the definition of those who started the reform in the United States and defined its goals.

LAST WORDS

The paper outlined the process of the adaptation of the New Math reform in Luxembourg and compared its discourse with the discourse around this reform in the United States. It explored how the expectations from the education and the ideal future citizen made new challenges for the process of the reform in this new context. For the sake of shortening this paper, I excluded the role of the international institutions in promoting this reform in Europe, for instance, by organising relevant conferences. Three of these conferences were held in Luxembourg (Conference1, 1965; Conference2, 1969; Conference3, 1973) and Luxembourgian delegates participated in other ones in other countries. Therefore, Luxembourg as a European country and a member of OECD played its role in the promotion of the reform at the international level, but at the national level, the country saw other issues on its agenda.

The story of the adaptation of the New Math reform in Luxembourg can be considered as an example of importing an educational ideology. The New Math reform indeed was one of the biggest ideological school reforms in the history of modern schooling. The reform accentuated the role of the mathematics in the life and the definition of the ideal future citizen of the Western World. However, these ambitious goal and promises did not radiate the same meaning in different contexts. A lessen that may be learned is that in the translation of an educational reform or model in a new context, it is necessary to see how its ideals promises can resonate with the cultural structure of the host context.

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USING CME TO EMPOWER PROSPECTIVE TEACHERS (AND STUDENTS) EMERGE AS MATHEMATICAL MODELLERS

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A critical mathematics education (CME) perspective (Frankenstein 1995) applied to mathematical modelling (MM) can support teachers and students in developing broader meanings for mathematics, MM, and their roles in the society. We integrate CME with a socio-critical approach to modelling, encouraging prospective teachers (PSTs) to think critically about mathematics and realize the potential it holds for modelling and interpreting the world. This work is part of a study aimed at identifying meaningful ways to promote connections between CME and MM in preservice teacher education. PSTs, in negotiating connections between community, critical, and classical knowledge (Gutstein, 2007), delineated modelling contexts embedded in real-life scenarios and developed instructional activities suitable for grades K-8. As a result PSTs learned to appreciate individuals within their own communities as valid practitioners of mathematics.

MATHEMATICAL MODELLING: A SOCIO-CRITICAL APPROACH

While many people understand and emphasize the realistic-applied modelling approach, Kaiser and Sriraman posit that the socio-critical modelling approach provides an "emancipatory perspective" [that] leads to a critical understanding of the surrounding world" (2006, p. 304). As individuals strive to make sense of the world around them, the information and data that they accept as personal knowledge becomes paramount for the way in which they interact and understand their world. Mathematical models often become the vehicle that represents and define numeric information, leading to assumptions and interpretations that become authoritative and unchallenged. Barbosa (2006) states that "mathematical models are not neutral descriptions about an independent reality, but that the modelling process has devices that are usually concealed to the general public" (p. 294).

Mathematical modelling has often been associated with advanced, higher-level mathematics. Whereas, the National Council of Teachers of Mathematics (NCTM, 2000) emphasize modelling as an application process in which K-12 students can engage with varying levels of sophistication. Students need to understand that models can be developed, revised, and reapplied within a given context. Reporting of mathematical conclusions then need to be interpreted within the context in which the question was initiated allowing the cause and effect of the modeled problem to be fully realized.

In this paper, we describe a study that inquires how we can teach mathematics that helps learners attain the content proficiencies, but beyond that, equips them with the empathy, consideration, and skills necessary to appreciate all individuals that comprise their community and to see these people as capable of learning and doing mathematics.

THEORETICAL CONSIDERATIONS

The modelling process can be delineated in two ways –the first is to focus on modelling as content where the focus is on the development of math competencies necessary to model a phenomenon; and secondly, modelling as a vehicle, where the focus is on the pedagogy of modelling content (Julie, 2002). Socio-critical modelling, the basis for this proceeding, empowers learners to use their communal knowledge to frame understandings that are central to their lived experiences, classical knowledge which promotes mathematical competencies, and critical knowledge that provides a more comprehensive understanding of the socio-political context in which the problem was posed (Gutstein, 2007).

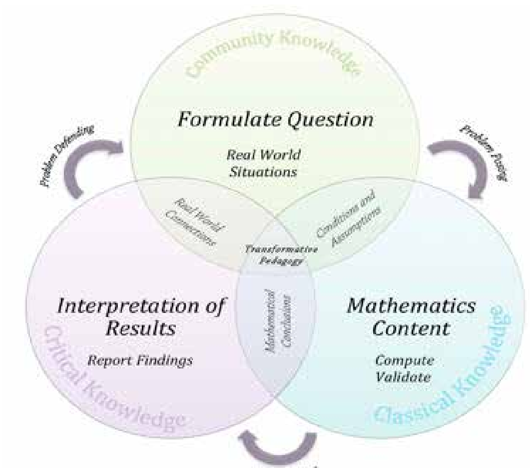

Figure 1: A Socio-critical approach to modelling

Figure 1 depicts the fundamental ideas embedded within socio-critical modelling. Barbosa (2006) posits that "critiquing modelling is part of the learning that takes place in the process of doing modelling, and one of the aims is to produce critical, politically engaged citizens" (p. 296). Researchers have paralleled mathematical modelling to the problem solving process (Cilitas, 2013) and in turn we connected it to the socio-critical perspective. The problem solving process requires that individuals; formulate question (Community Knowledge), collect data, analyze data (Classical Knowledge), interpret data (Critical Knowledge). Mathematical modelling applications help to expand problem solving performance, equipping students with the skills and knowledge to address and discern problems that they encounter in their daily life. (Niss, 1989).

MM IN TEACHER EDUCATION

MM has been advocated as being essential to enhancing K-12 students' mathematical understandings and for promoting good mathematical practices. Nevertheless, research indicates (Bautista, Wilkerson-Jerde, Tobin, and Brizuela, 2014) that many teachers view mathematical modelling as a topic reserved for students in secondary grades and hence, believe that it is a difficult topic for students to comprehend and an equally difficult topic for teachers to teach. Thus, it is imperative to attend to teachers' knowledge and beliefs about MM and it is necessary to begin such processes at the teacher education level. The work described in this proceeding is part of a broader research project aimed at identifying more meaningful ways to promote connections between MM and CME in preservice teacher (PST) education contexts. The key research goal is to support prospective elementary and middle school teachers (grades K-8) in realizing the value of MM in developing their students' interests in, and proficiency with, mathematics. In order to accomplish this goal, it was necessary to help the PSTs develop positive self-efficacy beliefs concerning their ability to apply MM and to teach others to do so.

PRACTICAL ENACTMENTS

Study Setting

The settings for the project were two mathematics teacher education courses situated in two different universities. One course was a senior level course for prospective middle school teachers; the second course was a master's level elementary education course and individuals in this course had not yet held a teaching position. In both courses, the key course goals were to enable PSTs engage in a deeper analysis of problems and concepts to gain new insights about mathematics and mathematical modelling. As part of course work, PSTs participated in a math modelling session focused on investigating real-life situations embedded in social/cultural/historical/political context. During these sessions, participants examined mathematical topics through activities that typically lie outside the focus of the traditional school mathematics curriculum. In addition, a math modelling research project was assigned to foster PSTs' mathematical modelling competencies. The project required PSTs to make connections to, or negotiate between, mathematical models of real-world situations and their own experiences or first-hand knowledge of these situations.

Math Modelling Project Overview

- *Introduction*. Describe an everyday activity; Identify co-collaborators and document your interactions with these professionals. Include pictures/diagrams/other artifacts as applicable to your work.
- *Math Exploration*. Develop a mathematical activity/task - Remember to situate your mathematical task in the chosen context. Keep your task open-ended so that there is ample scope to apply the modeling framework.
- *Math Modelling Framework*. Make sure to pay attention to the steps involved in math modeling such as: Making sense of a situation; determining given and needed information; making assumptions; problem posing; computing a solution; interpreting findings in context; validating findings; revising and repeating the process; reporting the solution.
- *Reflection*. Share your struggles and insights. Discuss opportunities and challenges both from a learner's perspective and a teacher's perspective.

A list of course project activities and several examples of PST developed MM activities are listed here: http://bit.ly/1bMbNwq. In this paper, the central focus is on PSTs' MM projects. We illustrate two examples to demonstrate how PSTs engaged in using mathematics in modelling real-world scenarios and how they used it as a vehicle to deepen their students' mathematical knowledge.

A CME APPROACH TO MATHEMATICAL MODELLING – TWO EXAMPLES

Social Context - Truck driving

PST Alan developed his modelling activity in collaboration with his uncle, Joe, a part-time truck driver. Joe, took pride in the fact that he has, in the past, delivered iPhones to different warehouses in different cities, on time, for the big launch. Alan chose to focus on this aspect of Joe's work-specific activity for his modelling task. Here is the task developed by Alan.

Mr. Joe is a part-time truck driver. As part of this job requirement, he has to deliver shipments to different warehouses across the country. He starts from [Choose your own location]. He starts from this city and delivers shipment to three other cities in the United States [outside of your chosen state]. What is the shortest route that he can take? How long will it take him to complete this trip? The release date for the IPhone 6s is January 1st at midnight (12:00 AM). When should Mr. Joe leave to make sure all of the stores got the IPhone 6s before the grand release?

<u>Student Work</u>

Three sixth grade students (I, J, K) participated in this task with Alan. Figure 2 illustrates their first attempt at modelling the situation.

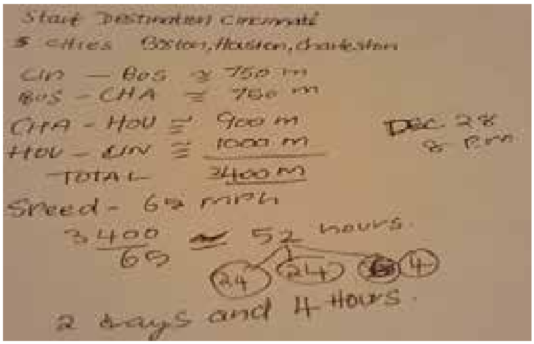

Figure 2: Modelling a route - Trial One

The shortest distance identified by the students spanned 3400 miles; at an average driving speed of 65 mph, a driver can cover this distance in about 52 hours. Going back in time (52 hours) from December 31, midnight, students reported that Mr. Joe should start his trip on December 28 at 8 p.m. Upon review of this work, Alan realized that the students have reduced this model-eliciting task to a mere computational problem. Determined to engage the students in a purposeful discussion on modelling, which would take into account the contextual parameters, Alan made an explicit reference to Mr. Joe and posed the following questions: How would Mr. Joe respond to this answer? What questions might he have? How would you convince him that this is indeed the best possible answer? What evidence should you present him? Then, he asked the students to develop an outline and document their problem-solving process so that they could present this evidence to Mr. Joe.

First, students outlined key steps that were instrumental in the model development. In parentheses they included an answer or a strategy that will lead to an answer.

- Choose four destinations (Cincinnati, Boston, Charleston and Houston as the four cities, with Cincinnati as the base destination)
- Find distance between two cities (make a table). Students used a ruler to measure the distance between two cities and rounded to the nearest quarter of an inch. Using the map scale (1 inch = 300 miles), they found the actual distance between the two cities. Since they did not know if this scale is accurate, they also used google maps to check the answer.

The second to last column in the table shows the 'google map distance' between the two cities. The last column gives a proportion of the two distances. Students were glad to note that this was constant (1.2) with minor fluctuations which they attributed to the rounding error that occurred while measuring the map inches.

Start	End	Distance		Google map (miles, rounded to the nearest 10)	Google map miles ÷ map miles
		Map inches (rounded to the nearest quarter of an	Actual miles (Map inches * 300)		
Cincinnati	Boston	2.5	750	880	1.2
Boston	Charleston	2.5	750	970	1.3
Charleston	Houston	3	900	1100	1.1
Cincinnati	Houston	2.75	825	1000	1.2
Houston	Boston	5	1500	1800	1.2
Charleston	Cincinnati	1.5	450	620	1.4

Figure 3: Modelling a route - Calculating distances.

- Find possible routes (use map). Once they created the table, the next step was to identify possible routes on the map (see Figure 4).

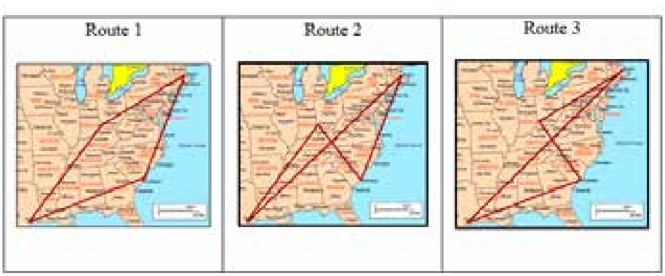

Figure 4. Modelling a route - Determining routes.

A careful analyses of data depicted in Figure 3, led them to identify several possible routes and calculate the distance for each route (see Figure 5).

Start	City 1	City 2	City 3	End	Total distance (miles)	
					Map	Google maps
Cincinnati	Boston	Charleston	Houston	Cincinnati	3400	3950
Cincinnati	Houston	Charleston	Boston	Cincinnati	3225	3950
Cincinnati	Charleston	Boston	Houston	Cincinnati	3505	4390
Cincinnati	Houston	Boston	Charleston	Cincinnati	3525	4390
Cincinnati	Boston	Houston	Charleston	Cincinnati	3600	4400
Cincinnati	Charleston	Houston	Boston	Cincinnati	3690	4400

Figure 5: Modelling a route - Determining the shortest route.

- Find the shortest route (use map and table). Using the table, students determined the shortest route (circled in table) and that it spans 3950 miles. At this time, Student J commented, "*We should exclude the distance to travel from the third city to the base destination Cincinnati, since our goal is to suggest a suitable start time for Mr.Joe so that he can deliver the iPhones by the deadline; the problem does not say that he has to be back home by that deadline*". Thus the total distance was re-calculated to be 3950 - 880 = 3070 miles.
- Find the duration of the trip (use rate formula). To determine the duration of the trip, students needed to know Mr. Joe's driving speed. Since they could not ask Joe rightaway, they looked up the speed limit guidelines on the website http://www.motorists.org/speed-limits/state-chart and found out that, in most states, the speed limit (for trucks) ranged between 55 to 65 mph. Assuming 60 mph as Mr. Joe's average driving speed, they calculated the total time needed as 51 hours (distance / rate = 3070/60 = 51)

With this information in hand, the students set out to answer the last part of the question: When should Mr. Joe start so that he can deliver the merchandise on time? Their response was crafter collaboratively through this discussion.

Student J: If the stores close at say 5:00 p.m. on New Year's Eve, then we have to deliver the phones before that time. Let us aim for 4:00 p.m. on Jan 31. Minus 24 hours, 30th 4:00 p.m.; minus 24 hours, 29th 4:00 p.m. Minus 2 hours leaves us at 1:00 p.m. on December 29th.

Student K: But wait, he cannot drive non-stop. He has to take some breaks for eating and going to the bathroom, and sleeping.

Student J: He may also stop to fill gas.

Student I: There may be accidents on the way too. So we have to factor in time for such things. Let us say, bathroom breaks take 3 hours, eating and filling gas takes 6 hours, and two nights of sleep is 16 hours. So the total is 25 extra hours.

Student K: let us add 5 more hours for other things, like delays and accidents. So the total is 30 hours to add to 51 hours, which is 81 hours.

Student J: Then we go back 1 day (24 hours) to 1 p.m. on December 28th and go back 6 more hours that day. 12, 11, 10, 9, 8, 7. So he should start at 7:00 a.m.

Student K: No it will be 8 a.m. since we are going back from 1:00 p.m, and we have to stop at 8.

The students reported this answer to Alan and were positive that Mr.Joe would approve this line of inquiry.

Social Context – Dance

PST Corrine developed her modeling activity in collaboration with a dance instructor and a chemical engineer. The instructor, Lydia has danced since age four and is now a dance teacher at a studio. She helped Corrine identify basic math concepts behind dance, explaining specifically about spatial reasoning, grouping by rows, how spotting works and helps you spin faster, and the angles of the arms and legs for certain movements. In any dance, transformations are the epitome of choreography. Dancers are constantly moving across the stage in a given direction, using the geometric transformation of translation. They are also rotating when they complete turns or flips in a routine. In group routines, when dancers reflect one another's' movements, they are using reflection and also creating symmetry. Incorporating transformations seamlessly into choreography is what creates dance, especially routines that are graceful and non-stagnant. Colin, the chemical engineer helped to identify what equations might be used to balance the two bodies out, which could include algebra and physics equations dealing with equilibrium and torque. In addition, one could use equations to figure the acceleration and velocity necessary to complete lifts that require a running start, or for turns. For this project, unfortunately, because of timing of the course there is no student work available, only the journey Corrine took to better understand the context of dance, the embedded mathematical concepts, and to see the mathematics in a "non-mathematical" context.

Figure 6. Symmetry and Transformations in dancing positions.

Here is the math modelling task that was posed.

The local dance studio has 7 dancers in its elite company. The choreographer needs to choreograph a dance to a portion of Megan Trainor's "Better When I'm Dancing" (first 44 seconds) for their spring recital.

A. Choreographers must think about space, timing, grouping of dancers, and movement.
B. Movements may follow certain patterns
C. Apply symmetry, rotations, and slides/direction changes, angles/lines of arms/legs into your choreography (don't let people move around like stiff poles!)
D. Dancers must think about the same things as a choreographer to produce the movements with grace

For this project you need to choreograph your dance! Make sure you use all the components (a-d above). You need to write out to explain how the math fits into your dance. How did you arrive at the dance you did? Did you start with the math, or start with the dance and then apply the math to what you came up with? Did these math concepts help you dance better? Why? What aspects of math did you have to use in dancing that you didn't think you'd ever use? With rotations during spins, but also when it comes to jumps (catches and releases) and flips, physics plays a huge role in the execution of the moves. When creating your routine, were you able to see these concepts in action?

DISCUSSION AND CONCLUSION

In this paper, we presented part of the data from the study in the form of PSTs' work samples, their discussions, and their reflections to argue for promoting both MM and CME and for explicitly connecting the two). The examples presented here illustrate that interactions of community, classical knowledge, and critical knowledge are instrumental in the development of an authentic modeling task, and in fostering a fruitful modeling experience for students. We use PSTs' work samples, their discussions, and reflections to argue for promoting and explicitly connecting MM and CME.

In the examples presented, we notice how students engaged in the steps of the modelling framework such as: making sense of a situation; determining given and needed information, making assumptions; developing mathematical models and using them to find a solution, interpreting the results in the context and validating their findings. Furthermore, we noticed how important it is to support students through their engagement in a modelling task. Giving too little information (or too much information) may result in a procedural or a simplistic answer that completely ignores the contextual parameters. Asking the right questions, maintaining a sustained focus on the context, allowing students to gather additional information as they see fit, and being willing to accept reasonable a line of thought are all keys a key to a successful implementation of a modelling task.

There were several tangible results for PSTs:

1. PSTs develop understanding that while there are few modelling tasks for middle school students (pre-existing) and it can be challenging to develop such tasks, they have learned that they are up to this challenge and that knowledge is empowering.
2. (Critical pedagogy) Their perceptions of who can and who cannot do mathematics changes. This ability to do mathematics assigns power to a person. These people become funds of knowledge. It is transformative because they learn that people they viewed as "plain folks" are capable of, and use mathematics.
3. PSTs view of mathematical modelling is transformed. They move from thinking of MM as only something that secondary students can do to recognizing that elementary and middle school students can also engage in this activity. They are no longer afraid to engage in MM (content / vehicle) perspective.
4. They contributed to a repertoire of knowledge by developing such MM tasks and thwarted the myth that meaningful math cannot co-exist when we attempt to teach according the tenets of a CME.

For PSTs, engagement in this modelling project gave them the much-needed confidence in their competency to develop MM activities and thereby enhance their content and pedagogy. On another level, this process empowers Joe, Lydia and Colin as they too are seen and valued as as key ‘sources of knowledge’. We end this paper with an excerpt from Alan’s Reflection.

> I was surprised to see how engaged the students were during the task. While the students knew the procedures, they were stumped when asked to interpret their answer in the given context. However, with a little guidance, they were able to do it. They were thrilled to know that I developed this task in collaboration with Mr. Joe. They said that they wouldn’t have thought of truck drivers as math teachers but have since changed their opinion. I too, have begun to question my own perceptions on mathematics, and my prejudiced view of who can and cannot have ownership of mathematics.

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FABRICATION OF NEWLY ARRIVED STUDENTS AS MATHEMATICAL LEARNERS IN SWEDISH POLICY

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In this paper we analyse Swedish policy documents by exploring discourses fabricating "newly arrived students" in relation to their learning of mathematics. As a response to the recent new laws on newly arrived students and the declining results in PISA, new policy regarding their schooling has been issued. From our analysis three themes emerged: the importance of a) mother tongue for learning, b) acknowledging newly arrived students' backgrounds as resources for learning, and c) deficiencies around immigrant students. This fabrication of newly arrived students seems to disrupt the plan for improving the Swedish PISA results in mathematics.

INTRODUCTION

In times of the refugee crisis in the Middle East, Sweden has received the highest proportion of refugees in EU in relation to its population (Migrationsverket [Migration Agency], 2016). Our focus in this paper is on the children and youth who have come recently to Sweden, the so-called "newly arrived students". The concerns around newly arrived students' schooling and their possibilities for learning are taken very seriously in Sweden. Moreover, Sweden's declining performance in PISA has led to investment in many of the measures that the OECD points to. One example is that the Government assigned Skolverket [the National Agency for Education] to implement targeted interventions aimed for school principals and municipalities that are deemed most in need of support and professional development in the reception and further schooling of the new comers. A huge amount of money is invested in the interventions (Utbildningsdepartementet [Ministry of Education], 2016), and a large number of new policy documents have been launched. In one way or another policy documents intend to steer curriculum and classroom practices, but they may also fabricate (Popkewitz, 2013) for example newly arrived students in various ways.

In this paper, we want to think about "newly arrived students" in relation to how the categorization of students and the policies about them contribute to fabricate these children and youth as particular kinds of mathematical learners. We analyse four recently produced policy texts by exploring discourses around newly arrived students. The guiding research question is: How are newly arrived students fabricated in policy texts on education of newly arrived in Sweden?

ANALYTICAL APPROACH

There is little research on newly arrived students' initial time in school in Sweden. There are no studies comparing Sweden with other countries. Research is in agreement about the positive effects of mother tongue support for newly arrived students, but most studies carried out have been focusing on newly arrived students' second language development (e.g. Bunar, 2010; McBrien, 2005), on social and individual perspectives, including identity formation among adolescents, power relations in society, and how students' integration is affected by these factors (e.g. Allen, 2006). In addition, some studies have adopted an institutional perspective focusing on transitions and measurements of the importance of migration age for students' achievement (e.g. Christie & Sidhu, 2006).

These studies show, however, that the categorization of students play an important role in their education. Categorization also involves how the educational system distributes opportunities for students to learn mathematics in school among various groups of students. Students categorized as immigrants, newly arrived or socioeconomically disadvantaged are often described as low achievers (Skolverket, 2012). We are aware that pointing out a particular group of students is problematic. Despite a good intention, categorizing students opens a risk that a particular group will be regarded as demanding and problematic in school since categorization can be counterproductive and reinforce stereotypes. On the other hand, it can provide understandings beyond deficit models regarding newly arrived students with various language and socioeconomic background. The categorization of students is a way to describe them, at the same time as it is telling stories "about the people in a group category" (Norén & Björklund Boistrup, 2013, p. 1). In Sweden, with the official categorization of students follows government money to support students with special needs. As mentioned, policy documents intend to steer curriculum and classroom practices, but they also fabricate and categorize students with a desire of "salvation" (Popkewitz, 2013); and in this case the salvation of newly arrived students is on the agenda.

According to Popkewitz (2012) the principles of curriculum have little in common with what is going on in a classroom. As an example, statements about the wish for learning more 'content' knowledge also embodies ideas about "how to see, think and act" (p. 177) on people. Popkewitz (2013) writes: "Theories and programs about particular kinds of people, for example, work their ways into schooling as children become those 'things' –adolescents, youth, urban, at-risk and disadvantaged!" (p. 440). Furthermore, PISA results link students' knowledge to teachers' professional knowledge and lifelong learning. Thus, quality indicators of PISA are not only measurements of performance, but also statements about "particular kinds of people" that are desired effects of education (Popkewitz, 2012, p. 177). The focus on PISA performance and desired outcomes that guide curriculum design aiming at producing such outcomes is placing newly arrived students in a particularly vulnerable position. It has been acknowledged in reports through the years that immigrant students are a heterogeneous group, but that group has generally more difficulties to achieve the learning goals (Skolverket, 2012). This indicates that schools do not succeed to provide "all" students with an adequate education. The PISA 2012[1] results in mathematics show a difference of 45 points between native students and students with foreign background born in Sweden; and a difference of 80 points between native students and students born abroad with a foreign background. Taking into account the socio-economic background reduces considerably the differences but they do not disappear (Skolverket, 2012 p. 18-19). The Swedish Government articulated that newly arrived students need to quickly advance in their knowledge in all school subjects trough instruction designed according to their conditions and needs (Utbildningsdepartementet, 2015). Thus, in the long run, the aim should be attaining higher scores in PISA.

EXAMINING POLICY TEXTS

We have chosen to analyse four recently published policy documents that are suppose to direct and support education for newly arrived students in Sweden. *General guidelines for the schooling of new arrivals* (here named Guidelines) was issued by Skolverket (2016) to give recommendations for how to organize and work with education for newly arrived students to meet the requirements/regulations in the Swedish Education Act, the Education Ordinance and the curriculum. The second and third analysed documents, also issued by Skolverket, are support materials that give example on how to organize education for newly arrived students and build on the *Guidelines.* The fourth analysed document is a report issued by Skolinspektionen [the Swedish Schools Inspectorate] (2014), in which an evaluation on education for newly arrived students is presented. This report was accounted for when developing the *Guidelines.*

1. We have not taken the PISA 2015's results (published in December 2016) into account, as students with foreign background did not affect the results in Sweden between 2012 and 2015.

Table 1. The analysed policy documents.

Document	**Short description of documents**
Allmänna råd för utbildning av nyanlända [General guidelines for the schooling of new arrivals] (Skolverket [National Agency for Education], 2016). Here named Guidelines.	General recommendations for municipalities, principals, teachers and other school staff saying how they should organize and work to meet the requirements/regulations in the Swedish Education Act, the Education Ordinance and the curriculum. It aims to affect the development in a certain direction and to promote a consistency in law. The recommendations should be followed.
Studiehandledning på modersmålet [Supervision in the mother tongue] (Skolverket [National Agency for Education], 2015). Here named *Supervision.*	This is a support material primarily for principals, teachers and supervisors in their work with organizing and implementing student tutoring activities in the mother tongue. It was revised in 2015.
Att främja nyanlända elevers Kunskapsutveckling – med fokus på samverkan, organisation samt undervisningens utformning och innehåll [To promote newly arrived students' knowledge development - with a focus on collaboration, organization and teaching design and content] (Skolverket [National Agency for Education], 2012). Here named Promote.	This is a support material primarily for principals, teachers and officials of the local education departments. The aim is to describe and give examples on how municipalities can promote newly arrived students' development of knowledge and skills with a focus on cooperation, organization and teaching design/ methods and content. This material has its starting point in the *Allmänna råd för utbildning av nyanlända [General guidelines for the schooling of new arrivals].*
Utbildningen för nyanlända elever. Kvalitetsgranskning. [Schooling for newly arrived students. Quality report] (Skolinspektionen [the Swedish Schools Inspectorate], 2014). Here named *Quality report.*	This document reports from an investigation on 10 different municipalities in Sweden where the purpose was to examine whether the schools visited provided newly arrived students' prerequisites to achieve sufficient knowledge in Swedish and simultaneously achieve proficiency in all subjects. Within this area, the review focused on whether the teaching was planned, implemented and adapted to the newly arrived students' abilities and needs, as well as on whether the visited schools were working on giving the newly arrived students' confidence in their own ability, motivation and influence.

When analysing the policy documents, we use two theoretical constructs: fabrication (Popkewitz, 2013) and discourse (Foucault, 1970/1993). The fabrication of human kinds concerns "practices about how to think about people, a way of acting on particular populations, and for people to act on themselves that excludes and abjects in the impulse to include" (Popkewitz, 2013 p. 440). According to Foucault a discourse unfolds in a discursive practice. In this text we define discourse as a systematic, recurrent and certain way to talk about and understand the world and the human kind (Foucault, 1966/73). Discourse shapes special knowing which also holds a practice. In social interactions between people common "truths" are built and struggles are fought about what is true or false. Following Foucault, we assume that there are discourses that govern these practices. In this paper we view the policy documents as discursive practices, since texts build on already existing discourses and texts are written within a certain discursive practice. Practices in institutionalized fields, such as making (political) decisions and writing policy texts on how to integrate newly arrived students in school, are also understood in terms of discursive practices. Regarding newly arrived students in schools, in the discursive practices of which policies are a part of, there emerge the ex- and implicit rule-based statements about newly arrived students, fabricating certain kinds of students (Popkewitz, 2013). The analysis process consisted of repeatedly and carefully reading of the documents, independently by the two authors with the aim of exploring discourses around newly arrived and the fabrication of newly arrives students. Thereafter, the findings were compared and two discourses were construed in three prominent themes. All policy documents are written in Swedish, thus all quotes are translated into English by the authors.

OPPOSING DISCOURSES

According to Lorentz (2007) the general school in Sweden between 1880-1980 was an arena where hardly any experiences of "strangers" who demanded a "different" education existed. The teaching was focused on creating a community based on equality and unity. "Good welfare" and "a strong state" dominated the Swedish public debate in the period 1930-1990. The talk of *multiculturalism* in Swedish policy documents was raised around 1975. At the same time media started reporting about ethnic discrimination, for example, immigrants were refused as guests in restaurants. In contrast the official immigration policy evolved into a non-assimilation, but integration policy. In 1976 the Parliament decided on the home language reform, which meant that municipalities imposed obligation to organize mother tongue tuition for students who were considered immigrant children. The aim of the reform was to build a holistic perspective on immigrant children's development, in addition to children's language development, even taking into account emotional, intellectual and social development (Municio, 1987). Later research suggests that these two discourses still exist today. One operationalizes as a discursive practice that encourages multilingualism and multiculturalism, the second operationalizes as a discursive practice that wants to preserve Swedish-ness (Bagger, 2015; Nilsson & Bunar, 2015; Norén, 2010; Svensson, 2014).

In our analysis of the four documents mainly three themes emerged. These themes resonate with earlier research as noted above. The themes are: a) the importance of mother tongue for learning, b) the importance of acknowledging newly arrived students' background (prior knowledge and experiences), both a and b as resources for learning, and c) deficiencies around immigrant students, for example their lack of "proper" knowledge in Swedish.

The importance of mother tongue for learning

Since 1976 when the Parliament decided on the then so-called home language reform, a student who has a guardian with a mother tongue other than Swedish should be offered mother tongue tuition, if the language is daily used in interactions in the home, and the student has basic knowledge in the language (10 §, School Act, 2010). Following the School Act, the first heading in the *Supervision*, after the foreword and introduction is: "Multilingualism –a resource" (Skolverket, 2015, p. 8). Multilingualism is stated to be a resource for both the individual and society. Research has shown that mother tongue is important for learning, and the emphasizing in the document can be interpreted as directing a way to make newly arrived students to do better PISA results in mathematics, because one aim for the governing is to raise these results (SOU 2016:38, 2016). Better results are a win situation for the individual but also for the society. To do better results the newly arrived student is in need of his/her mother tongue. Therefore, it is important that the teaching in school affirms students' multilingualism. This message reappears in all of recently published documents issued by Skolverket on newly arrived students. In the *Promote* it is stated:

> Another piece of the puzzle for the school to be able to seize the student's skills and develop them further, is that the student has the opportunity to express themselves in their mother tongue. It takes several years to acquire a new language so that it can be fully used in different school subjects. Research also shows that the development of knowledge benefit from the terms used in subject teaching entrenched on both students' mother tongue and in Swedish (Skolverket, 2012).

The use of mother tongue for learning i.e. 'mother tongue –a resource' represents an inclusive orientation towards pedagogy, and incorporates language as right orientations (Schecter & Cummins, 2003). In the *Guidelines* mother tongue supervision and mother tongue education is explained to be important for many students' language and knowledge development.

> When a student must demonstrate their knowledge in their strongest language, which otherwise cannot be expressed in full due to insufficient Swedish, strengthens the confidence and motivation to study. It also enables teachers to gain a better understanding of the student's actual knowledge in each subject. (p. 29, 2015)

This is in line with a discourse on multilingualism/culturalism as an asset.

The importance of acknowledging students' background

The policy documents all emphasize the importance to acknowledge newly arrived students' strengths instead of focusing on their shortcomings and deficiencies. "In all teaching, both in the introductory class and in the regular teaching group, teachers need to emphasize students' strengths and challenge students, instead of pointing at deficiencies" (Skolverket, 2016, p. 31). The statement can be interpreted as this has not been the case in Swedish schools, and that teachers have viewed newly arrived students as students with deficiencies, which has been a common discourse around immigrant students in Sweden for many years (Norén, 2010; Svensson, 2014). In *the Quality report*, it is stated that a major part of the teaching in regular classes are not based on students' prior knowledge, experiences and interests. This means that the newly arrived students experience a teaching that seems to be completely unaffected by the existence of newly arrived students in the class. Thus, according to Skolinspektionen (2014), the subject matter is characterized by a usually unconsciously, narrowly Swedish cultural perspective that assumes a familiarity with the specific Swedish which the newly arrived students most often lack. Skolinspektionen makes the following conclusion:

> If the school has a focus on what the newly arrived students can or focus on what the newly arrived students cannot is central. An important part of this is if the school views newly arrived students as a problem or as an asset. To create a good approach, the perception of newly arrived students as an asset has to be spread by politicians, through administration and principals to the teachers. (2014, p. 31)

In the documents it is understood that newly arrived students have to get opportunities to learn. To get those opportunities, besides their mother tongue, mapping of their prior knowledge is highlighted as important. The law regarding newly arrived students in Swedish school states that two months after a newly arrived student start compulsory school in Sweden a mapping of the student's prior knowledge has to be done.

Immigrant students and resources tied to deficiencies

In the *Guidelines* it is stated that they are written to support the newly arrived students' development, as far as possible. To reach this aim the school's compensatory mandate is of importance because it "means to take into account students' different needs. One aim will be to compensate for differences in students' ability to benefit from the education" (p. 13). In relation to allocation of economic resources, it is important that principals take into account the needs of each school units and possibly reallocates economic resources. It is pointed out how important it is that "An important instrument for greater equality in education is that students are given support and incentives, as well as the resources allocated based on students' different abilities and needs" (p.16). Another quote is putting the newly arrived student in a deficit position, not knowing enough Swedish: "It takes a conscious effort for a long time, for a newly arrived student to be able to conquer the linguistic knowledge required in the different school subjects" (p. 28). In the *Guidelines* the newly arrived students are fabricated as in great need, because they lack knowledge in Swedish. A discourse that preserves Swedish-ness is operating. The required is knowledge in Swedish because Swedish is the language of instruction. To compensate for difference doesn't mean to use mother tongue, but develop Swedish(ness). The normal is to use Swedish for teaching and learning. This is constructed within a discourse of normalization, and a way to fabricate newly arrived students as a certain kind of people (Popkewitz, 2012). The same is applicable in the *Quality report*. According to the report many teachers and principals declare that: "/…/ it is important for the newly arrived students to learn the Swedish language during their introductory time in school" (p. 7). Saying that, it's a fact that most teachers in introductory classes are Swedish as second language teachers. This fact makes students loose knowledge in other school subjects than Swedish, and constructs mother tongue as less valuable. Another aspect is that a low degree of individualization and scaffolding, as well as teachers' low expectations, is leading to students that are almost never challenged and stimulated in their learning. "Many of the difficulties that have emerged in the investigation can be traced to that schools do not do a proper mapping of the newly arrived students previous knowledge and experience" (p. 18). This document shows that a normalizing discourse "Swedish only" preserving Swedish-ness (Bagger, 2015; Norén, 2010) is affecting the school introduction for newly arrived students.

NEWLY ARRIVED STUDENTS AS MATHEMATICAL LEARNERS

Mathematics appeared scarcely in the documents, and is completely missing in the *Guidelines*. So are the rest of the school subjects. The *Quality report* mentioned mathematics five times, very briefly saying that some teachers document if newly arrived students have received mathematics teaching or not and that some of the visited schools map the newly arrived students' prior knowledge in mathematics. In the *Supervision* mathematics occurs in examples on how supervision in mother tongue can be organised, using mathematics as an example. The created fictional examples all use mathematics when exemplifying different ways of organising supervision in the mother tongue. Using mathematics as the example may relate to the influence of Swedish students' low PISA-results, and the lower achievement of immigrant students, that has to be fixed. The same rational can be found in the *Promote*, in which a few teachers and principals describe different projects, that all are related to mathematics. The reason is, as one teacher put is: "Mathematics was chosen because many newly arrived students bring math skills to the classroom" (p. 59). This aspect can be interpreted as taking students' prior knowledge in mathematics into account, at the same time pointing to students missing skills in other school subjects. And mathematics is still at high stake in PISA.

The intentions of the policies are inclusive, since it aims at creating better future for newly arrived students, but at the same time it is important for the society that newly arrived students succeed. The Swedish PISA results are not acceptable, for example the Swedish Government appointed an expert committee to make proposals aimed at raising academic achievement, improving education quality and greater equality in Swedish schools, since the achievement in international tests (like PISA) has decreased for several decades (SOU 2016:38, 2016). However, policy documents govern, that is, they conduct people in a specific direction inscribing the desirable and thus "inscribe differentiation and ordering on groups of people" (Valero & Knijnik, 2016). Organising the schooling and teaching for newly arrived students according to the policy documents may lead to success for these students. However, the policy documents also fabricate (Popkewitz, 2013) the newly arrived students as the ones that are in need of rescuing. In this fabrication of the newly arrived students it also becomes visible what is missing to succeed in school and exclusion is created. That is, they are fabricated as students with resources such as their mother tongue and prior knowledge, though at the same time it becomes obvious that they lack the most valued resource in the Swedish school system, the Swedish language. Not knowing Swedish when taking the PISA test and the National tests in mathematics usually means lower results (Utbildningsdepartementet, 2015) because the tests are in Swedish.

The plea for mother tongues' importance for learning, is in line with an inclusive discourse, but at the same time fabricates newly arrived students as kinds of people that are in need (Popkewitz, 2012; see also Bagger, 2015). The policy for immigrant students in Sweden has for a long time promoted the use of students' first languages, as well as subject content and language integration for learning (Municio, 1987; Norén, 2015). Despite the inclusive view on newly arrived students' education it seems like discourses that preserve Swedish-ness and deficiency fabricate newly arrived students as in great need in the everyday life of Swedish schools.

The Swedish Government has the last couple of years invested a significant amount of money to develop mathematics teaching and to improve students' achievement in Swedish schools through *Matematiksatsningen* 2009-2012 and *Matematiklyftet* 2012-2016 (Utbildningsdepartementet, 2012). Mathematics is the only school subject that such a large financial investment has been made in, giving school mathematics a high value and thus opportunities to raise the PISA scores. If raising the mathematics scores in PISA becomes the aim for schooling the fabrication of newly arrived students as mathematical learners in need of rescue will go on and exclude newly arrived students from being "normal", since they are fabricated as mathematical learners that may disrupt the plan for improving the Swedish PISA results in mathematics.

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DEVELOPING CRITICAL AND REFLECTIVE DIMENSIONS OF MATHEMATICAL MODELLING

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Among the many diverse and innovative teaching methodologies, this theoretical paper highlights the importance of using the critically and reflective dimensions of the mathematical modelling process to solve problem situations that afflict contemporary society. In the last three decades, mathematical modelling, particularly, research related to the critical and reflective dimensions of this approach has been seeking an identity, is defining its own objectives, and is developing a sense of its own nature and potential of the research methods used in order to legitimize pedagogical action. The importance of both philosophical and theoretical perspectives found in these dimensions of mathematical modelling as well as the importance of a learning environment that helps students to develop their own critical and reflective efficiency is discussed in this paper.

INTRODUCTION

Many of the main goals for schools are related to the development of creativity and criticality in students, which helps them to apply different tools to solve problems faced in their daily lives. The development of competencies, abilities, and skills also can help to reflect on problems faced by fellow members of contemporary society, cultural group, or community.

According to D'Ambrosio (2015), in most cases, these goals are established in the school without the participation of communities in planning curricular actions, which may contribute to an authoritarian education whose main purpose is to promote a lack of motivation and passivity in students. Thus, an educational focus must be used to prepare students to be active, critical, and reflective participants in society.

However, in order to reach this goal, it is necessary that teachers promote teaching and learning processes that help students develop a critical-reflective efficiency. To become more active participants in society, educators should be encouraged to adopt pedagogical practices that allow learners to become critical and reflective analyzers of problems and practices that surround them.

CONCEPTUALIZING CRITICAL AND REFLECTIVE EFFICIENCY

One of the most important characteristics of teaching for a critical-reflective efficiency is the emphasis on the critical analysis of students using the phenomena present in their daily lives. One important feature of this form of teaching is related to the reflections the students themselves make about social elements that underpin their increasingly globalized world. The critical perspectives in relation to social conditions that affect a students' own experience help them to identify common problems and collectively develop strategies to solve these problems. This is a type of transformatory learning based on previous experiences of students that aims to create conditions that help them learn to challenge worldviews and values predominant in society by using data. In this regard, by using their own experiences, and the data they collect, and the critical reflection they learn to make on the data and experiences, students are able to develop their own rational discourse in order to create meanings necessary for the transformation of society (D'Ambrosio, 2015).

Rational discourse is a special form of dialogue in which all parties have the same rights and duties to claim and test the validity of their arguments in an environment free of prejudice, fear, and social/political domination. It provides an action plan that allows participants to enter into dialogue, resolve conflicts, and engage collaboratively to enable the resolution of problems in accordance to a set of specific rules. In this type of discourse, intellectual honesty, elimination of prejudices, and critical analysis of facts and data are important aspects that allow dialogue to happen rationally (Rosa & Orey, 2007).

This context is related to a rational transformation that involves the critical analysis of social phenomena. In this kind of educational environment, discourse, conscious work, intuition, creativity, criticality, and emotion are important elements that work to help students to develop their own critical-reflective efficiency.

TEACHING FOR CRITICAL-REFLECTIVE EFFICIENCY

Education towards a critical-reflective efficiency places students back at the center of the teaching and learning process. In this regard, classrooms are considered as learning environments in which teachers help or coach, students to develop their own creative and sense of criticality by applying transformatory pedagogical approaches. However, in order for this form of pedagogy to be implemented in classrooms, it is necessary for educators to discard transmissive traditional pedagogical approaches. In other words, teaching is a social and cultural activity that should introduce students to the creation of knowledge instead of passively being recipients of its transmission. This means that pedagogical transformatory approaches are the antithesis of approaches that seek to transform students into passive containers filled with academic information in what Freire (2000) called the *banking mode education*.

Currently, the debate between these two teaching approaches continues, but discussions are centered in relation to the content to be taught, transmitted, and limited to in relation to the time required to teach of this content. Regarding this discussion, Rosa an Orey (2007) state that there is a need to elaborate upon a mathematics curriculum that promotes critical analysis, active participation, and reflection on social transformation by students. There is a need for curriculum changes that seeks to prepare students to become critical, reflective, and responsible citizens. This mission aims to find practical solutions to the problems faced by society, which must be in accordance to the values and beliefs practiced by communities. This means that it is impossible to teach mathematics or other curricular subjects in a way that is both neutral and sensitive to the reality experienced by students.

Thus, one important objective for schools in a democratic society is to provide the necessary information they need through relevant activities so that learners have the necessary tools to both discuss and critically analyze curricular content and at the same time to enable them to solve daily problems and phenomena. In our point of view, mathematical modelling is a teaching methodology focused on a critically-reflective efficiency by students because it engages them in relevant and contextualized activities, which allows them to be involved in the construction of their own mathematical knowledge.

DETERMINING AN EPISTEMOLOGY OF THE CRITICAL-REFLECTIVE DIMENSION OF MATHEMATICAL MODELLING

Currently, there is no general consensus on a specific epistemology for critical-reflective dimensions of mathematical modelling. However, it can be described as a process that involves the elaboration, critical analysis, and validation of a model that represents a system taken from reality. In this regard, Rosa an Orey (2007) argue that mathematical modelling is considered an artistic process because in the process of the elaboration of a model, the modeler needs to possess mathematical knowledge as well as a dose of intuition and creativity to be able to interpret its context. Students need to work in learning environments that provide the necessary motivation so that they develop and exercise their own creativity through the critical analysis and generation and production of knowledge. Research related to critical-reflective dimensions of mathematical modelling has been defined by establishing the nature and potential of their research methods and investigations. In this dimension, the intersection of theory and practice assists students in understanding systems taken from their own reality and to acquire tools they need in order to exercise citizenship and to actively participate in society.

The main objectives of this approach provide students with the mathematical-pedagogical tools necessary to act, modify, change and transform their own reality. Thus, teaching mathematics processes can start from social and cultural contexts of the students by providing them with the opportunity to practice and develop logical reasoning and creativity. This approach facilitates the learning of mathematical concepts that help students build their knowledge in mathematics so that they are able to understand the social, historical and cultural context in which they live (Rosa & Orey, 2007). The use of critically and reflective modelling dimensions are based on comprehension and understanding of reality in which students live through reflection, critical analysis and critical action. When students borrow existing systems, they study and learn to use mathematics in symbolic, systematic, analytical and critical ways. From a given problem-situation, students are able to develop and share hypotheses, test them, fix them, draw inferences, generalize, analyze, conclude, and make decisions about the objects and phenomena under study.

According to this context, mathematical modelling is the paradigm for a learning environment in which students are invited indeed encouraged through the use of mathematics, to inquire and investigate problems that come from other diverse areas of reality. In this learning environment, students work with real problems by using mathematics as a language for understanding, simplifying, and solving these situations in an interdisciplinary manner. This means that mathematical modelling is a method of applied mathematics that has seized and transposed the field of teaching and learning as one of the ways we use reality in the mathematics curriculum. This enables them to intervene in their reality by obtaining mathematical representations of given situations by means of reflective and critical discussions in relation to the development and elaboration of mathematical models (Rosa & Orey, 2007). From this educational paradigm, there are three distinct mathematical modelling practices that may be used in school curricula (Barbosa, 2001).

Case 1: Teachers Choose a Problem

In this pedagogical practice, teachers choose a situation or a phenomenon and then describe it to their students. According to the curriculum content to be developed, teachers provide students with necessary mathematical tools suitable in the elaboration of mathematical models in order to solve proposed problems (Rosa & Orey, 2007).

In order to determine the height of an object, teachers choose a problem, situation or a phenomenon and then describe it to the students. Mathematics is used in order to stimulate students' skills by using problem solving techniques during the modelling process. In this perspective, problems and situations are authentic since they are also taken from other knowledge areas (Shiraman & Kaiser, 2006).

For example, it is necessary to consider a typical exercise given in trigonometry: *From the top of a cliff, whose height is 100 m, a person sees a ship under a depression angle of 30°. Approximately, how far is the ship from the cliff?*

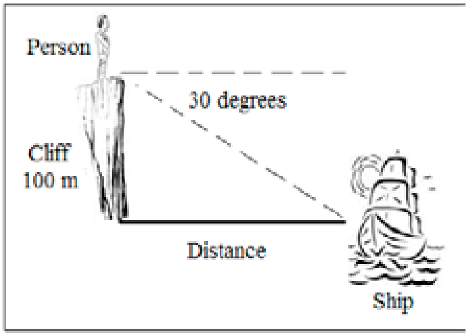

Figure 1: Representation of the problem presented by the teacher

Students can use the tangent function, tg30°=100/d, in order to determine the distance from the base of the cliff to the ship. In this regard, this trigonometric equation represents a simple mathematical model that demonstrates an application of trigonometry that illustrates the use of mathematics in order to solve a problem situation that occurs in reality. It is important to discuss with students the assumptions that have been previously established as a critical analysis of the solution because this is an important aspect of the construction of a mathematical model. During the process of mathematization of this problem, some generalized simplifications of reality were established that are not critically discussed nor reflected with the students.

In the process of problem solving, it is assumed that the ocean is flat, the cliff is perfectly vertical to the straight line chosen to represent the distance from the base of the cliff to the ship, a straight line can reasonably approximate the distance from the base of the cliff to the ship, and the curvature of the Earth is ignored. On a small scale, this fact is not so problematic, however, on a larger scale it can lead to significant deviations in the process of preparation and resolution of mathematical models. It is also assumed that the height of the person is approximately equal to 1.70 meters, which is negligible compared to the height of the cliff, which is 100 meters; the angle of depression was exactly measured, and the ship is a significant distance from the cliff (Rosa, Orey, & Reis, 2012).

In this regard, a point can reasonably represent the position of the ship in the ocean. However, this point can become another mathematical meaning if the ship gets closer to the cliff. These assumptions are considered logical simplifications of the problem because it provides a reasonable estimate in determining the distance between the base of the cliff and the ship. It is important to discuss with students how answers to these types of problems or situations are never absolutely accurate –they are models. The analysis of mathematical models allows students to determine accurate solutions by using increasingly detailed representations of reality.

These assumptions are related to Halpern's (1996) *critical thinking* that involves a wide range of thinking skills leading toward desirable outcomes and Dewey's (1933) *reflective thinking* that focuses on the process of making judgments about what has happened. This approach allows students to solve word problems by setting up equations in which they translate a real situation into mathematical terms, involve the observation of patterns, the testing of conjectures, and the estimation of results, and combines to help students to mathematize systems taken from their own reality.

Case 2: Teachers Suggest and Elaborate the Initial Problem

Here students investigate a problem by collecting data, formulating hypotheses, and making necessary modifications in order to develop the model. Students themselves are responsible for conducting the activities proposed in order to develop the modelling process. One of the most important stages of the modelling process refers to the elaboration of a set of assumptions, which aims to simplify and solve the mathematical model to be developed. In order to work with activities based on the critical-reflective dimensions of modelling, it is necessary that students relate these activities to problems faced by their community.

For example, a teacher proposed the following problem and questions that students investigated: *A company discharges its effluent into a river located near their facilities. These waters contain dissolved chemical substances that can affect the environment in which the river flows. How can we determine the concentration of pollutants in that river? How can we make sure that concentrations in the river are below the standard limits allowed by law?*

Students then investigated the problem by collecting data and were responsible for conducting activities proposed in order to develop the modelling process. One of the most important stages of the modelling process referred to the elaboration of the set of assumptions, which aimed to simplify and solve mathematical models to be elaborated as well as the development of a critical reflection on the data that will be collected (Rosa et al., 2012).

It is important to discuss with students certain modelling variables such as: a) if the average velocity and rate of water flow was constant, b) if there is a seasonal change in the water level of the river, c) if the rate of pollutant concentration in the river was constant, d) if the pollutant and the water are completely miscible regardless of the changes in temperature, e) if there was any further precipitation during the period of data collection, e) if the pollutant and water mixed completely, g) f the pollutant does not solidify in the sediments of the river, h) f the solid particles were deposited in the sediments of the river, i) if the pollutant is volatile because it can be reduced to gas or vapor at ambient temperatures, j) if the pollutant is chemically reactive, and k) if the shape of the river bed is uneven.

It is necessary to determine the key questions that affected the final concentration of pollutants in the river, as well as the rate of flow of pollutants on its waters. This activity helped students to reflect on the mathematical aspects involved in this problem, enabling them to understand phenomenon they encounter in their daily lives so they can critically solve a situation by focusing on the data and the using mathematics to resolve conflict.

Case 3: Teachers facilitates the mathematical modelling process

In case three, teachers allow students to choose a theme that is interesting to them. Students are encouraged to develop a project in which they are responsible for all stages of the process, that is, from the brainstorming and formulation of the problem to the validation of the problem and the solution. The supervision, or coaching, of the teachers is constant during the mediation of the teaching and learning process. This process enables students to become more critically-reflective and engaged in the proposed activities (Rosa et al., 2012).

However, even though there may be some disagreement regarding the use of a specific mathematical modelling practices, it is possible to conduct activities, experiments, investigations, simulations, and research projects that interest and stimulate students at all educational levels. Thus, the choices of pedagogical practices used by teachers depend on the content involved, the maturity level of the students and the teachers with the use of the modelling process. On the other hand, we can emphasize how critical analysis of the results obtained in either approach must be highly encouraged and developed.

During the development of mathematical modelling processes, problems chosen and suggested by teachers or selected by students can be used to get them to critically reflect on all the aspects involved in the situation modeled. These aspects are related to interdisciplinary connections, the access to and use of technology, and the discussion of environmental, economic, political, and social issues. Thus, the use of mathematical content in this process is directed towards the critical analysis of the problems faced by the members of the community.

For example, the results from a conversation during a morning walk with students along a street in Ouro Preto, Brazil encouraged exploration and developed some simple models that enabled the exploration of the relationships between mathematical ideas, procedures, and practices by developing connections between community members and formal academic mathematics. By observing the architecture of the façade of the school, professors and students were able to converse and explore and to determine ways to relate functions of three types of curves: exponential, parabolic, and catenary to the patterns found on its wall (Rosa & Orey, 2013).

Figure 2: Curves on the wall of the school

After examining the data collected when they measured various curves on the wall of a school and by fitting them to exponential and quadratic functions through mathematical models, they came to the conclusion that the curves on the wall of the school closely approximated a catenary curve function. The reflective aspect of this dimension is related to the emancipatory approach of the mathematics curriculum because its pedagogical practices offer *open* curricular activities.

In the Wall problem above, the professors and students applied multiple perspectives to solve the problem. This required constant critical reflection and questioning as they moved towards solutions. The mediator role of the instructor/coach is extremely important during the modelling process; where the *open* nature of modelling activities is be difficult for many learners to establish and develop a model that satisfactorily represents the problem under study (Barbosa, 2001).

THE PROCESS OF CRITICAL-REFLECTIVE DIMENSIONS OF MATHEMATICAL MODELLING

According to the Brazilian National Curriculum for Mathematics (Brazil, 1998), students need to develop their own ability to solve problems, make decisions, work collaboratively, and communicate effectively. This approach is based on increasingly emancipatory powers, which help students face challenges posed by society by turning them into flexible, adaptive, reflective, critical, and creative citizens. This perspective is also related to the sociocultural dimensions of mathematics, which are closely associated with an ethnomathematics program (D'Ambrosio, 1990). This aspect demonstrates the power and role of mathematics in society by highlighting the necessity to analyze the role of critical and reflective thinking about the nature of mathematical models as well as the role of the modelling process to solve everyday challenges present in the contemporary society.

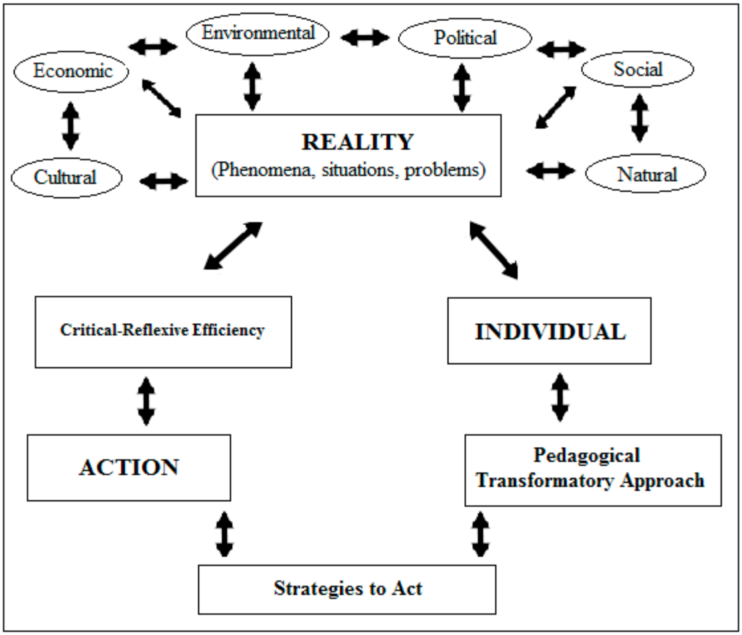

Figure 3: Critical-Reflective mathematical modelling cycle

Mathematical modelling provides real and concrete opportunities for students to discuss the role of mathematics as well as the nature of mathematical models (Shiraman & Kaiser, 2006). It many ways it could be understood as the direct use of a language used to study, understand, and comprehend problems or phenomena faced daily by a community. For example, mathematical modelling is used to analyze, simplify, and solve daily phenomena in order to predict the results of or modify the characteristics of these phenomena (Rosa et al., 2012).

In this process, the purpose of mathematical modelling is to develop students' critical and reflective skills that enable them to analyze and interpret data, to formulate and test hypotheses, and to develop and verify the effectiveness of the mathematical models. In so doing, the reflection on reality becomes a transformative action, which seeks to reduce the degree of complexity of reality through the choice of a system that it represents (Rosa & Orey, 2007). This isolated system allows students to make representations of this reality by developing strategies that enable them to explain, understand, manage, analyze, and reflect on all parts of this system. This process aims towards supporting the optimization of pedagogical conditions for teaching so that educators can enable students to understand a particular phenomena and to act effectively to transform it according to the needs the community.

The application of critically-reflective dimensions of mathematical modelling makes mathematics to be seen as a dynamic and humanized subject. This process fosters abstraction, the creation of new mathematical tools, and the formulation of new concepts and theories. Thus, one effective way in which to introduce students to mathematical modelling in order to lead them towards the understanding of its critical-reflective dimension is to expose them to a wide variety of questions, problems and themes. As part of this process, questionings about the themes are used to explain or make predictions about the phenomena under study through the elaboration of mathematical models that represent these situations (Rosa & Orey, 2013).

However, the elaboration of mathematical models do not mean to develop a set of variables that offer qualitative representations or quantitative analysis of the system because models are understood as only approximations of reality. In this direction, to model is a process that checks whether the parameters are critically selected for the solution of models in accordance to the interrelationship of selected variables from holistic contexts of reality. It is not possible to explain, know, understand, manage, and cope with reality outside the holistic contexts (D'Ambrosio, 2015).

This aspect of traditional learning prevents students access to creativity, conceptual elaboration, and the development logical, reflective, and critical thinking. According to this perspective, any dimension of mathematical modelling facilitates the development of competencies, skills, and abilities that necessary for students to play a transformative role in society (Rosa & Orey, 2007).

FINAL CONSIDERATIONS

The fundamental characteristic of teaching towards a critical-reflective efficiency is the emphasis on the critical analysis of students in problems faced by members of contemporary society. Thus, a critical perspective of students in relation to the social conditions that affect their own experiences can help them to identify common problems and collectively develop strategies to solve them (D'Ambrosio, 2015).

This is paradigm that incorporates a type of transformatory learning that aims to create conditions that help students challenge their worldviews and values dominant in society. They are better able to reflect critically on these experiences in order to develop data-based rational discourse by creating their own meanings necessary for structural transformation of society (Freire, 2000). This presents a rational transformation because it involves critical analysis of sociocultural phenomena through the elaboration of mathematical models. Hence, Rosa and Orey (2007) affirms that mathematical modelling is therefore a teaching methodology that focuses on the development of critical-reflective efficiency and engages students in contextualized teaching-learning processes that allow them to become deeply and actively involved in the constructions of social significance they perceive in their world.

The critical-reflective dimensions of mathematical modelling are based on the comprehension and understanding of reality (Barbosa, 2001). When we borrow systems from reality, students begin to study their symbolic, systematic, analytical and criticality. In this regard, starting from problem situations, students can make hypotheses, test them, correct them, make transfers, generalize, analyze, complete and make decisions about the object under study. So, critically reflecting about reality becomes a transformational action that seeks to reduce complexity by allowing students to explain, understand, manage and find solutions to the problems that arise therein. This approach helps to move the field forward because it provides critical reflections on the role of mathematical models elaboration for the resolution of problems, situations, and phenomena that afflict contemporary society.

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THE SUBJECT OF MATHEMATICS EDUCATION RESEARCH

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In this paper, I speculate on the analogies between science, society, mathematics and education, through engagement with elements from Lacanian psychoanalysis. I explore how science and society's dreams of totality find an echo on modern mathematics, particularly in the way both seek in their endeavours to suture the subject of their investigations. This flattening of the subject has important implications for how we conceive the role of mathematics education. While the reporting of positive experiences where students apparently learn important mathematics for their lives is seen as the most prolific way to carry research, I will argue that such an approach leaves unaddressed both the real student, and what are the real conditions of today's schooling.

SCIENCE AND SOCIETY'S DREAMS OF TOTALITY

> "If you have something that you don't want anyone to know, maybe you shouldn't be doing it in the first place."
>
> (Eric Schmidt, former Google CEO, 2009)

To know everything is the drive that moves science. Johnston (2012) calls it a "death-drive-like compulsion toward knowledge at all costs" (p. 111). This compulsion towards completion is particularly noticeable today in the fields of genetics, neuroscience and physics. The famous Human Genome Project (HGP) is the world's largest collaborative biological project with the goal of identifying and mapping all of the genes of the human genome. Its completion was announced in 2003, after 99% of the human genome has been mapped. The consequences of this new knowledge are still unpredictable, as many private companies are using genomics for purposes that resonate with the ones of eugenics. Genomics, through the improvement of the genetic quality of human production, is seen by many as a way to complete the human species, by reducing or even eliminating that "which fails" in achieving an ideal human being. The ultimate horizon seems to be a world where, through genetic manipulation, people could perfect themselves. In neuroscience the drive is to understand every aspect of the nervous system, including how it works, how it develops, how it malfunctions, and how it can be altered or repaired. Ultimately, neuroscience seeks to map the entirety of human cognition and emotion in terms of neuronal processes occurring inside the brain. This reduction of human life to synapses occurring between neurons is seen by many scientists as a way to overcome human and social science's limitations, by offering a rigorous and complete understanding of human behaviour, free from all the vicissitudes of concrete human life (e.g. Harris, 2010). The chaotic reality of being a person is reduced to formulae, liable to prediction and control. Another field where the drive towards completion is palpable is physics. The search for a theory of everything (e.g. Hawking, 2003) that fully explains and links together all physical aspects of the universe has become the Holy Grail of physics. This theory struggles to unite in one single narrative the two theories upon which modern physics rests: general relativity and quantum mechanics. The challenge is to incorporate into one consistent, cohesive model the infinitely large (stars, galaxies, etc.) and the infinitely small (atoms, subatomic particles, etc.), thus providing a unifying account of how the universe works.

The drive towards totality that characterises the scientific endeavour is also present is society more generally. The Nazi and Stalinist dreams of a full "predictable" society, where people actually become the ideal subjects aspired by those in power, was made possible through the deployment of ideologies intended to close the gap between what one is and what one should be, on behalf of man's dream of totality. Twentieth century literature has provided us good insights into what a "closed" society can be, in which the novel "1984" by George Orwell is only the best-known example. At the time of its publishing, Orwell's novel was received as an alarming but relatively futurist view of humankind. Somehow, Orwell had that miraculous instant of seeing the future, at the time where the future was still to come. We are now in Orwell's future, and only in moments like Snowden's and Manning's testimonies, or Wikileaks' disclosures do we realise the oddness of what is going on.

Eric's statement above is a clear Orwellian sign of the "knowledge society" we live in today. It presupposes a society where everybody follows the letter of the Law, a complete society where there is no place for "misbehaviours". You should only do what is supposed to be known. In the horizon lies the idea of a total society –each one in his or her own place, causing no friction, no alarm, doing only what the Other knows.

MATHEMATICS: A FLY-BY-WIRE SCIENCE

The archetypical case of this scientific search for completion is Hilbert's formalist program, aimed to stablish a secure foundation for mathematics that could avoid the paradoxes and ambiguities of common language. This cleansing of mathematical language was particularly noticeable in the efforts towards the formalisation of the notion of limit made by mathematicians such as Bolzano and Weierstrass, as well as the formal foundation of the notion of number carried out by Cantor. While previous approaches to limit (e.g. Newton's) made use of common language, with the use of (subjective) expressions such as "as close as desired", "as small as desired", "close enough, but not equal", the "epsilon-delta" definition of limit deals only with mathematical formulae. The process of mathematical formalisation seeks to assure that mathematics can function without a subject, making sure that nowhere one can find a trace of what is usually called the "subjective error".

However, the very efforts to solidify mathematics in a secure foundation led to Gödel's theorems of Incompleteness. That is, the colossal efforts made by mathematicians such as Frege, Whitehead, Hilbert, Russell and Gödel to reduce mathematics to a symbolic machinery ended up in proving the impossibility of such endeavour. It ended up in a piece or irrefutable real: the impossibility of mathematics to totalise itself and the all of reality. The process of mathematical formalization engenders its own impossible spots. These spots, although not "beyond" the formal structure, are impossible to reach (that is, to demonstrate). This is why mathematics, according to Lacan (1999), was the first discourse to perceive that the symbolic order itself contains elements of the real.

When, in our mundane activities, we say something as trivial as "I am Alexandre", we always have the impression that the signifier "Alexandre" never quite well encapsulates everything that I am. I can say I am a teacher, a father, an academic, and so on, but, ultimately, there will always be a sense of mismatch as if I am not only the sum of all these attributes. The more signifiers I use to describe myself, the more I feel as if something is escaping. There is some real about myself that cannot be completely mapped by the chain of signifiers. Every relation that the subject has to itself is rooted in its impossibility of coinciding with itself (that is, to the symbolic representations of it). According to Lacan (2006), "science turns out to be defined by the deadlock endeavour to suture the subject" (p. 731). It is a deadlock endeavour because there will always be subjective elements "in which the imbalance of the structure manifest itself" (Maniglier, 2012, p. 46). What is rejected ends up returning, thus the paradoxes of set theory and formalisation as such. The function of the whole structure is precisely to conceal the original imbalance, that is, to conceal the fact that mathematics is nonetheless made by people who err. However, notwithstanding Godel's results, mathematics continues to thrive as the paradigmatic example of a science without a subject, a *fly-by-wire* science (Laurent, 2013, p. 30), which can be applied independently of the subjects who carried it.

THE SUBJECT IDENTICAL TO ITSELF

The drive towards totality that characterizes modern science is also present in social sciences, especially those relying on so-called "evidence-based" assumptions and "big data" assets. These endeavours rest on the underlying logic that one can elaborate with precision what society needs and purpose solutions accordingly. It is assumed that one day soon it will be possible to calculate all human activity reduced to objectifiable behaviour (Laurent, 2013, p. 29). In order for such endeavour to become possible, subjects need to be conceived as *identical to themselves* (Fochi, 2013, p. 40), that is, reduced to their symbolic identification.

A subject identical to itself is a subject completely predicable, a normalised subject. A subject who does not screw up, that is identical to the identity assign to him or her by the symbolic order –he or she is only what the Other is supposed to know. Such a society completely flattens the constituting trace of human subjectivity: the fact that we always miss to completely fit in the symbolic demand. As noticed by (Fochi, 2013, p. 40), the paradox of apparatus like Google is that the world of seemingly extreme personalisation that Google creates (each user has its own personalised information, according to his or her recorded choices), "can only function by flattening the subject onto its identity with itself" (p. 41). That is, by limiting the subject to the information one has of the subject.

THE REAL OF MATHEMATICS EDUCATION

As we previously seen, M20[1] is arguably the most elaborated human artifice for concealing the subject. As argued by Lacan, it is precisely this foreclosing of the subject that allowed science, and M20 in particular, to flourished massively in the last two hundred years. Mathematics education, as a science, is not immune to this drive towards completion. Researchers have been criticizing the way in which mathematics education research tends to leave aside what Brown calls the real of schools: the little failures, obstacles, constraints, that populates the lives of many students and teachers. Mathematics education research is often looking at what Skovsmose (2005) calls a prototypical classroom and ignoring everything that somehow does not fit the picture of a well-organised and equipped class, with a teacher desiring to teach and students willing to learn. In much of the research into mathematics education, students and teachers are depicted as fully assuming the symbolic mandate conferred upon them. When problems appear, they tend to be ignored by research, or solved

1. M20 is a term coined by Roberto Baldino to designate the developments in mathematics that resulted from the work of mathematicians such as Cauchy, Weierstrass, Dedekind, Cantor, Hilbert, Frege and Russell, in their attempts to substantiate mathematics in a secure axiomatic system, whereby previous faulty notions of number, limit, continuity, infinity and infinitesimals can be elaborated in a secure way. That is, in a way that does not require any reference to common language (smaller as we can, infinitesimal approach, etc.).

through the implementation of a better practice (Pais, 2015). Perhaps we can say that the reigning ideology in mathematics education research is one that obliterates the real of schools for the sake of research, thus allowing it to flourish.

As I explore elsewhere (Pais, 2016), mathematics education research makes sense in itself. Researchers believe in a subject supposed to know the truth about the successful union between students and mathematics. Using a diverse panoply of well-grounded theorisations and extensively tried methodologies, researchers seek to seize this relationship, to "write" it. The problem arises as soon as one confronts the research's consistency with the crude reality of schooling, where mathematics is often considered a meaningless, useless and boring school subject, cause of anxiety for many students as well as teachers, and a harmful instrument in the reproduction of race, gender and class inequalities. When confronted with obstacles to the teaching and learning of mathematics that cannot be controlled by research –poverty, inequality, economic constraints, and governmental decisions, but also students' refusal to assume the symbolic mandate conferred upon them– researchers tend to forsake them for the sake of research. Instead of conceiving these "external" circumstances as the very arena in which the true nature of research's inner potentials is to be "tested", researchers conceive them as empirical impediments, thus keeping the presuppositions of research intact.

Why does research creates a reality so at odds with the one experienced by many students and teachers worldwide? It seems that the more mathematics education leaves behind the pressing problems of today's schooling, the more it thrives intrinsically. When asked about the concrete conditions that need to be met so that mathematics in school can change in the way desired by researchers, research reaches a deadlock. Lerman (2014) calls it "the internalism of research" (p. 193), to signal the disavowal by researchers of the concrete circumstances that make the real of schooling. Lacan attributes science's prodigious fecundity to the fact that it wants to know nothing about truth as cause (Skriabine, 2013, p. 52). In the case of mathematics education, mathematics education does not want to know anything about the cause of students' desire (Pais, 2015).

SINGULAR STUDENTS

We all know, either through our own experience as students or our work as teachers, that for most students engagement with mathematics is not motivated by the intrinsic characteristics of this science –its beauty, its power to generalise, its utility, etc.– but derives instead from a will to satisfy some Other's demands (say, parents' demand for good grades, teachers' demand for learning, academic or professional demands, etc.). The fact that people fail in school mathematics means that students do not always, (and in some cases, almost never) identify themselves with the mathematical learner envisaged by the curriculum and pursued by the teacher. However, as I explore elsewhere, research tends to disavow this incongruence, by creating and reporting situations where students do indeed learn meaningful mathematics, and appreciate its beauty, utility and value (Pais, 2016b). Research is animated by a sense of “positivity”, and values situations where, notwithstanding all the difficulties, a breakthrough was possible (Gutiérrez 2013; Presmeg & Radford 2008; Sriraman & English 2010). As posited by Gutiérrez (2013), “it is important to highlight the features of practice that coincide with certain kinds of students engaging/ succeeding in school mathematics (and this form is much more productive than focusing on failure and/or disengagement)” (p. 52).

There is the implicit assumption that the results of research are valid independently of what constitutes for a subject his or her relation with the truth of school mathematics. As a result, mathematics education threats the subjects of its research as being identical to themselves, that is, as fully endorsing the identity conferred upon them by the symbolic (that of being a student, with the task of learning). Through these efforts, we, researchers, perceive ourselves as doing the good, bringing mathematics for all to enjoy and to love (Boaler, 2010). The result is a society where students fully endorse the symbolic mandate conferred upon them –that of being a good student, willing to learn and in love with mathematics. What is lost in this endeavour is the *real* subject, a research approach that takes each individual in its own *singularity*.

Within physics, a singularity is a one-dimensional point which contains infinite mass in an infinitely small space, where gravity become infinite and space-time curves infinitely, and where the laws of physics as we know them cease to operate. As the eminent American physicist Kip Thorne describes it, it is "the point where all laws of physics break down"[2]. This entity, although impossible, is a necessary part of the explanatory scheme of physics. A singularity is the name for something that, although a result of the laws of physics, it ultimately leads to the breakdown of physics its entire edifice. This is the real at its purest, not something that exists out there outside our knowledge, but an inherent product of our knowledge that remains unhittable, thus threatening the entire system to collapse. The systematic failure in school mathematics, the student who never learns notwithstanding all the efforts of the teacher, the worldwide school credit system that guarantees exclusion by means of promotion, are examples of singularities within mathematics education research. These features force us to question not only local practices, but also the

2. Information retrieved from http://www.physicsoftheuniverse.com/topics_blackholes_singularities.html at 8th January 2015.

entire system that puts "mathematics for all" as a goal to be achieved. If forces to question our dream of completion, of a world where mathematics will totally reach all students.

Although positing symbolic systems as incomplete might be seen as undermining the power of science, it can be argued that such incompleteness is indeed a source for radical emancipation. Symbolic systems "necessarily include one possibility which is at the same time the impossibility of the system itself, i.e. not only an alternative option within the same system, but also the possibility of an alternative system of alternatives" (Maniglier, 2012, p. 38). The physical notion of singularity is the bedrock whereby physics is forced to question itself, thus pointing towards the possibility of a new rationality about the nature of the universe. The point of impossibility is at the same time negativity, in the sense that it is what fails in the system, what prevents any system from being complete, but also, and perhaps more importantly, of "positivity", because this same impossibility is the gateway for an entire new field of possibilities.

IMPLICATIONS FOR RESEARCH

In terms of research, the choice is between building a kind of encyclopaedic endeavour to exhaust the field –to find both an explanation for the problem of failure in mathematics and a "formula" that could be applied so that everyone could become mathematically proficient, thus assuring the constitution of a truly enlightened society– or coming to grips with the real so schools, with the fact that failure is endemic to current schooling and that there will always be a mismatch between what society wants and what a subject becomes. These are not easy tasks to carry. We can however, in our discourses, refuse to participate in the farce. Instead of running after the hysterical societal demand of mathematical equity, developing increasingly refined stratagems to better teach and learn mathematics that only seem to function in the controlled reality of a research setting, perhaps we should acknowledge the crude reality that mathematics is not for all. Schools, however uncomfortable such awareness may be, are places of selection and teachers are agents of exclusion. These are the conditions of today's schooling, and research cannot afford dismissing them as being beyond its field of action. Secondly, by positing the importance of school mathematics in terms of knowledge and competence, research provides an ideological screen against the role school mathematics plays within capitalist schooling. While presenting school mathematics as an important subject in terms of knowledge and competence –that is, in terms of what Marx called the use-value– the other, surreptitious, functions of mathematics, its exchange-value, can actually become operative. My suggestion is to conceive the importance of mathematics not in terms of mathematics itself, but in terms of the place this subject occupies within a given structural arrangement (Pais, 2015). That is, to conceptualise the importance of mathematics not in terms of its inherent characteristics –problem solving, utility, beauty, cultural possibilities, etc.– but in terms of its attendant submissions to political as well as economic criteria and goals. In short, I suggest that school mathematics should be investigated as a crucial element in the accreditation system, and not so much, as it is today, as a precious knowledge aimed to empower people and to enable societal development.

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ACADEMIC AGENCY: A FRAMEWORK FOR RESPONSIBILITY

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What does it mean to be responsible for learning in the mathematics classroom? How teachers define and enact their sense of responsibility for student learning and achievement is central to this investigation and application of a novel theoretical framework, Academic Agency. This paper articulates underlying dispositions of teachers' sense of responsibility to educate all children by attempting to identify constructs that impact instructional choices made in mathematics classrooms. Academic Agency is established as the framework for this study, situating the constructs of action, commitment, efficacy, and mathematics knowledge for teaching within the notion of teachers' sense of responsibility.

ACADEMIC AGENCY

Figure 1: Academic Agency Framework

Academic Agency, the framework for this study, amalgamates constructs previously identified through a myriad of educational studies and situates them within the notion of teacher's sense of responsibility in teaching mathematics. The novel framework of Academic Agency situates the constructs of action, commitment, efficacy, and mathematics knowledge (Hill, Schilling & Ball, 2004) within the notion of teacher's sense of responsibility. Figure 1 depicts interactions and movement toward Focused Academic Agency, the point where commitment and knowledge, while influenced by efficacy, intersect with action. Agency, the pinnacle element of the framework encompasses the responsibility of action and becomes the changing factor that moves personal responsibility into professional accountability. Along with the identified constructs, four types of responsibility dispositions can also be described through the framework: Custodial Emphasis, Commitment Emphasis, Knowledge Emphasis and Academic Agent.

SENSE OF RESPONSIBILITY

To begin the discussion regarding responsibility as framed by this study; two paradigms related to responsibility must be addressed. Silverman (2009) suggested that responsibility might be categorized as virtue-responsibility (VR) or accountable-responsibility (AR). She posits that a responsible and virtuous person will engage in areas that are driven by morality and that they regulate their behavior via morals-based choices. Conversely, an accountable and responsible person is driven by rules, outcomes, and punishment. (See Table 1)

Table 1. Virtue Responsibility (VR) versus Accountable Responsibility (AR) From On Responsibility: Teachers' Conceptions of Promoting Social Justice (p. 41), by S.K. Silverman, (2009). Unpublished doctoral dissertation, The Ohio State University, Columbus.

	Responsibility	
	Virtue	Accountable
Forum of Judgment	Conscience	Law
Answerable To	Self	Others
Defined By	Discovery	Decision
Codified By	Morals	Ethics
Agency	Individual	State
Judged According To	Intention	Outcome
Consequence of Error	Guilt	Punishment

Lee and Smith (2001) identified three components to measure a sense of collective responsibility: 1) teachers' innate sense of responsibility for student learning, 2) a teacher's willingness to modify teaching strategies to address students' needs, and 3) a teacher's sense of efficacy in their teaching practices. Collective responsibility therefore exists on a continuum, on one end are environments where teachers acknowledge their success and failures in the classroom and accept responsibility for student success and at the other extreme, teachers who take little responsibility for student success and blame the failure of student advancement on conditions such as student ability, socio economic level, or lack of motivation of the student (Lee & Loeb, 2000; Lee & Smith, 2001).

DISPOSITIONS OF RESPONSIBILITY

As mentioned four types of responsibility dispositions emerged as the framework of Academic Agency developed; custodial, commitment, knowledge and academic agent. Each exemplar is defined below.

Custodial Emphasis. Custodial emphasis refers to those instructional situations that appear to be more directive. Custodial emphasis does not mean that a teacher who subscribes to this instructional emphasis does not feel a sense of responsibility to children, rather it means that they may adhere to a more accountability driven sense of responsibility, where external expectations and influences outweigh the personal, long term instructional needs of the student.

Knowledge Emphasis. When an educator subscribes to knowledge emphasis, they may tend to be more concerned with overall summative assessments and see the classroom as one entity, instead of multiple identities making up one classroom. Instruction is typically more formal, with goals and outcomes based upon a common indicator. Knowledge is coupled with action, but a missing piece is commitment to the individual, and an inability to see an individual as a unique participant in the acquisition of knowledge.

Commitment Emphasis. The line that separates commitment emphasis and knowledge emphasis may be somewhat difficult to distinguish. One factor that separates them is noting how the teacher addresses the personal learning needs of the student. Commitment, when used as a descriptor, does not replace or contradict the need for knowledge but shifts the focus of classroom instruction to the student and away from summative results. Commitment to students' growth and understanding become the foundation for all classroom instructional and personal decisions.

Academic Agent. Academic Agency, initiates both commitment and knowledge in the classroom. The role of the teacher becomes facilitator -facilitator of knowledge, of student independence, of enhancing self-esteem, and of the promotion of critical thinking skills.

EFFICACY

Efficacy's influence on personal commitment, personal knowledge and action is irreplaceable in an educational setting and how an individual defines their personal and professional responsibility. Ashton and Webb (1986) found that teacher expectations and commitment to responsibility were altered by student characteristics such as socioeconomic class, race, or classroom behavior. Specifically, teachers that demonstrated a low sense of efficacy failed to accept any responsibility for student achievement. Teachers with high efficacious feelings cited more positive relationships with students and took a greater responsibility in reaching all children. What a teacher believes about the nature of mathematics directly impacts their belief system and what it means to actively engage in doing mathematics (Mewborn & Cross, 2007).

RESEARCH DESIGN

As an integrated design mixed-methods study, data from the quantitative phase informed the manner in which the qualitative study was addressed. The quantitative component incorporated survey instruments to address specific constructs of Academic Agency, specifically; action, commitment, efficacy, and mathematics knowledge. The qualitative element extended the knowledge gained from the quantitative study to provide a more concise picture of individuals and their personal grounding within the Academic Agency model. The qualitative section incorporated the use of interviews and observations to expand and provide a platform for individual voice.

QUANTITATIVE RESEARCH METHODS

The survey instruments asked teachers to respond to forced-response questions, establishing common boundaries for responses making the interpretation of data cohesive.

Teacher efficacy. To quantify an individual's sense of efficacy, two instruments were used: Teacher Efficacy (TE) (Woolfolk & Hoy, 1990) and the Mathematics Teaching Efficacy Belief Instrument for in-service teachers (MTEBI-A) (Enochs, Smith & Huinker, 2000).

Teacher responsibility. To explore teachers' sense of responsibility and commitment, Teachers' Beliefs Form I, developed by Silverman (2009) was administered. Silverman's original instrument looked at various components of responsibility including multiculturalism and diversity, economic class, gender, faith, disability, sexual orientation, and culture. For this study an abbreviated 33-item version of the Silverman (2009) Teachers' Beliefs was adapted to measure teacher beliefs regarding specific aspects of responsibility in the classroom setting. The modified instrument looked at the sub-groups of race, socio-economic status, disability, gender, and culture.

Mathematics content knowledge. A version of the Learning Mathematics for Teaching (LMT) instrument was constructed for quantitative data collection (Hill, Schilling & Ball, 2004). The adjusted multiple-choice instrument contained 14 items focused on number, algebra, and geometry in the middle grades. The participant responses were identified as either correct (1) or incorrect (0) from a scoring key.

QUALITATIVE RESEARCH METHODS

Stepping away from the quantitative methods, the qualitative methods provided a descriptive picture of what was occurring within a sample population. The semi-structured interview allowed more specific questions to be addressed and provided the vehicle for the educators' voice to be heard.

Observations. Hour long observations were performed with each of the participating teachers. Prior to each observation, a pre-observation conference was conducted. This conference addressed the upcoming lesson to be observed, goals for the intended lesson, and how the instructor hoped to address student misconceptions during the lesson. Following the observation, a post-observation conference was held. During the post-observation conference the teacher had time to reflect upon the successes and challenges that the lesson presented.

Interviews. Semi-structured interviews were conducted with each consenting teacher in the sample. The interviews lasted approximately 30-45 minutes, and were audio recorded. Interview questions were structured around sense of responsibility and overall teaching practices. During the interview, aspects contained in the knowledge section of the Academic Agency model was central: items such as pedagogical content knowledge, mathematics knowledge for teaching, as well as instructional emphasis.

QUANTITATIVE DATA ANALYSIS

The research question to be answered through quantitative data, "What is the relationship that exists between a teachers' sense of efficacy, mathematics teaching efficacy, mathematics content knowledge and responsibility?" Pearson Correlation was used to measure the degree and direction of the relationship between variables. The correlations between pairs of variables are reported in Table 2.

Table 2. Pearson Correlation Matrix

	RESP SUBGRP	OVERALL TEACHER RESP	TE PERSONAL	TE TEACHING	MTEBI PMTE	MTEBI MTOE	*MEAN MKT*
RESP SUBGRP	1	.069	.098	.101	-.100	.216	*-.196*
OVERALL TEACHER RESP		1	.311*	.345*	.242	.311*	*.069*
TE PERSONAL			1	.474**	.660**	.419**	*.101*
TE TEACHING				1	.212	.596**	*.080*
MTEBI PMTE					1	.345*	*.284**
MTEBI MTOE						1	*.234*
MEANMKT							*1*

**Correlation is significant at the .01 level (2-tailed)
* Correlation is significant at the .05 level (2-tailed) (n=49)

QUALITATIVE ANALYSIS

With the framework of Academic Agency in place, the initial coding structure for the sample interviews looked at overall efficacy, mathematics teaching efficacy, classroom environment, instructional emphasis, academic emphasis, Pedagogical Content Knowledge (Shulman, 1986), and Mathematics Knowledge for Teaching. Once the initial coding was completed in NVivo, the larger groupings were then broken down into sub-categories.

INTERVIEW AND OBSERVATIONS

During the interview process, the questions purposefully probed more deeply into the responsibility that one feels towards teaching middle grades mathematics and what choices are made to instruct all children. Questions such as, "In your mathematics classroom, do you try to implement various instructional strategies?" and, "In learning mathematics, who is ultimately responsible for equipping students with the skills and knowledge necessary to move forward?" were asked of all participants.

This section addresses specifically how a sample of individuals from the study defined responsibility and whether their espoused and enacted responsibility was grounded in virtue or accountability. Artifacts from participant interviews are presented to sustain the conjectures regarding individual teaching structures.

CUSTODIAL EMPHASIS

Samuel is a fourth year suburban middle school mathematics teacher. Both quantitative and qualitative results depicted Samuel having an orientation that focused upon summative results. Samuel scored in the lowest of the selected participants for responsibility and mathematics teaching efficacy, in the middle for overall teaching efficacy, and scored the highest out of the sample for mathematics content knowledge for teaching.

Espoused Disposition. Episodes from the interview provide additional evidence in support of custodial emphasis. When posed with the question about a teacher's sense of responsibility in the classroom, Samuel responded:

> For a student that just doesn't, doesn't have that want, it is kind of hard for me to give them the time. That doesn't mean I totally ignore them that doesn't mean that I say oh yeah you don't have to whatever you know I continue to say, you need to get this done, this is going to translate to your grades, this is going to translate to you being on probation from this advanced class, this is going to translate to you not getting a good grade you know when you get to your next class in high school. But, again I can say that as many times, but if they don't pay attention to it, then, that is not going to matter much.

He continued by stating that, "I tend to spend more of my time with students that are receptive to my help because if I don't, first of all it gets me down personally, you know, oh well I am putting all of this work into them and they just don't care…"

The second aspect that became apparent during the interview was Samuel's belief that mathematics students should be held accountable for summative results. When asked about how he deals with assessing individual students, he commented, "If they don't master the learning target or at least have some progress in the learning target, it is hard to justify giving them a higher grade just for effort." To extend this idea, the questions was posed about how mathematics curriculum should be implemented, he stated:

> I mean it is really kind of, you know we teach more than just to the test, but in the end if that is what you are being evaluated on, that is what your students are being evaluated on, that is what your school district is being evaluated on that has to be one of the focuses you have.

Samuel's personal responses of how he defines academic success seem to indicate that summative assessment scores outweigh the individual learning process.

COMMITMENT EMPHASIS

Lydia, a teacher with more than six years of teaching experience, articulated that commitment to her students is the one aspect that she most subscribes to in her role as a mathematics teacher. Her classroom was comprised of children from a myriad of cultures and languages. Lydia scored in the highest section for responsibility, teaching efficacy, and mathematics teaching efficacy. She scored in the lowest for mathematics knowledge for teaching.

Espoused Disposition. During the interview, when urged to discuss her sense of responsibility and how she viewed classroom instruction, the following thoughts were expressed. The question posed to Lydia was, "What is important in your classroom?" Her reply incorporated a recent interaction with a young girl in her classroom. The young girl had been successful in turning in the required work but Lydia had noticed the young girl's dependence on her cousin to help her complete the tasks. She chose to broach this concern with the girl. Lydia's stated:

> I told her, I said if I didn't care about you I would ignore this and let you go on because you're turning in completed stuff. I said, but to me it's not just doing it, it's do you understand? The whole year, I want you to know I care whether you know this or you understand or not. So we're going to work together and you're going to learn this stuff.

Lydia went on to support this commitment sentiment when asked about how she evaluates work presented by the students. She stated, "Our philosophy in the building is that we shouldn't have students failing. We should be continuing to work with them and their grade guides our instruction and to keep intervening and helping and getting them to understand those concepts."

Lydia's interview is saturated with espoused beliefs that support the descriptor of commitment emphasis, each one providing insight into her personal belief system. When asked, "Who's responsibility do you think it is to make sure kids are learning?" She replied, "Definitely the teacher's responsibility...It is your responsibility to teach that student as much as you can teach them in that year. You know, move them as far forward as you can. So I think that our success ultimately would fall on the teacher."

KNOWLEDGE EMPHASIS

Anna has taught in the same rural district for 30 years. Anna stated that she is no longer taking any type of professional development classes and that during the day she dedicates substantial amounts of time to working with students struggling in mathematics.

Espoused Disposition. In each category Anna scored in the middle except for teaching efficacy. Her score for teaching efficacy was the lowest amongst the participants.

During the interview, Anna made several references to personal aspects of her students, such as being a hard worker or good in mathematics, but failed to identify the individual learning needs of the students, seeming to be unaware that this differentiation must be made and recognized. Knowledge of the learner, when coded for Anna, did not mean knowledge of student learning style, but instead, referred to statements such as "as you need to know your student."

During the interview, when asked about what makes her classroom effective, Anna stated, "Everyone has the same work, the same problems, the same everything. I guess you just give verbal praise when they reach certain steps, to give them encouragement to go on to the next level, until they reach the level that they are capable of." With this assertion, it became apparent that individuality in student learning is not being emphasized.

ACADEMIC AGENT

Madelyn has taught for over ten years in the same suburban school district. Her teaching responsibilities include teaching mathematics and science to 6th grade students. Students entered the room at ease, casually talking with one another and knowing exactly what was expected of them. Students immediately began working on their tasks and approached Madelyn if a question arose. Madelyn's survey results showed her scoring high in all areas recorded.

Espoused Disposition. Madelyn's qualitative results showed that student learning, student centered, and open classroom were the most coded nodes, with curriculum knowledge and teacher accepts responsibility, accounting for similar values. Her classroom structure differed because of the manner in which she incorporated both knowledge and commitment into the classroom environment allowing components of problem solving and democratic education to infuse the educational setting. All student responses were validated and Madelyn pushed them to fully articulate their understanding before proceeding.

Madelyn, as an instructor, sees herself as an agent of change. One of the first instances occurred when asked about student learning, she stated that,

> I believe in experiences. Students learn from experiences and experiences that we chose to have our students work through is, helps either to cement their knowledge in a way that is easily accessible to them or makes it knowledge that just kind of there for a moment and then gone. So, I think a teacher that provides rich experiences is just helping their students have knowledge at a deeper level.

When asked about her sphere of influence, Madelyn responded:

> I think my attitude towards what we do in class and towards students individually affects them as they're learning in class...I really, really try to accentuate the positive so students know when they walk in here this is a safe place... you know my goal is to have you try to make improvements for where you are.

Madelyn appeared to find her own actions and reactions to student learning unequivocal. Looking through the lens of agency and knowledge, Madelyn recognizes that her task is far reaching. She acknowledges that mathematics "is essential because, the processes involved in math are processes that a person needs to have in their life" but also recognizes that "students do need to know that they have ownership as well." She identified herself as operating under the constructivist paradigm adhering to the notion that teachers support learning.

KEY FINDINGS

Two major discussion points organize the key findings section. The first finding addressed is the positive correlation between teacher efficacy, mathematic teaching efficacy and overall responsibility. The second finding examines how a teacher's sense of efficacy and mathematics teaching efficacy can moderately predict overall sense of responsibility.

EFFICACY AND OVERALL RESPONSIBILITY

The quantitative data indicates a positive relationship existing between efficacy and responsibility. As a teacher's sense of overall responsibility increased, his/her teaching efficacy also increased. Sam and Anna, who self-reported low teaching efficacy and low overall responsibility, aligned themselves with a more accountable sense of responsibility. The qualitative data from the interviews and observation establish that these two teachers also held a teacher driven instructional practice, falling prey to district pressures for achievement and were, therefore, unable to address individual learner needs.

EFFICACY AS AN OVERALL PREDICTOR OR RESPONSIBILITY

The study's results point to the notion that a person's efficacious feelings, whether related to self, teaching or mathematics, impact behavior and classroom context. This key finding suggests that efficacy may be a valid predictor for responsibility. When mathematics teaching efficacy and teaching efficacy were regressed against overall responsibility, a moderate ability to predict results was recorded.

One explanation is that those individuals who fail to understand the fundamental process of engaged pedagogy lack the skills necessary to empower children through the process of learning. The act of self-actualization requires responsibility to one's person; therefore, if a person's sense of responsibility is wavering, he or she is unable to move forward in personal growth, inhibiting the acts and processes they chose to incorporate in a classroom context.

FUTURE RESEARCH

This project sought to begin a dialogue that has not been addressed sufficiently in mathematics education literature concerning teacher agency as it relates to student achievement. To provide children the opportunity to compete in a global society, teachers must fully understand their level of responsibility in the classroom, and their sense of agency. Continued research will attempt to succinctly define responsibility in the mathematics classroom and advance the framework of Academic Agency.

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THINKING FORWARD: USING STORIES FROM THE RECENT PAST IN MATHEMATICS EDUCATION IN ENGLAND

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In mathematics education, the neoliberal project has been extraordinarily successful in England, and beyond, in framing how we currently think and work. In this paper we explore the role of radical history in supporting action in the present to overcome neoliberalism's hegemony and consider how it might enable shaping an alternative vision for the future. We describe a project with which we are presently engaged which looks back to the Smile curriculum development project (1972-1990) and uses both historical archive material and current remembered accounts to provide web-based resources intended to provoke critical thinking. In particular, we discuss the use of evocative, shared, personal stories in achieving these ends.

INTRODUCTION

Dominant discourses largely hold sway through instilling the conviction that what they propose is natural, common-sense and the only possible way for things to be (Gramsci, 1971). In England, and beyond, neoliberalism has been extraordinarily successful in this respect. Indeed, Perry Anderson goes so far as to say 'there are no longer any significant oppositions -that is, systematic rival outlooks- within the thought-world of the West' (2000: 13). This discussion paper considers one of the consequences of this and argues for a particular strategy which we can adopt in the struggle for more democratic and socialist thinking in mathematics education.

We have come to the view that neoliberalism both silences our histories and excludes any possible futures except its own. It shuts us off from imagining a different world - neoliberal capitalism represents "the end of history", with no further development possible in social and political thought. And it cuts us adrift from our past and de-historicises our lived experience of the present. John Berger (2016) uses the metaphor of *no-fixed-abode* or homelessness to capture the experience of the absence of a sense of history:

> Any sense of history, linking past and future, has been marginalised if not eliminated. People are suffering a sense of historical loneliness. (p. 17)

This is as true in mathematics education as in the wider social and political sphere. To take just one example, we have been startled to discover that, for those who have entered the profession in, say, the last fifteen years, the idea both that teachers might be trusted to write their own examination papers for the public school leaving examination, as they were in England in the 1970s -and also that widespread cheating did not take place- seems unbelievable.

In this discussion paper, we seek to explore the contribution of historical awareness to a radical politics of mathematics education. In particular, we consider how to use history to engender a radical consciousness and a critique of contemporary neoliberal educational discourses and to provide a meeting place (Berger, 2016) from whence to understand, interrogate and oppose the dominant discourses currently shaping society.

In the first section, we begin by providing the background and context for the discussion which follows, giving a brief overview of the current neoliberal social and political context in education in England and considering its impact on teachers' sense of self. Next we introduce the reader to *Smile Mathematics*, a secondary mathematics curriculum development project of the 1970s and 1980s which forms the vehicle for our thinking about the significance of history. Lastly in this section, we briefly describe a research and action project based on the history of *Smile* which will produce a web resource to include amongst other material retrospective stories based on extended conversations with a number of participants who were involved in *Smile*. In the second section, we discuss the radical history tradition; our own deep investment in the stories being told; and the risks and "pay-offs" of telling historical stories. We work with the notion that history is about the present and argue for the significance of the everyday. In our conclusion we acknowledge the utopian nature of the whole enterprise.

BACKGROUND AND CONTEXT FOR THE DISCUSSION[1]

The sense that viewing contemporary mathematics education through the lens of recent relevant history might contribute to achieving critical distance from the current neoliberalism in education led us to initiate a project to explore this possibility: *A study of teacher led curriculum change: the case of* Smile Mathematics*, 1972-1990*. The aims of the project are to record an historical example of teacher-led curriculum change in mathematics in England; to analyse how and under what social, political

1. A more extended version of this section can be found in Povey, Hilary & Adams, Gill with Everley, Rosie (2016) "Its influence taints all": mathematics teachers resisting performativity through engagement with the past. Paper presented for 13th International Congress on Mathematical Education, Hamburg, 24-31 July 2016.

and cultural circumstances such teacher autonomy becomes possible; and to provide a 'public resource' (Nixon, Walker & Clough, 2003: 87) to review current practices and understandings.

Before describing the project, we give a brief overview of the current social and political context in England and its impact on teachers' sense of self. The impact of neoliberal thinking on education in England is well known (for example, Ball, 2003; Day & Smethem, 2009; Groundwater-Smith & Mockler, 2009; Stronach, Corbin, McNamara, Stark, & Warne, 2002; Macpherson, Robertson & Walford, 2014). An 'epidemic of reform' (Ball 2003: 215) based on constant surveillance and ranking of performance has changed who teachers are as well as what they do, with education recast as a consumer good rather than as a moral enterprise and a public service. The independent thinking of teachers is challenged, as are their individual and collective professional and personal identities. Systems of testing and auditing shape, order, position and hierarchise those in the field (Sachs, 2001) through systems of comparison, evaluation and documentation, making everything calculable:

> it is impossible to over-estimate the significance of this in the life of the school, as a complex of surveillance, monitoring, tracking, coordinating, reporting, targeting, motivating (Ball, Maguire, Braun, Perryman & Hoskins 2012: 525).

Currently, in England, pupil performance in mathematics examinations at age sixteen usually operates as the single most important item of data in judging and ranking (and then punishing) secondary schools, with mathematics teachers therefore routinely experiencing greater pressure and coming under more scrutiny than most, if not all, of their colleagues. We are indeed in the grip of the terrors of performativity and are engaged in a struggle over the (mathematics) teacher's soul (Ball, 2003).

Faced with this struggle, Judyth Sachs calls for teachers to take on an 'activist identity' (2001), one which arises from democratic discourses and has social justice at its heart; and Ian Stronach and colleagues have called for teachers to 're-story' themselves (Stronach, Corbin, McNamara, Stark, & Warne, 2002: 130) in ways which challenge their positionings by neoliberalism. Many teachers are engaged in this re-storying in a variety of ways:

> not all teachers are convinced by the rhetorics of performance, and many teachers are not convinced all of the time. (Ball, Maguire, Braun, Perryman & Hoskins, 2012: 588)

It is in support of this process that we are proposing a role for historical "companionship", overcoming the destruction of organisational memory (Goodson, 2014).

Here we will say a little about the mathematics education project the history of which we are exploring. *Smile Mathematics* was a secondary mathematics curriculum development project initiated by teachers and funded and supported by the Inner London Education Authority's (ILEA). It came into being in the 1970s, a time of reconstruction in the English school system characterised by a commitment to social justice (Goodson, 2014). It stood for all attainment teaching, teacher creativity and an investigative, problem-solving pedagogy. It saw itself as learner centred and gave considerable responsibility to students for organising and shaping their own learning and that of their learning community.

Teachers were released from school one day a week over many years to form a working collective to create, refine and publish materials for use in their own classrooms and beyond. The structure of the project instilled a deep democracy, with decision making resting with a consensus of those who participated. Against the advice of the ILEA Chief Inspector for Mathematics, who advocated a more traditional democratic structure, *Smile* adopted an open authority structure which placed the teacher at the heart of decision making - and was allowed to do so. *Smile* afforded opportunities for democratic professionalism, a concept which has collaboration at its core (Whitty, 2006).

In the current project, we are creating an archive using digital media which will include archive material and both accounts written contemporaneously with *Smile* and recent accounts which have been generated as a result of the project. We intend to create a 'systematic narrative' based on historical documents (Goodson, 2014, p. 34-35), some already archived and some collected as part of the project. This will provide a framework within which to view the recently generated material.

In this paper, the focus of our attention is on the retrospective accounts. Our main source for this has been three extended conversations which we have conducted with groups of *Smile* teachers from an earlier era. We also participated -both of us were/are *Smile* teachers. The conversations involved between six and eight participants each including ourselves and each lasted around three hours. They were audio recorded and have been transcribed. We have also conducted an individual interview and participants have been encouraged to provide further personal commentaries and archive material. It is the vividness, the strong sense of lived experience, the humour and vitality of these texts, that will support the potentially transformative function we have in mind.

DISCUSSION

So, we have become interested in the idea that adopting an historical perspective can make more apparent the fact that different regimes of truth hold sway at different times and in different places (Hall & Noyes, 2009: 851). Once this is recognised, the current taken-for-granted can be more easily seen for what it is: a temporary set of hegemonic assumptions, historically contingent and fragile (Ball, Maguire, Braun, Perryman & Hoskins, 2012: 514); and a set assumptions that serve some people's interests more than others. However, reweaving the threads of significance between then and now is a delicate and difficult task (Anderson, 2000: 13-14).

We are not historians; but we do not think that this, in and of itself, should disqualify us. We have found helpful in trying to characterise our historical endeavour ideas which inform the Radical History Workshop movement, in particular, the thinking of Raphael Samuel and his seminal paper *On the methods of History Workshop: a reply* (1980). Like them, we want a space in our writing for political or moral commitment. The radical history movement grew out of a desire of a group of historians to challenge the conventional academic treatment of history: ' an academic mode in which the historical subject was subsumed in the methodological preoccupations of the historian' (164). Rather, the subject was herself / himself allowed a voice and encouraged to speak. We see ourselves as relatively untrained "barefoot" historians whose authority comes from knowing the terrain.

Thus, we are deeply invested in the subject of study. The radical history tradition does not describe itself as a value free social science nor claim historical neutrality; rather it asserts that truth is partisan and is a weapon in the battle of ideas (Samuel, 1980: 168) - 'so far from attempting to bury our beliefs, or to claim that they did not exist, we have preferred openly to proclaim them' (168). However, our involvement with our data does mean that we have to be constantly alert to the possibility of reading it in ways which just suit our own purposes and our sense of our own personal and professional histories; or which sentimentalise; and we need to acknowledge discontinuities, ambiguities and contradictions in our data. The intention is not to valorise the past (although we do believe that the period under study was a time of more democratic mathematics education, sometimes informed by stirrings of socialism) but to use it to see the present more clearly.

Alongside other forms of representation (archive material, edited transcripts, aphoristic fragments), we intend to tell stories from the past. Doing so will inevitably implicate us in the conventions of this particular genre for representing reality (Stronach and Maclure, 1997: 49), leading us to 'resolve contradictions, smooth over inconsistencies and achieve a sense of closure' in our stories (53). We accept this limitation and hope to employ textual features so that visitors to the website are alerted to it. We intend our stories to be honest and to be faithful to our material and to our interlocutors; but we also want them to "perform" in the ways we have suggested above for those who visit the website.

Indeed, we claim that our use of stories helps make explicit the fact that 'the past is constituted in narrative' (Hodgkin & Radstone, 2003: 2): it is always a representation, a construction, and can never be the "original" of the phenomena being studied (Passerini, 2000: 134). Any account privileges some things and ignores others:

> As in a still life picture, some objects will have been blown up out of all proportion, others reduced in scale, while the great majority will have been crowded out of the frame. Historians thus do not reflect the past -they signify and construct it; meaning is in the eye of the beholder. (Samuel, 1980: 171)

It is this provisionality which is foregrounded in creating history as the remembered. Recounting memories gives a 'more cautious and qualified relation to the past than the absolute assertion that for some is associated with history' (Hodgkin & Radstone, 2003: 2). Much recent memory work has been concerned with traumatic experiences, ones which have been either previously hidden from historical accounts or recounted from perspectives other than those of the survivors. In these cases, the intention is to contest the accepted -or, at least, the previous- version of history. Our concerns are rather different. There is an occasional negative reference back to the period we are studying but in general it is the absence of any account at all that is so evident. The traces of the past are unseen, ignored and forgotten (Passerinin, 2000: 135).

Central to our enterprise is the contention that 'history is about the present' (Hodgkin & Radstone, 2003: 1). We argue that to look backward is not backward-looking but, rather, forward-looking. Our use of history in this project is 'present-minded' (Samuel, 1980: 168). The accounts from *Smile*, both archived stories and stories created by shared memory, 'historicise understanding of the present' (169), shedding light on contemporary realities in mathematics education. We assert that 'our understanding of the past has strategic, political, and ethical consequences' (Hodgkin & Radstone, 2003: 1). The meaning with which we invest the past is also the meaning with which we invest the present and shapes how we take the past and the present forward:

> an understanding of subjective experience and everyday social relationships can be used to pose major questions in politics and theory, and to transform our understanding of some of the leading phenomena of our time. (Samuel, 1980: 173-174)

We note here the significance that is attached to the "everyday". Although we are committed to interrogating the "big story" of neoliberalism, our stories will do this through evoking a different way of being in the world from that offered by the current neo-liberal project, presenting as it does the individual before the community and reducing educational relationships to ones of exchange value. If our stories are to realise the power we invest in them, they will need to recover 'the texture of daily life in the past' (Samuel, 1980: 172). We note that such memories are a fragile possession that will vanish with the bearers unless set down and recorded; but that 'detached from the self who remembers, memory can become a property to be inherited and passed on' (Hodgkin & Radstone, 2003: 10).

CONCLUSION

We appreciate that there are real dangers in our project.

> Memory, because of its powerful pull towards the present, and because of its affective investments, allows more readily for a certain evasion of critical distance. (Hodgkin & Radstone, 2003: 8)

We want our stories to summon up the sense of a life being lived, of the personal and of subjective experience. But it is essential for our project that it also enables critical distance and an acknowledgement of the political dimensions of the past, present and future. We want to find and walk along the fine line between affective, empathetic engagement and a robust commitment to acting for the future. We have written elsewhere about our 'enchantment' with our data because it 'allows recognition of the excessive, the ebullient, the vivid and the felt' (Burnett & Merchant, 2016: 30); but we do not want - and argue that the enchantment does not need to be - an empty nostalgia that entails no practice. Timothy Bewes (2002) describes nostalgia as being a one-way relationship to the world, its typical effect being to 'reify the past into a frieze of clichés, incapable of releasing inventive action in the present' (172). One of the striking things about the research conversations which generated the stories was a strong sense of a continuing engagement by the participants with the politics of the present. This came across in both positive and negative reflections on our shared past. These people looked back to what was in general conceived of as a better place but with a continuing commitment to a better future.

We acknowledge that our intentions are utopian. Luisa Passerini (2000) has written about the utopian use of history:

> not in the sense of a ready-made scheme projected onto the real world, but rather as a criticism of existing conditions springing from an intuition of changes potentially immanent in the present. (138)

It is precisely this sense of using history to see the possible held within the present that has motivated us in our work. In the search for alternatives, both other places and other strands of thought lying outside the ones to which we have become habituated can work to challenge the discourses that dominate us and, despite our best endeavours, shape our thinking. We have argued here that *other times* can do this too. But the past alone, of course, is not itself agentic.

> Only a genuine transformative passion can weave lost experience into the finding of a more liveable future. (Bewes, 2002: 172)

It is in the quest for a more liveable future that we find our purpose. On a personal level, we have found that our engagement with this history project has given us greater clarity about the English mathematics education present (and reinforced hope for and commitment to a better mathematics education future); for us, it has reduced our historical loneliness. At the time of writing, we are about to begin work on the website and to struggle to find ways to make our historical material "work" for those who were not there.

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CREATIVE INSUBORDINATION ASPECTS FOUND IN ETHNOMODELLING

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A dilemma in mathematics education is related to its overwhelming bias against a local/cultural mathematical orientation in most research paradigms. A search for innovative methodologies as found in ethnomodelling is necessary to record historical forms of mathematical ideas, procedures, and practices developed to be used in diverse cultural contexts. Ethnomodelling is not an attempt to replace global academic mathematics, however, at the same time, it is necessary to acknowledge the existence of local mathematical knowledge as well as its dialogical approach. The insubordination as triggered by ethnomodelling is creative and evokes a disturbance that causes a review of rules and regulations in the mathematical modelling process by applying an ethnomathematical perspective in the development of this approach.

INTRODUCTION

The acknowledgement of the relation between culture and mathematics can be interpreted as a reaction to *cultural imperialism* that has imposed its version of mathematical knowledge on colonized cultural groups around the world with the expansion of the great navigations from the fifteenth century (D'Ambrosio, 1985). In order to be in resonance with contemporary Western developmental models, other cultures have been forced to adapt to these paradigms or perish. Hence, mathematics perpetuates to some extent imperialist goals, thus, it can be perceived as a secret weapon that maintains the imposition and domination of Western cultural values on the development of local cultures (Bishop, 1990).

School mathematics has also been criticized since it helps to reinforce a capitalistic-western and Eurocentric[1] approach to prevailing school curricula as well as for helping to globalize particular kinds of mathematical technologies and ideologies (D'Ambrosio & D'Ambrosio, 2013) that support the maintenance of cultural imperialism. However, the development of non-prescribed[2] strategies used to solve problems in diverse societal

1. In this paper, Eurocentrism is considered as the conscious or unconscious practice of placing emphasis on European or Western concerns, culture, and values at the expense of the members of other cultures.

2. Non-prescribed strategies cover a range of procedures and techniques that allow members of distinct cultural groups to determine and communicate solutions to problems they face daily without the imposition or enforcement of specific rules or methods. It is

domains is an alternative method as well as an important tool for identifying innovative problem-solving techniques and mathematical ideas, procedures, and practices in ethnomodelling research.

The reaction to this cultural imperialism can be also related to the development of the concepts of *creative insubordination*[3] (Crowson & Morris, 1982), *responsible subversion* (Hutchinson, 1990), and *positive deviation* (Zeitlin, Ghassemi, & Mansur, 1990). These concepts are equivalent as they "relate to the flexibility of rules and regulations in order to achieve the welfare of the members of distinct cultural groups" (Rosa & Orey, 2015a, p. 133). These three terms can be applied interchangeably because the amplitude of their concepts embraces innovative solutions in the pedagogical action of ethnomathematics through ethnomodelling, which helps to confront the belief that persists in contemporary society that mathematics is a culturally neutral knowledge.

The historical and contemporary relations between culture and mathematics illustrate that this knowledge area is not culture-free. Accordingly, the culturally specific nature of mathematics should be acknowledged in order to describe mathematical ideas, procedures, and procedures practiced among the members of distinct cultural groups such as tribal societies, labor groups, professional groups, social classes, and children of a certain age group (D'Ambrosio, 1985). Consequently, it is important to search for alternative methodological approaches as Western mathematical practices are accepted worldwide in order to record historical forms of mathematical ideas, procedures, and practices that occur in different cultural contexts. Therefore, the members of distinct cultural groups apply innovative mathematical solutions to the challenges faced by society, which are identified and refined from the ideas, procedures, and practices they develop from generation to generation.

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a process in which they come up with different mathematical tools that can help them to organize, analyze, comprehend, understand, and solve specific problems located in the context of their real-life situation. These tools allow them to identify and describe specific mathematical ideas, procedures or practices in general contexts by schematizing, formulating, and visualizing a problem in different ways in order to discover patterns, relations and regularities (Rosa & Orey, 2010).

3. The concept of creative insubordination was elaborated by Crowson and Morris in 1982 to describe how sch. administrators (principals and vice-principals) circumvent or make rules and institutional norms flexible in order to better serve the needs of their students, teachers and parents. These professionals use alternative and creative ways so that they are able to reach good results for the common good of the school community through the adoption of anti-bureaucratic behaviors. It is important to highlight that the concept of creative insubordination arose in the 1970s, aiming to make positive changes in public health policies by applying practices developed by the members of local cultural groups. Although most of the research and investigations carried out were related to nursing practices and in the management of school systems, this approach can also applied in any oppressive system such as education (Rosa & Orey, 2015a).

One alternative methodological approach is ethnomodelling, which may be considered as the practical application of ethnomathematics that adds cultural perspectives to the modelling process (Rosa & Orey, 2013). The "application of ethnomathematical techniques and the tools of mathematical modelling allow us to see a different reality and give us insight into science done in a different way" (Orey, 2000, p. 250). In this context, the methodological approach that connects the cultural aspects of mathematics to mathematical modelling is *ethnomodelling*.

ETHNOMODELLING AND CREATIVE INSUBORDINATION

As a creative insubordination process, ethnomodelling seeks to change the "outside existing paradigms and conflicts with prevailing values and norms" (Marzano, Waters, & McNulty, 2005, p. 113) since it stands for the development of mathematical ideas, procedures, and practices that have its roots developed within distinct cultures. In this context, ethnomodelling binds contemporary views of ethnomathematics and, simultaneously, recognizes the need for a culturally based view on the modelling concepts and processes. Studying the unique cultural differences in mathematics encourages the development of new perspectives on the scientific questioning methods.

Research on culturally bound modelling ideas may address the issue of mathematics education in non-Western societies by bringing local cultural aspects into mathematical teaching and learning processes (Eglash, 1999). This approach reveals responsible aspects of subversion in the ethnomodelling process that are identified with an ongoing movement that challenges the *status quo* of academic mathematical knowledge by aiming at to alter the system in creative ways (Lyman, Ashby, & Tripses, 2005) in order to better serve the needs of the students. Essentially, it involves looking at issues from perspectives outside of existing educational systems and pedagogical models.

Similarly, ethnomodelling can be considered as insubordinate and creative educational approach because it disrupts the existing order in an academic system, as it does not follow the linear modelling approach prevalent in many school curricula. It develops the study of ideas, procedures, and mathematical practices found in distinct cultural contexts, thus it attempts to break bureaucratic rules of academic mathematics in order to recognize different techniques and value diverse modes of producing mathematical knowledge developed by the members of diverse cultural groups (Rosa & Orey, 2015a).

This context allows ethnomodelling to challenge the prevailing traditional mathematical ways of thinking since it works as a positive deviance approach that involves thoughts and/or actions that differ *from* the imposed norms and regulations (Dehler & Welsh, 1998). From the anthropological and sociological points of view, this act of responsible subversion examines how individuals solve problems in spite of or in opposition to the formal system or the commonly accepted rules (Hutchinson, 1990). Historically, mathematical knowledge takes different forms in different cultures. Western worldviews on the ideas of modelling begin to shift in order to acknowledge that this process in the ethnomodelling-based curriculum is culturally bound.

AN ETHNOMODELLING-BASED CURRICULUM

The concept of positive deviance is useful, offering researchers and educators alike a basis for decision making when expected actions collide with perceptions regarding mathematics curriculum. It involves an intentional act of breaking some curricular rules in order to serve the greater good of students. Researchers and educators who are positive deviants must question and discuss the opposing *status quo* of mathematical knowledge in order to enact meaningful changes into mathematics curriculum. The main objective of this positive deviance procedure that departs from established norms or rules is to modify these regulations by applying innovation, creativity, and adaptability (Walker, 2005).

One of the goals of ethnomodelling-based curriculum is to add cultural components to the modelling process. Hence, instead of being another research paradigm itself, ethnomodelling aims at encouraging the search for mathematical ideas, procedures, and practices that are culturally bound as well as to examine their adoption into the mathematics curriculum (Rosa & Orey, 2015b). Traditional mathematical modelling methodologies in school curricula do not fully take into account the implications of the cultural aspects of local systems.

Mathematical curriculum conceived in an ethnomodelling approach helps students to develop mathematical concepts and practices that originate in their own cultural traditions by linking them to formal academic mathematics. The understanding of conventional mathematics then feeds back and contributes to broader understandings of culturally based mathematical principles (Rosa, 2010).

Classrooms should not be isolated from the communities in which they are embedded, thus, they are part of a larger community with defined cultural practices. In this context, classrooms may be considered as learning environments that facilitate the application of pedagogical action developed through the application of an ethnomodelling-based curriculum, which allows for a broader analysis of the school context in which pedagogical actions transcend school environments since curricular practices embrace sociocultural contexts of students.

The objectives for developing an ethnomodelling-based curriculum include: a) to assist students to become aware of how people mathematize and think mathematically in their own culture, b) to use this awareness to learn about formal mathematics, and c) to increase the ability to mathematize mathematical practices in distinct cultural contexts. This curriculum applies cultural experiences as vehicles to make mathematics learning more meaningful and, more importantly, to provide students with the insights of mathematical knowledge as embedded in their own sociocultural environments (Rosa & Orey, 2007).

This curriculum leads to the development of cultural activities that enable students to become aware of the mathematical potential found in their own communities and cultural traditions so that they are better able to understand the nature, development, and origins of academic mathematics (Rosa & Orey, 2010). This implies that an ethnomodelling-based curriculum is not just about the application of relevant connections in learning mathematics, but it is also about generating mathematical knowledge from cultural ideas.

The comprehension of the acts of creative insubordination in the process of learning mathematics generated from ethnomodelling processes enable the development of teaching strategies that help researchers and educators to apply methodological decisions related to teaching practices (Rosa & Orey, 2015a). This approach helps to improve the mathematics performance of students by modifying, adapting, and flexibilizing curricular practices.

IMPLICATIONS OF AN ETHNOMODELLING-BASED CURRICULUM

One of the primary issues regarding the mathematics curriculum is concerned with the position of both researchers and educators in relation to the global and local approaches. In this regard, the pedagogical work on mathematical content developed in classrooms may be based on the researchers and educators' own worldviews, which relates to culturally-universal, culturally-specific, or culturally-dialogical approaches to mathematics education.

Both researchers and educators who operate from more positions that are more global have been taught to perceive that mathematical ideas, procedures, and practices occur the same way in every culture. Ethnomathematics tells us that there is a beautiful diversity in the way people have come to order, count, classify, infer, and model. However, researchers and educators can learn how to base their beliefs in relation to Western versus non-Western mathematical traditions in which the members of distinct cultural groups have come to construct, develop, acquire, accumulate, and diffuse the same kind of mathematical knowledge (Rosa & Orey, 2013).

Researchers and educators who connect to local/cultural perspectives believe that many factors come into play when mathematical ideas, procedures, and practices are developed in regards to the cultural backgrounds of the members of distinct cultural groups. These factors include the diverse sociocultural values, morals, and lifestyles of their learners and communities. For example, different cultures have developed different ways of doing mathematics in order to understand and comprehend their own cultural, social, political, economic, and natural environments. It is necessary to highlight that students also operate from local approaches.

Since these professionals have come to believe that cultural backgrounds and life experiences greatly influence the overall development of the mathematical knowledge of learners, they are able to use culturally specific strategies in their teaching and learning practice. Therefore, they come to see that current worldwide guidelines and standards for mathematical instruction are very much culturally bound (Rosa, 2010). It is important that both researchers and educators understand that the diverse experiences, lifestyles, cultural values, and overall worldviews influence the development of mathematical knowledge (Rosa & Orey, 2013).

Other issues necessary to discuss here are related to the belief systems of researchers and educators in relation to cultural universality, especially that which focuses on similarities and minimization of cultural factors. Characteristics of dialogical approaches in ethnomodelling research provide the conditions in regards to the development of intercultural competence, which are the "ability to communicate effectively in cross-cultural situations and to relate appropriately to a variety of cultural contexts" (Bennett & Bennett, 2004, p. 149).

Dialogical approaches (glocalization) of ethnomodelling help us become more mindful of forms of the hegemony prevalent in mathematics classrooms. Thus, it is necessary to incorporate cultural-based forms of knowledge and the notion of these approaches and the continuous changes that arise in the process of teaching and learning mathematics. It is necessary to state here that we are not enforcing another form of dualism, which is globalization versus localization. Indeed, our intention is to contest the narrow view of globalization that allows for new or alternative traditions and developments of mathematical ideas, procedures, and practices (Rosa & Orey, 2015a). This is important in order to demonstrate how global approaches that may not necessarily be exclusive construct rather than coexist with localization.

It is important to highlight here that the dialogical approach is a reaction to globalization, or a reinforcement of cultural identity of the members of local communities. This means that, in the ethnomodelling processes, mathematical knowledge might consider the worldwide connections, but also with the specific conditions of the local knowledge. This aspect of insubordination in relation to ethnomodelling is creative as it allows participants to adapt rules in order to change, challenge, or even subvert the regulation of the implementation processes of the norms. This context allows for the development of the learner's own cultural competences, which are abilities to "develop targeted knowledge, skills and attitudes that lead to visible behaviour and communication that are both effective and appropriate in intercultural interactions" (Deardorff, 2006).

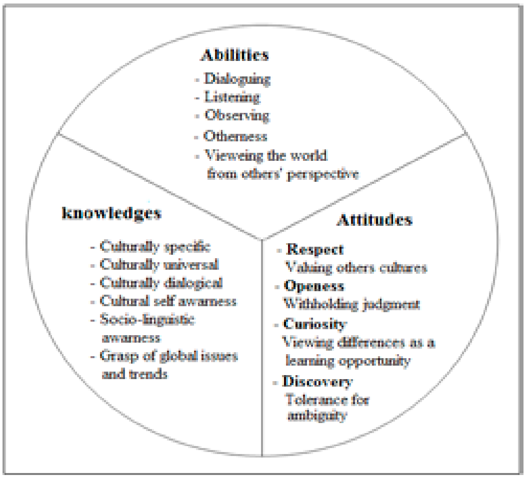

Figure 1: Intercultural competence in the ethnomodelling process adapted from Deardorff (2006)

The question is, then, how necessary it is to understand cultural specificity (local) against the background of universal theories and methods (global) that are susceptible to cultural differences and cultural contextualization. Results from culturally specific investigations encourage more cross-cultural research that supports the development of local perspectives (D'Ambrosio, 1985; Eglash, Bennett, O'Donnell, Jennings, & Cintorino, 2006; Rosa & Orey, 2010).

Insubordination strengthens the notion that mathematics cannot be conceived as universal because its principles, concepts, and foundations are not the same everywhere (Rosa & Orey, 2007). Conversely, it is naïve for us to state that members of distinct and diverse cultural groups do not share universal mathematics ideas, thus, some mathematical activities are widely practiced across cultures and are basic to the human condition.

As mentioned earlier, many of the everyday activities members often perform daily involve a substantial amount of mathematical application. In this context, counting, measuring, designing, locating, explaining, and play are six universal activities that are practiced by members of every cultural group. These universal activities provide fundamental facets used to probe traditional daily living practices (Bishop, 1993). It is also important to highlight that these activities have become intertwined with other aspects of daily routines of these members.

Even though these activities are often thought of as universal, it is important to recognize that they are merely universal to those individuals who share the same cultural characteristics and historical perspectives. On the other hand, Rosa (2010) argues that it is equally naïve to believe that *universal* mathematical ideas and procedures do not reflect all cultural values and lifestyles of learners.

By applying an *innovative insubordinate* pedagogical approach to these opposing views, it may be helpful to understand the universality of mathematical ideas, procedures, and practices that could be relevant to researchers, educators, and the members of a specific cultural group (Rosa & Orey, 2015b). Hence, it is necessary that this approach takes into consideration the relationship between cultural norms, values, attitudes, and the manifestation of mathematical ideas, procedures, and practices developed in different knowledge fields as well as in the context of distinct cultures.

In this direction, Rosa (2010) states that if researchers and educators become self-aware of themselves and their worldviews, cultural paradigm, and values, then they may be more open to apply aspects of ethnomathematics and modelling in their pedagogical practices through the application of ethnomodelling.

FINAL CONSIDERATIONS

The tragedy of the impending disappearance of local knowledge is most obvious to those who live it, but the implication for others can be detrimental as well when mathematical procedures and techniques, technologies, artifacts, and problem solving strategies are lost during the development of contemporary society. Defined in this manner, the usefulness of both local and global knowledge is evident and necessary to the development and implementation of the school curricula. Thus, one possible reason for many failing educational systems around the world could be that both policy makers and curriculum developers have ignored local approaches in relation to school curricula, especially when it suggests the recognition of other epistemologies and of holistic and integrated natures of the mathematical knowledge developed by the members of other cultural traditions. It has been hypothesized that low attainment in mathematics could be due to lack of cultural consonance in the mathematics curriculum (Rosa, 2010).

When students come to school, they bring with them the values, norms, procedures, and techniques that they have acquired in their own sociocultural environment and some of these elements are mathematical in nature. However, mathematical concepts of the school curriculum are presented in a way that may not be related to their cultural background. Moreover, the inclusion of cultural aspects in the mathematics curriculum has long-term benefits for mathematics learners (Rosa & Orey, 2010). Current mathematics curriculum lacks a dialogical approach in regards to the preparation of students for a living in dynamic and diverse societies. The lack of awareness of local knowledge and the alienating effects of Western educational norms indicates a need for the development of an ethnomodelling-based curriculum.

Mathematical knowledge created based on local approaches is a form of intellectual decolonization, and provides us with a major contribution to mathematics education and in the development of contemporary society. Ethnomodelling becomes the “joining of a new field of inquiry, which might be called communal transformation” (Block, 2010, p. vii). An ethnomodelling-based curriculum provides a theoretical basis for the teaching and learning process because it combines key elements of local knowledge with a global approach. The main goal of ethnomodelling is the acquisition of both local and global knowledges by applying dialogical approach.

Similarly, it is possible to define dialogical approach “by the social actor’s fluid and critical engagement with, and reconstruction of, local and global phenomena” (Giulianotti & Robertson 2007, p. 173). When analyzing the diffusion of mathematical knowledge, it is necessary to look at the local and global reconstructions together. Therefore, cultural aspects contribute to recognizing mathematics as part of daily life, enhancing the ability to make meaningful connections, and deepening the understanding of mathematics (Rosa, 2010).

In this context, creative insubordination is necessary to serve students’ learning (Ayers, 2001), which is the main concern of the educational system and, then, rules and norms may need to be bent to achieve this goal. In this context, it is important that researchers and educators address student’s cognitive and pedagogical needs into the mathematics curriculum through ethnomodelling.

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COMPETENCE-BASED TEACHER EDUCATION – REVISITED

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The research discourse on mathematics teacher education in Germany is strongly influenced by the idea of competence-based teacher education. This contribution seeks to critically explore what the notion of competencies tells about teacher education at university. For this purpose, the dominant notion of competence in the German discourse will be presented and the analytical lens derived from German critical psychology and used to examine this notion in a socio-political manner will be made explicit. In revision, the competence discourse relocates challenges that are inherent to mathematics and mathematics education into the individual learners' private sphere and obscures inherent contradictions.

INTRODUCTION

The term competence-based teacher education goes with the promise of providing a strong focus on the individual learners' need and a link to the teachers' professional practice. These two issues are seen as major challenges in the current teacher education system in Germany. At least in Germany, the scientific discussion is strongly influenced by the idea of competence-based teacher education.

German teacher training is split into two phases; the first phase takes place at university and the second phase is a practical training which takes place in school [1]. Hence, the university phase has a strong focus on building up a substantial knowledge base. It is important to note that the teacher students usually make little actual teaching experiences; they are not in school-practice[2]. When enrolling at university, teacher students choose two school subjects and the school type (Primary and lower secondary school, secondary school, vocational school) they want to teach later on. Studies are then split into elementary (for future primary and lower secondary school teachers) or university math (for future secondary or vocational school teachers), math didactics, general didactics, the other chosen subject and its related subject matter didactics. Math didactics is usually presented as a bridge between math courses and general didactics

1. Teachers on probation are supervised by specific seminars outside the university.

2. Even in practical phases in the first phase of teacher education, teacher students might only teach a few lessons.

courses. It is a debate what constitutes the knowledge base of math didactics, rather subject-oriented didactics or didactics oriented on empirical research findings (Jahnke, 2010).

The competence-based teacher education paradigm claims to shift the focus, already in the first phase of teacher education, away from a discipline-based oriented teacher education towards an orientation that takes into account the learner's need by defining concrete learning outcomes that are relevant for professional practice. The crack between theoretical knowledge and practice, that seems to be institutionalized in the two different phases of German teacher education, shall be closed by orienting teacher training on competencies.

In aiming at understanding the learning of teacher students in the institutional setting of university, this contribution seeks to explore what can be learned by this specific discourse about the learning of teacher students and also wants to point out what is missing in the competence-based approach. First, the dominant notion of competence in the German discourse will be presented. Second, an analytical lens that conceptualises the relation between the individual and societal and political structures will be explained as it is believed to be crucial for understanding the following, third, revision of the competence discourse.

ON COMPETENCE-BASED TEACHER EDUCATION

The call for a competence-based teacher education is omnipresent in the German discourse. The most dominant approach is strongly oriented on the framework of the international study TEDS-M (Blömeke, Kaiser, & Lehmann, 2010; Tatto & Senk, 2011) and the broader teacher competence framework by Baumert & Kunter (2006).

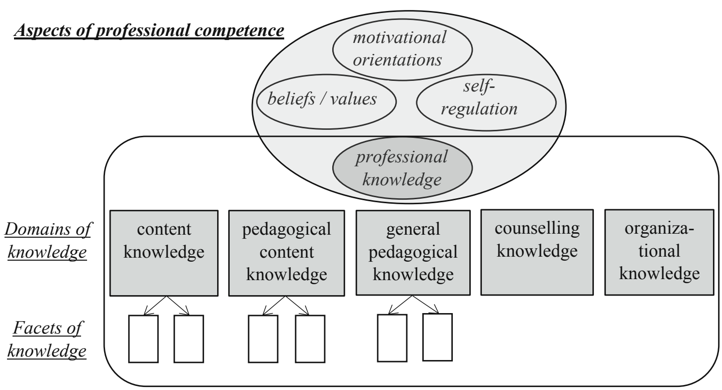

Figure 1: Competence Model by Baumert & Kunter

The TEDS-M framework provides an effect model that relates *opportunities to learn* to teacher students competencies which are measured by standardized test scores. The competence framework by Baumert & Kunter (2006) claims to provide a systematic overview of competence research –also referring to TEDS-M– and to clarify what facets are relevant and should be learned by teacher students. The central category is *professional knowledge*, influenced by *beliefs and values*, *motivational orientation* and *self-regulation skills*.

The core-activity of a teacher is described as "*providing lessons and systematically initiate and support students´ insightful learning*". This short description of the essential competence of teachers' activities is taken for granted by the discourse. Furthermore, it is suggested that it is possible to identify general objective categories that identify "good" teachers and those, in principle, should be measurable.

In the following, the categories and central findings relevant for German teacher education are presented in more detail.

Professional knowledge

The research on professional knowledge of mathematics teachers and teacher students refers to Lee Shulman's distinction between the following knowledge components; *content knowledge*, *pedagogical content knowledge*, *curricular knowledge* (Shulman, 1986), *general pedagogical knowledge*, *knowledge of learners and their characteristics*, *knowledge of educational contexts* and *knowledge of educational ends, purposes, and values, and their philosophical and historical grounds* (Shulman, 1987). These knowledge components are constituted by three qualitatively different forms of knowledge; *propositional knowledge*, *case knowledge*, and *strategic knowledge* (Shulman, 1986). He argues that the actual professional knowledge of teachers is greater than the sum of its parts and extends to wisdom of practice[3].

The German discourse mainly focuses on *content knowledge*, *pedagogical content knowledge* and *general pedagogical knowledge*[4]. Aspects of the other knowledge components are either subsumed under these constructs in form of sub-branches (facets of knowledge) or ignored. Specifications of *curricular knowledge*, *knowledge of learners and their characteristics* and *knowledge of educational contexts* are subsumed under the terms of *pedagogical content knowledge* and *general pedagogical*

3. The reading of Lee Shulman's work in the German discourse differs from the Anglo-American discourse.

4. These are usually similarly translated to the three parts of math teachers studies; math courses are superficially related to content knowledge, math-didactics to pedagogical content knowledge and general didactics to general pedagogical knowledge. This translation is misleading.

knowledge. *Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds* is usually not addressed and only remains in stated norms.

The aspect of mathematic-specific curricular knowledge is subsumed under the concept of *pedagogical content knowledge* in the TEDS-M study. That Shulman's essential point in describing *curricular knowledge* also contained the context-component of knowing other subjects curricula is ignored in today's notion. Additionally, the knowledge branches of organizational and counselling knowledge (cf. Rambow, & Rambow 2000) are prominent in the German discourse. These shall foster the expert role of the teacher. These two components likewise subsume aspects of *knowledge of learners and their characteristics* and *knowledge of educational contexts*. In this notion, the teacher's role and educational contexts are not seen to inherent contradictions but the teacher has to handle exterior challenges (technically). Most research has been dedicated to further differentiate *mathematical content knowledge* and *mathematical pedagogical content knowledge* (Schreiner, 2015), which are seen as essential cognitive components of mathematics teachers' professional development (Tatto & Senk, 2011).

Content knowledge: Especially *content knowledge* is placed in a prominent position in the German discourse. This is justified by the COACTIV-study which shows a strong interconnection between *content knowledge* –predominantly based on school-mathematics knowledge– and *pedagogical content knowledge* (Krauss et al, 2008). The interaction of those two sub-branches of professional knowledge in the learning process remains unclear. The TEDS-M study (Blömeke, Kaiser, & Lehmann, 2010) sees deficits in the school-mathematical knowledge of teacher students especially in relation to their comparably many *opportunities to learn* in an international comparison. Math courses at university count as *opportunities to learn*. It is not explicitly modelled in what ways university-level-math courses have an influence on school-mathematics knowledge.

Lee Shulman (1986) highlights the difference between *substantive* and *syntactic structure* concerning *content knowledge*: The *substantive structure* describes the variety of ways organizing basic mathematical concepts and principles while the *syntactic structure* refers to how truth and falsehood, validity and invalidity are established. This difference is ignored in the current discourse and is relocated to another subcomponent, described thereinafter/ further down.

Pedagogical content knowledge: The TEDS-M study attests German teacher students relatively few math-didactic courses (Blömeke, Kaiser, & Lehmann, 2010) and therefore few *opportunities to learn pedagogical content knowledge*. This finding leads to a back reference to mathematical *content knowledge* and the need for modelling its relationship with other knowledge branches. More and more sub-branches of knowledge are tried to be modelled and quantified in standardized testing. The construct of *school-related content knowledge*, for example, seeks to capture the connection between academic, school-mathematical and pedagogical content knowledge (Heinze et al, 2016).

Beliefs and values

Epistemological beliefs concerning mathematical knowledge and subjective theories about teaching and learning mathematics are seen as an essential component of the belief and value structure of the individual teacher student. Beliefs are predominantly seen as an agent of change to sustainably adjust the practice of teachers (cf. e.g., Holzäpfel et al, 2012). More and more different dimensions of beliefs are proclaimed in math teacher education research. Katja Maaß (2006) for example takes a closer look at beliefs related to the usefulness of mathematics which shall foster a teaching practice with a focus on modelling tasks and shall contain plenty of references to “reality”.

The TEDS-M study marks beliefs about the nature of mathematics, beliefs about learning of mathematics and beliefs about achievement of mathematics (Tatto & Senk, 2011) as important factors that have to be taken into account. The study states (Blömeke, Kaiser, & Lehmann, 2010) a positive relationship between a *construction view* and teacher students’ successful learning in western countries[4]. The COATIV-study (Krauss et al, 2008) correlated mathematical world views and general educational goals of mathematics teacher and also identified two dominant views of mathematics teachers in Germany: *transmission view* and *construction view*. “Inadequate” mathematical world views are used to explain deficits in building up content knowledge (Blömeke, Kaiser, & Lehmann, 2010).

Part of what Lee Shulman (1986) refers to as *syntactic structure* of *content knowledge* is relocated to the notion of beliefs and values about the nature of mathematics. The philosophical stance of what constitutes truth is seen as an individual belief about the nature of mathematics.

Motivational orientations and self-regulation skills

Explanatory variables of teacher student’s learning success are self-esteem, self- efficacy and intrinsic motivation (Blömeke, Kaiser, & Lehmann, 2010). German mathematics teacher students are identified with high instrinsic-pedagogic motivation, but a neutral intrinsic-

4. These are usually similarly translated to the three parts of math teachers studies; math courses are superficially related to content knowledge, math-didactics to pedagogical content knowledge and general didactics to general pedagogical knowledge. This translation is misleading.

mathematical motivation (Baumert & Kunter, 2006). The so-called professional motivation of mathematics teachers' students is artificially divided into a mathematical and a pedagogical motivation. The interplay of a variety of different reasons is not taken into account and the specificity of the mathematics teacher occupation is ignored.

ANALYTICAL LENSE - THE INDIVIDUAL IN RELATION TO SOCIETAL ARRANGEMENTS

A crucial point in research that considers learning in an institutionalized context is the conceptualization of the relation between the individual learner and the institution, which is standing for a specific societal arrangement. Hence, in the following, the theoretical approach providing this link is made explicit.

Following German critical psychology, the relation between the individual and societal conditions is seen as being dialectical: Human beings (re-)produce their life conditions in society and, at the same time, are subjected to their life conditions and societal arrangements. The two sides - life conditions and the individual - are seen as disjunctive categories only on an analytical level. The categories presented subsequently are disjunctive on an analytical level only; nevertheless the ascription of research findings to one category or another is crucial for its interpretation and implications.

In summarizing the German critical psychology agenda, Holzkamp (2013a) states:

> "[ ] we are attempting to elaborate this two-sided relation as an interrelationship, i.e. to analyse human beings as producers of the life conditions to which they are simultaneously subject, and to conceptualise the mediation between the vital necessities of sustaining the societal system as a whole and these necessities on the subjective level of the discrete individuals" (p. 20)

This statement entails a paradigm change: Instead of focusing on behaviour- determination [e.g. university lectures provoke learning mediated by specific mathematical world views of the individual learner] it is proclaimed to focus on meaning-reasoning-relations: Specific societal arrangements provide a landscape of objective meanings that the individual can relate to. Contents of mathematics and mathematics education courses are seen as *objective meaning structures* that involve opportunities and constraints to learning –both at the same time. These objective meaning structures are situated in the *societal sphere*, present on a manifest and/or latent level. These are recognised from the learners' own perspective, interpreted and subjective meanings are ascribed to it – relying on past and future prospects of one´s own unique biography. The individual builds up her or his own *subjective meaning-reasoning-relations.*

These relations constitutes the learners own premises for his or her actions and are located in the *private sphere* of the individual (Holzkamp, 1985)

The individual leaner is not transfigured to be purely autonomous, rationally acting on purely cognitive reasoning. The individuals' reasoning takes places in the *cognitive* and *affective-motivational sphere*. Emotions are seen as a subjective assessment that potentially alerts me about fundamental contradictions of interests –reflecting an imbalance in dominance of power– that I cognitively do not want to be aware of (Holzkamp, 1985, Tolman, 2013).

Learning is a specific form of activity initiated by the attempt of (re-) gaining control over life-sustaining conditions. To Klaus Holzkamp (1995), knowledge entails a critical emancipatory potential by capacitating the individual to actively position oneself to contradictory societal demands[5]. Yet knowledge is controlled by institutional power structures. The attempt to gain more control over life-sustaining conditions is always accompanied by the risk of getting in conflict with the agents of power and the provocation of restrictions[6] (Holzkamp, 2013a). This has to be anticipated by the individual prior to acting.

A research practice (Holzkamp, 2013b) that reduces human being's actions and their societal conditions to variables is blind for inherent coherences and contradictions of the field of interest. This blindness possibly leads to a *mistake of categories.* The term *mistake of categories* (Holzkamp, 1985) is used to mark a mistake of confusing different levels of analysis; private and societal sphere, cognitive, affective-motivational sphere.

REVISION

The TEDS-M study and the competence framework by Baumert & Kunter follow a behaviour-determination paradigm and do not state meaning-reasoning-relations of teacher students in university setting explicitly. The question arises what this strand of research can tell about ...

- ...*objective meaning structures* in mathematics education at university
- ...the individual learners' *subjective meaning-reasoning-relations*

The competence framework works on the basis of multiple mistakes of categories. First categories belonging to the private sphere are relocated into the societal while second categories belonging to the societal sphere are relocated to the private sphere. The third mistake of

5. Contradictions are not seen as negative per se. Contradictions can be both, a hindrance or an opportunity for learning.

6. These relations are specified for the specific historic conditions of capitalist societies' antagonistic class conditions.

categories is the swapping of categories that belong to the cognitive sphere to the affective-motivational sphere.

Objective meaning structures: Mathematics and mathematics teaching

Competence research depersonalises one's own learning and instead of highlighting the individual reasoning creates a self-referential system of different knowledge types [mistake of categories I]. It proclaims a universal knowledge structure, even though the interconnections are not specified. Inherent values of mathematics (e.g., abstract vs. applicable) and their contradictory relations are ignored.

Contradictions inherent in the content matter are obscured by splitting the contradictory elements into different components that constitute professional knowledge and variables that are assigned to the private sphere [mistake of categories II], determining the successful building up of these knowledge components.

On the on hand side, mathematics teacher students ought to have a mathematical world view that sees mathematics as a process and appreciates mathematics for its application in reality, on the other hand side, they shall also perceive themselves as mathematicians who love the abstract formal side of mathematics that is presented at university.

The former emphasis on understanding the knowledge organization needed for teaching mathematics is lost by particularising knowledge into different branches of studying which then follow their particular interests. The professional motivation of a future mathematics teacher shall be driven by her or his mathematics interest, highlighting the abstract logical science and the vision of a teacher being a mathematician. At the same time, the professional motivation shall also be driven by pedagogic ideals and not by “mathematical values”.

This leads to an even deeper crack between theory and practice. The theoretical knowledge organization is disrupted by superficially assigning relevant knowledge components to different categories (and branches of studying) and only fits to a practice in which the socio-political is computationally eliminated by the control of confounding variables. The practice of teachers is reduced to a quite narrow picture of teaching that is taken for granted by the discourse.

Subjective meaning-reasoning-relations: Learning and Beliefs

Societal demands –in an idealized form– are personalised and relocated in the sphere of the individual learner [mistake of Categories II]. Contradictory parts of the knowledge dimensions are relocated into the sphere of the individual and degraded to an affective-motivational conditional factor of learning [mistake of Categories III]. Through this, the knowledge is constituted as consistent in itself. At the same time, the affective-motivational sphere is only seen as an agent of change that has to be manipulated to a specific standpoint.

Different elements of mathematics that seem contradictory are split into different aspects and marked as mathematical world views. The philosophical question of what constitutes mathematics is relocated into the private sphere of the individual learner. Either the learner adapts (or already has) to the “right” view of mathematics and so becomes a successful learner, or the teacher student does not. The teacher student becomes responsible for finding a way through these contradictory societal demands. This closes a discourse –potentially beneficial for the individual’s learning process- about the nature of mathematics and also its inherent contradictions. Interestingly, inherent values of mathematics are no longer part of the professional knowledge about mathematics. They are merely seen as an individual relation to mathematics that is either a benefit or a hindrance for learning and teaching mathematics.

CONCLUSION

Competencies describe “desirable” outcomes of learning processes that are not specified in their relations to each other. Competence based teacher education can be read as an idealized vision, highlighting specific aspects of mathematics and reducing the mathematics teacher to a description of specific characteristics. For this purpose, specific societal arrangements, contradictions and different standpoints are ignored or reduced to confounding variables. The dominant competence notion in the German discourse constitutes a landscape of objective meanings that is contradictory which is masked by the creation of more and more specific competence constructs. Challenges that are inherent to the subject matter are relocated into the private and even affective-motivational sphere, closing down a discourse that might be relevant for the learning of teacher students and positioning to the discourse.

The question remains what subjective meanings teacher students ascribe to these societal demands and how they [explicitly and implicitly] position themselves to this discourse.

Furthermore, the competence discourse is not the only one that regards mathematics teacher education. How are other discourses (e.g., teacher identity, professional development) related to the competence notion and what do they tell about meaning-reasoning-relations of teacher education in university settings?

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MATHEMATICS LEARNING AND SOCIAL BACKGROUND: STUDYING THE CONTEXT OF LEARNING IN A SECONDARY SCHOOL IN A SEMI RURAL AREA OF MAHARASHTRA

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This study attempts to understand the social context of mathematics learning in the 9th standard of a secondary school in a semi-rural area of Maharashtra, India. It demonstrates that the assignment of students to divisions in the class exactly duplicates the hierarchy of castes in the area. Moreover the classroom processes –both in terms of tone of the discourse and pedagogical procedures– profoundly affect the students' learning, and reinforce the gap between the high-ranking Division 9A and the low-ranking 9D. Once students are assigned to a particular division in class 5 they stay there until they leave school. They are denied the possibility of moving into a different division or studying mathematics in English. The school in this study is a decent one. It is a typical middle level school in Maharashtra. At the same time the scenario in the school exemplifies the crisis of education in the state, and the endemic caste discrimination in the system.

INTRODUCTION

It is widely recognized that at least in many countries, poor and marginalized children's performance in school tends to be lower than that of children from better off families. In India, however, there have been few studies of the social correlates of mathematics performance.

Three studies have specifically focused on the learning of mathematics by poor and marginalised children in India. A study conducted by Farida Khan (1999) in India explored mathematical knowledge of vendors (*paan*[1] and newspaper sellers) and school children. She observed that vendors had a better understanding of mathematical word problems in addition and subtraction and had better strategies to solve them, such as regrouping of numbers, whereas school children followed the conventional algorithms given in textbooks. On the other hand, the lack of algorithms reduced the ability of the vendors to solve problems as compared to school children when the numbers are large and calculations are lengthy (Khan, 1999). In another study, Arindam Bose and K. Subramaniam (2011) studied school

1. Betel leaf (a mouth freshener).

children's everyday mathematical knowledge of number and tried to observe how students utilize this knowledge to do arithmetic operations on numbers.

Jayasree Subramanian, Mohammad Umar and Sunil Verma (2015) studied the mathematics learning achievement of socioeconomically marginalised students at the upper primary level studying in government schools and private schools catering to low income families. These students, mostly first generation learners coming predominantly from SC, ST and OBC caste categories studying in government schools, were denied the opportunity to learn mathematics because of indifferent teachers and the uncongenial physical atmosphere of the schools. Looking at the mathematical skills of students in standards 5 and 6, the authors found that many students could not write numbers, and had difficulty in basic operations such as addition and multiplication of three-digit numbers though they were fairly capable in oral computations.

Subsequently, in a longitudinal study they took up as part of Eklavya[2] to try out an alternative approach to teach fractions, the team worked with children from one of the government schools. The study reported a range of complex mathematical reasoning abilities (such as comparing fractions, representing fractions as a measure and as a number on the number line) that the children developed and demonstrated when given an opportunity learn meaningfully[3].

THEORETICAL FRAMEWORK

This paper is based on the work I did for my M.Phil dissertation for the Tata Institute of Social Sciences, Mumbai in 2013. My aim was to study the social context of mathematics learning. In that study I looked at caste, socioeconomic class and gender. For the present paper I have concentrated on caste, with some reference to socioeconomic status.

From the start of this work I have found Skovsmose's (2005) distinction between foreground and background a useful way of looking at the problem. By 'background' he means what a person has experienced and by 'foreground', the opportunities which the social, political and cultural situations provide. In terms of the Indian situation, I would say that a student's caste is one of the most important aspects of their background. In the 'foreground' I would include school policies and classroom processes (including teachers' attitudes and skills). Skovsmose's comment about what he calls a "ruined foreground" is particularly relevant to this study.

2. Eklavya is non government organization started in 1982 Hoshangabad Madhya Pradesh State. This organization is working on developing primary education.

3. http://www.eklavya.in/gallery/video-gallery/material-developed-by-eklavya

> I find that one key to understanding students' achievements at school is their foregrounds, including their interpretation of possibilities. In particular it is difficult to fight for something that appears unattainable (Skovsmose, 2012, p. 6).

THE CONCEPT OF CASTE

It is generally known that caste is a basic unit of traditional Hindu society. However, for the purposes of this paper we need to look at the concept a little more closely. The caste system is often described as a four-fold hierarchical division of society into the following groups or *varnas*: Brahman, Kshatriya, Vaishya and Shudra –that is, priests, warriors, merchants and workers. Below them are the Ati-Shudras: the communities that had been labelled Untouchables. Tribal (or adivasi) communities are more marginalised, and do not fit into the classification system.

This four-fold system (called *chaturvarna*) has been criticized by the late Irawati Karve (1968). In Karve's view the real social reality is the *jati*. In Berntsen's opinion, this view enables us to make sense out of data relating to caste interaction. The caste (*jati*) system developed separately –though on similar lines– in each region of the country. Small social groups with a common lineage interacted with other groups in order to exchange goods and services.

All the *jatis* in a particular region were ranked on a hierarchical scale of ritual purity, with the Brahman *jatis* on top, and the Untouchables on the bottom. Each *jati* had a traditional occupation –though over time the link between *jati* and occupation became less. In order to preserve their lineage, each group permitted marriage only within their *jati*. In order to preserve their status of ritual purity each *jati* also observed rules regarding interdining and the exchange of cooked food.

A person born into a particular *jati* could not change his/her caste identity except by rejecting Hinduism altogether and converting to another religion. They could, however, improve their socioeconomic status through education, business, or employment. Moreover, the *jati* as a whole could improve their status on the caste hierarchy by changing their lifestyle (say, for example, by giving up eating meat.) In this way, education, wealth and political power have created new hierarchies, but generally speaking, those on the bottom have tended to stay there.

Purity and pollution

A key concept in the idea of caste is ritual purity, and its opposite, pollution. From time to time every person is in a polluted state. Defecation, contact with childbirth or with death makes a person ritually impure. This impurity is cleansed by bathing. A woman is in a state of pollution during her menstrual period. But there are castes (*jatis*) for whom the state of impurity is a permanent condition. These are the Untouchables. Traditionally they lived outside the village. They were not allowed to draw water from the well used by the other villagers, or to touch a higher caste person.

Untouchability and reform in Maharashtra

In Maharashtra the largest Untouchable caste are the Mahars. Their traditional role was that of village servant. Their duties included repair of the village wall, disposal of dead animals, the flesh of which they dried and stored. They also served as the messenger for the village headman. In the event of a death they had to walk miles in the middle of the night to inform the relatives of the deceased.

In the 1930s Dr. Babasaheb Ambedkar started a campaign against this system of village service. By the time of his death in 1956 all Mahars had rejected this role. What Ambedkar accomplished in Maharashtra was nothing short of a revolution. But it was the culmination of a history of reform efforts that went back seven hundred years to the Mahanubhav Panth and the Varkari Panth, the two bhakti (devotional) sects were established. The Mahanubhav Panth, which is still extant, though with a low profile, is a monotheistic sect which rejects caste, gender bias, idol worship, and the ritual and spiritual authority of the Brahmans (Feldhaus, 1988). The Varkari Panth, which produced a stunning series of saint poets from the thirteenth to the seventeenth century, is still a vibrant part of Maharashtrian religious life. It has been a democratizing force. The poets used Marathi instead of Sanskrit, and most of the saint poets were non-Brahmans. There was even one Mahar poet. All of them acknowledged the equality of people in the realm of the spiritual. But they did not challenge the Brahman hegemony or the practice of Untouchability. It was only in 1947 after the fast of Sane Guruji, the well known freedom fighter, that the temple of Vithoba at Pandharpur, where the Varkari pilgrimage culminates, was open to Untouchables.

One of the most pernicious aspects of Untouchability has been the denial of educational opportunities. Significant changes, however, began taking place in the latter half of the 19th century. The British colonial government was anxious to extend public education to the lower castes. In 1862 they appointed the Hunter Commission to look into problems of education. Jotiba Phule an activist of the middle-ranking Mali (Gardener) caste, made a deposition before the Commission, arguing the need for providing education for Untouchable children. Both the government and Christian missionaries took up this challenge. The fiercely anti- Brahman Phule established his own school and continued his struggle to give social justice to the lower castes in Maharashtra. Phule died in 1890, and the following year Ambedkar was born.

Ambedkar devoted his whole life to the annihilation of caste. He was a brilliant student, who faced caste discrimination in schooling life. He was sent to Columbia University in New York, where he did an M.A. and Ph.D. in political economy. Later he went to the London School of Economics, where he received the degree of Barrister of Law. Back in India, he devoted himself to the uplift of what he called the depressed classes. His activities ranged from direct agitation and institution building to political organization. After India gained its Independence he became Law Minister in Jawaharlal Nehru's cabinet. He was in charge of the committee to draft the Constitution, and it was at his insistence that the practice of Untouchability was outlawed. He wrote prolifically, both in English and in Marathi. Over and over he advised his followers *"shika, sanghatit vha, sangharsh kara"* ("Study, Organise, Agitate"). He had also drafted the Indian constitution which was adopted in 26 November 1949. Throughout the years it became increasingly clear to Ambedkar that Hindu society was not going to be willing to abandon the practice of Untouchability and give social justice to the oppressed. He declared that though he had been born a Hindu he would not die a Hindu. He considered converting to Islam or Christianity, but finally decided that Buddhism was the best choice for India. On October14, 1956 he publically renounced his Hindu identity and embraced Buddhism, along with large numbers of his followers. Ambedkar died on December 6th 1956.

One of the most important policies initially by the government of India in response to Ambedkar was to reserve a certain percentage of seats in higher education, and in government, and in state and central government jobs for the ex-Untouchbles. These reservations made it possible for many ex- Untouchables (whom Ambedkar had started calling Dalits[4] to get higher education degrees and government jobs. Unfortunately, as more and more Dalits made socioeconomic progress, there has been a backlash of resentment on the part of many caste Hindus. Sometimes this has resulted in riots. In addition, other castes started arguing it is not only the Dalits who are suffering from backwardness; other groups should also be granted reservations. It must be said, however, that not all Dalits have gone ahead many of them are still suffering from poverty, illiteracy and discrimination. More and more it is also becoming clear that it is the tribal people who most desperately need attention and protection. This is particularly true regarding those belonging to small tribes and remote villages.

Terminology of caste categories

For administrative convenience, *jatis* within each state are usually clubbed

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4. Dalit means 'ground down'. The word is used to convey a sense of pride.

together in categories of those castes who have similar status – that is, a comparable degree of backwardness. The ex-Untouchable castes are called Scheduled Castes (SC) –that is, castes whose names appear in the approved government list (thus enabling them to avail themselves of certain benefits). There is also a category of Scheduled Tribes (ST). These are tribal/adivasi communities who have remained outside the caste system. The majority of them are impoverished.

In addition there are categories labelled Nomadic Tribes (NT) and Denotified Tribes (DT). These generally are communities, many of whose members continue to roam the countryside. There are some pastoral communities included in this category, so they are not uniformly poor. The Denotified Tribes are those whose members were branded by the British as criminal tribes. The Indian government has rescinded (denotified) the label of criminality, but the people in these communities continue to eke out a precarious existence.

The list of caste categories used in Maharashtra is generally like this. It is clear today there are many castes who are eligible for benefits or reservations on account of some degree of backwardness. In the list above it is only the Non-Backward category that is not eligible for reservations. This group is sometimes referred to as the 'Open' category. The list starts with the least backward and then goes down from there.

Non-Backward (NB also called Open)
Other Backward Castes (OBC)
Nomadic and De-notified Tribes (NT/DT)
Scheduled Castes and Tribes (SC/ST)

METHODOLOGY

I approached this study from the standpoint of a Dalit woman who has grown up in Maharashtra and studied in schools similar to the described here. I have been aware of the complex of factors that impact learning outcomes in schools like this. Perhaps because my father is well-educated and is prominent in the community, I never experienced the kind of discrimination and outright cruelty described by many Dalit writers.

Because I wanted to get a holistic picture of what was happening in the school, and wanted to gather quantitative and qualitative data, I decided to employ the mixed methods research design described by John Creswell (2009). I looked at the school records for enrolment data and caste affiliation, and also obtained the students' marks on one unit test given in classes 9A, 9D, 10B and 10C. Using a stratified sampling technique, I selected 70 students out of 310 and gave them a questionnaire to fill out. This questionnaire included items on students' background, their personal profile, parents' occupation, and their views about learning mathematics. In addition I selected 9 students for semi-structured interviews. I also conducted semi-structured interviews with the head master of the school, two mathematics teachers and two parents. To triangulate the data I conducted classroom observations using Patton's (1990) five dimension method (Creswell, 2009).

THE SCHOOL

I conducted the study in a government-aided secondary school situated in a semirural area of Maharashtra state in India. The school was started in 1951; hence it is quite old and prestigious in that region. Students' enrolment is quite high. For the academic year 2014 to 2015 the total enrolment was 1663 students. The school runs classes from 5 to 12. Classes 5 to 7 are upper primary, classes 8 to 10 are secondary and classes 11 to 12 are higher secondary. The total enrolment at the secondary level (Classes 8 to 10) was 830 students.

Student population by caste category

To understand the social dynamics of school in India, particularly in a rural or semi-rural setting, it is necessary to look at the percentage of students belonging to each caste category. Fig. 1 below shows the percentage of students by caste category at the secondary level (classes 8 to 10, total number 830).

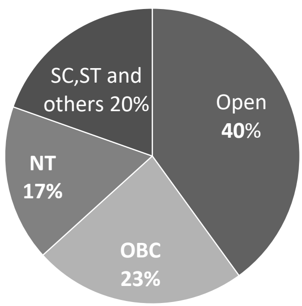

Figure 1: Percentage of students by caste category in secondary school

This chart is very striking. What it shows is that the ranking of caste categories according to number of students admitted from each category exactly matches the hierarchical ranking of the categories as we have listed them on p. 6. The category with the highest number of students is the Open category, followed by OBC, NT and SC, ST and Others.

Semi-English

We have already seen that the caste composition of the divisions reproduces the traditional caste structure. Now today the State government and the school have added another structure of discrimination: Semi-English. Though the medium of instruction in the school is Marathi, under this policy, mathematics and science are taught in English in Divisions A and B (from the 5th standard on). So the student admitted in Division A or B has the privilege of studying in a new building, and also enjoys the prestige of studying mathematics and science in English.

Caste category breakdown within divisions

When students enter the school in Class 5 they are assigned to a division based on their performance in the entrance test. Table 1 below shows the percentage of students by caste category and divisions at the secondary level (830 students).

Table 1. Percentage of students by caste category and division at the secondary level (830 students)

Division	Open	OBC	NT	SC, ST, Others
A	60.3	17.7	13.4	8.7
B	39.2	27.4	17.5	16.0
C	29.6	24.8	18.7	27.0
D	12.5	26.8	22.3	38.4
Percentage	40	23	17	20
Total number	332	194	142	162

This table is even more striking than Fig.1 above. It clearly shows that in Division A, 60.3 percent students are from the Open category while only 8.7 percent are from SC, ST and Others. As we read down the table, the percentage of students from the Open category decreases, and the percentage of SC, ST and Others increases. In short, the assignment of students to a particular division perfectly maps onto the caste hierarchy. Moreover, a teacher in the school under study confirmed that almost without exception a student assigned to the D division in Class 5 will stay there throughout his school life.

Generally Divisions A and B are perceived by parents and some teachers to be '*good classes*' and the students are described as '*bright*'. Class D is seen as the '*dull class*' Students are described as '*weak*'. Even the school infrastructure clearly indicates the status of the two classes. The 9A division is housed in a new building while 9D is in the old building.

TEST MARKS AND CASTE BACKGROUND

Now we can ask ourselves if performance on a unit test is correlated with caste category. Table 2 below gives the mean score out of a total of 40 in a mathematics test conducted in 9A, and table 3 the means score for 9D.

Table 2. Mean score (out of 40) on unit test in 9A

Category	No,	Mean
Open	61	23.84
OBC	17	21.71
NT	12	24.42
SC, ST, Others	6	21.33
Absent	0	
Total	96	

Table 3. Mean score (out of 40) on unit test in 9D

Category	No,	Mean
Open	6	16.00
OBC	10	17.60
NT	6	16.50
SC, ST, Others	13	15.78
Absent	24	
Total	60	

There is a very clear difference between the scores of the two divisions. The tables give ample evidence that the students in 9A do much better than the students in 9D. It is interesting however to look more closely at the averages across category in each division. At first glance there is practically no difference between the means of the various caste categories. This might suggest that classroom processes can override caste category. When we look however, at the actual scores of the students in each division, a starker picture emerges. In 9A about 1/3 of the students of the upper caste are in the highest interval 31-40, while there is not even one SC or ST in the highest interval. In 9D the picture is somewhat different. First of all, out of 60 there were 24 students who were absent. Of the students who were present, all except two scored in the 11-20 interval. There was one who scored in the 21-30 interval and one in 1-10.

How do we read this table? It seems to suggest that the atmosphere and pedagogy of 9D has ground down all the students to a minimum level. There is however one factor that we have not discussed evidence from the interviews from the students suggest that every one in 9 A attends private tuition classes in mathematics while nobody in 9D takes tuition. It is difficult to assess the impact of these classes. The only thing that we can say certainty is that the ability of a family to afford outside tuition fees gives their children and added advantage.

CLASSROOM ATMOSPHERE AND PEDAGOGY

In the data on test scores we saw two things. First of all, there is a significant difference between the mean score of 9A and 9D. Secondly, even within each division there is a correlation between the ranking of the caste and the mean score. This suggests what Skovsmose calls a "ruined foreground" –a situation which makes students feel that success is an unattainable goal. With this in mind let us look briefly at classroom atmosphere and pedagogy.

In the course of the study I observed a total of 35 mathematics periods –from 9A, 9D, 10B and 10C. I will comment here only on what I saw in 9A and 9D. We have already seen that there is a striking difference in regard to absenteeism at the time of the unit test. In 9A out of 96 students not a single one was absent. In 9D, out of 60 students 24 were absent. I have not been able to talk to these students. It is possible that some did not attend school because they had to work. But it is probably true that the majority found the classroom atmosphere so uncongenial and the hope of success so elusive that they chose to stay away.

The teacher in 9D was a young person with a B.Ed. degree and seven months of teaching experience. She told me –most likely expressing her own view– that everybody said the D division was dull. In the classes I observed she was constantly irritable, and seemed insecure about her ability to keep the students' attention and to control the class. There was actually relatively little disturbance, but she would punctuate her explanation with comments like, "Pay attention", "You don't study", "You don't finish your homework", "You are behind in your studies". She spent a lot of her time in seeing whether or not the students –particularly boys– were writing correctly in their notebooks. She rarely asked questions, and if she did, only one or two students volunteered to answer.

On the other hand, the teacher in 9A has M.Sc. and 20 years of experience. He had a positive attitude, and I never heard him say anything disparaging about backward class students. He managed to handle the class of 96 students without difficulty. For instance in one 9A class period the teacher was teaching a lesson in geometric construction. He would draw a figure on the blackboard and explain the step by step procedure required for solving a particular problem. From time to time he would ask a question, and more than a dozen students would raise their hands to answer. On occasion he would ask a student –always a boy– to come to the board and work out the problem. At no time did any of the students raise their hands to ask the teacher a question. This is a time-honoured method and the teacher used it competently. Nevertheless, even this fair-minded person was unable to break down the correlation of caste category and performance. Of course, in his class the student-teacher ratio was impossible. In the case of 9D the number of students actually present should have made it possible for the teacher to accomplish something.

DISCUSSION

At the beginning of this paper I commented that Skovsmose's distinction between students' background and foreground is useful theoretical framework for looking at our data. In our case, the background of the students consists of their caste, the socioeconomic and educational status of their families, and their previous schooling from Class 1 to 5. All these factors provide an extremely challenging background for students' learning.

The foreground is the education that the students are receiving in this school. It should be understood that, on the whole, this school is a decent one. It is typical of many private schools in Maharashtra that have been functioning with some government aid over decades. We see nothing of the terrible ill treatment of marginalised children such as that described by Murali Krishna (2004) in his autobiography. Nevertheless, the school is clearly failing to help the marginalised to survive in the educational system. The problem is more than the teaching of mathematics.

One of the most disastrous things that happens in the school is the assignment of incoming students in to divisions whose composition clearly reduplicates the caste hierarchy of the community furthermore the a provision of Semi English classes for 9A and 9B along with the newly constructed building for these two divisions give a strong message these are the students who counts are expected to succeed. For the students in the D division, on the other hand the only message is that they don't count. This is exactly what Skovsmose was saying in his description of ruined foreground.

Of course, in a way it is unfair to point a finger at this particular school. The syllabus, the textbooks and the structure of the education system have been developed, and are running basically for the middle class.

There is unmistakably a crisis in the education in the state of Maharashtra, and it is probably fair to say, all over India. If we have to address this crisis we must first of all be willing to recognise the caste bias that is endemic in the education system. We must find ways of revising the curriculum so that it is more relevant to the marginalised, without in any way watering down the content. We also need to find ways to increase the sensitivity and social commitment of educational administrators, teacher educators and teachers. This is a herculean task. Success will not come over night. But we have to start somewhere.

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MATHEMATICS TEACHER PROFESSIONAL DEVELOPMENT TOWARD EQUITABLE SYSTEMS: WEAVING TOGETHER MATHEMATICS, DISCOURSE, COMMUNITY, POSITIONALITY, AND ACTION RESEARCH

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This paper describes the process of designing a teacher professional development aimed at situating mathematics teaching and learning within broader social and political systems and structures. We designed the professional development to incorporate five strands of an equitable mathematics system -mathematics, classroom discourse, community and culture, positionality, and action research. We discuss how the design process was informed by our interactions with district stakeholders, historical and contemporary sources, our previous work and the current literature on mathematics teaching and learning. We include discussion questions to consider implications for incorporating these strands in teacher education.

PROFESSIONAL DEVELOPMENT TOWARD EQUITABLE SYSTEMS

Inequities with respect to students' opportunities to access and learn mathematics persist for many students in today's mathematics classrooms. Mathematics teaching and learning do not, however, exist in a vacuum. Rather, these practices are situated within broader structures, including inequitable structures that privilege some and oppress others. Teacher development, then, should support mathematics teachers in seeing the ways in which their practice, and their efforts to improve their practice, are part of larger social and political histories and structures (Gutierrez, 2010/2013).

Mathematics teacher professional development (PD) typically emphasizes reforming specific mathematics teaching practices (e.g., supporting children's mathematical thinking -Carpenter, Fennema, & Franke, 1996; enacting high cognitive demand tasks in ways that maintain the demand -Smith, Silver, & Stein 2005a, b). Many of these PD efforts effectively support teachers in aligning their practice with high-quality mathematics teaching, which promotes, for example, sense making and mathematics in context. Reform-based teaching approaches, however, often fall short of equity goals because equity issues in mathematics education are multifaceted and occur at more than just the individual teacher or classroom level. In response, the *Access, Agency, and Allies in Mathematical Systems* project endeavors to design PD, which positions equity at the systemic level. By "equitable system" we refer to intersecting levels of mathematics education that function synergistically to support the fair distribution of opportunities to learn (Hand, Penuel, & Gutiérrez, 2012). Research indicates that PD that fails to approach change from a systems perspective often falls short of expected goals (e.g., Cobb, McClain, Lamberg, & Dean, 2003). Addressing issues of equity from the point of view of a system entails inviting community leaders, district personnel, school administrators, mathematics teacher educators, mathematics teachers, and students to work collaboratively to improve the teaching and learning of mathematics (Cobb & Jackson, 2011). Importantly, we focus on the coherence and alignment of the components of an equitable system of mathematics education from the perspective of classroom mathematics teachers (Coburn, 2005), since teachers are the lynchpin to systemic change (Grossman & McDonald, 2008). As researchers and mathematics educators concerned with issues of equity in mathematics education, we concur with Zeichner (1993) that PD design should make explicit questions of how teachers' everyday practices challenge or support various oppression and injustices.

In addition to a one-week 40-hour teacher institute that took place in July 2016 (prior to the school year), the PD we have been designing involves a longitudinal plan to support teachers to do cycles of action research during the 2016-2017 school year. Furthermore, during the process of action research, teachers will be encouraged and supported to identify stakeholders (e.g., administrators, coaches, parents, community leaders, business leaders) in their work who have the potential to become allies (i.e., people who help, support, or act in solidarity with another in a particular effort) in future work. Thus, the overall, long-term aim of the PD is to foster the development of equitable systems that support fair distribution of opportunities to learn mathematics in urban schools. Because this project is ongoing, we focus specifically here on our design decisions related to the one-week teacher institute with the goal of engaging attendees in critical discussion about the work. We hope that we can both spur cross-national discussion about PD design and also learn more about our own perspectives, assumptions, and biases from engaging in a discussion with participants from other countries and contexts.

Description of professional development strands

We identified and designed the PD in relationship to five individual strands that coherently align components of an equitable system of mathematics education from the perspective of mathematics teachers: (1) *mathematics*; (2) mathematics classroom *discourse*; (3) *community* and culture; (4) *positionality*; and (5) *action research*. The *mathematics strand* aimed to deepen teachers' pre-algebraic and algebraic reasoning. We also designed activities for teachers to learn to adapt curriculum to engage students in high-cognitive demand tasks (e.g., Stein, Grover, & Henningsen, 1996), social justice math tasks (e.g., Gutstein, 2003), and group tasks that foster positive interdependence (e.g., Boaler & Staples, 2008). The *discourse strand* focused on scaffolding and developing mathematical discourse practices to challenge what it means to know and do mathematics. It highlighted particular cultural assumptions embedded in expected mathematical discourse and built understanding related to how these discourse practices contribute to students' positioning and developing identities (Herbel-Eisenmann, Steele, & Cirillo, 2013). The *community strand* focused on developing teachers' recognition of the resources for mathematics learning that students' bring from their community and cultural experiences (e.g., Turner et al., 2012). PD experiences were designed to challenge deficit discourses about students, their families, and their communities. The *positionality strand* gave teachers an opportunity to explore their own racial, gender, class, etc. identities and consider the systemic influence of privilege and oppression on mathematics teaching and learning. Activities encouraged teachers to develop political knowledge, beliefs, and dispositions necessary to challenge systems that marginalize particular groups of students in mathematics education. This strand also included opportunities for teachers and project team members to develop a sense of community and trust. Finally, the *action research strand* was designed in a way that respects teachers' knowledge and expertise (Herbel-Eisenmann & Cirillo, 2009), supports their intellectual leadership capacity (Caro-Bruce & Klehr, 2007), and is situated in the work of teaching (Whitcomb, Borko & Liston 2009). It emphasized the process of systematically examining, reflecting on, and making important changes to teaching practices that the teachers want to improve. During the teacher institute, the first four strands were woven together, and teachers were given a very brief overview of action research. At the end of the week, we asked teachers to identify which key ideas they wanted to continue learning about and these ideas -modifying tasks, facilitating classroom discourse, and learning from/with families and communities- have become the focus of the study group meetings in September and October, 2016. Toward the end of October, the teachers will learn more about action research and will select from among the ideas offered to design and enact their first cycle of action research.

Information about context

Thirteen teachers from a small urban district in the Midwestern United States volunteered to partner in this work. In the midst of a water crisis with many systemic health, economic, and political implications, the district reached out for support to build a university partnership. Demographically, the students in the school district are approximately 56% African American, 35% White, 4% Hispanic, 4% two or more races, 0.5% American Indian, 0.5% Asian, and the median household income is $24,834 (as compared to the state median income which is about $48,000). The teacher participants had a range of years of experience, job titles, and job responsibilities. All were experienced teachers who had worked in the district for multiple years. Their grade level expertise spanned early childhood to secondary grade levels. In addition to teachers who focused on their traditional classroom role, there were teachers who specialized in special education, intervention, mathematics coaching, and there was one long-term substitute teacher without a teaching license. The summer teacher institute was held over five consecutive days for seven hours each day and was designed to introduce teachers to the five strands. All participants elected to continue with the PD into the school year.

PROCESS FOR PROFESSIONAL DEVELOPMENT DESIGN

Informing the process based on interactions with district stakeholders and other sources

The PD design team consisted of seven mathematics teacher educators (MTEs) from six universities across the United States, who are part of the larger project team drawn from eight universities[1]. The design team also periodically met with consultants who specialize in anti-oppressive education and resources[2]. These meetings were used to gain perspective on the consultant's experiences with planning and facilitating activities for social change specifically focused on concepts of privilege and oppression. Through these consultations, the design team discussed rationale for the use of specific activities, was given feedback on proposed order and combination of activities, and engaged in dialogue about situations to consider during facilitation that take into accounts identities of facilitators and participants. For example, one discussion of particular

1. The members of the A3IMS research team are Joel Amidon, Tonya Bartell, Sunghwan Byun, Mary Q. Foote, Victoria Hand, Frances K. Harper, Beth Herbel-Eisenmann, Durrell Jones, Courtney Koestler, Gregory V. Larnell, Carlos LópezLeiva, Ashley D. Scroggins, Anita Wager, and Ayse Yolcu.
2. Consultants are facilitators for Allies for Change. http://www.alliesforchange.org/

significance was related to considering who was facilitating and who was participating in the PD by paying particular attention to how project team members identify racially compared to how the participants might identify. This was a point of interest when considering the dynamics of having a person in a place of privilege facilitating activities about oppression with participants who identify with those oppressed categories. Through this process the design team discovered that many members had assumed that in this urban context the majority of the teachers in the PD would be White, and had not considered how a person of color may prefer to connect with another person of color surrounding these issues. This caused the group to be more thoughtful about considering who would facilitate each part of the PD by considering our positions in relation to the content of the activities.

Two of the PD design team members and two additional team members (referred to as the "local team") live in close proximity to the school district and were able to visit the district, meet with the superintendent, the principals, district math coaches, and some of the potentially interested teacher participants. These visits allowed us to get a sense of individual and collective concerns, solicit feedback about structure and timing of the PD, and to provide information about how the project was described in the funded grant proposal, as well as answer questions various stakeholders had. For example, our original proposal included a two-week teacher institute. We learned from one of the teachers, however, that finding daycare for two weeks would make participation in the PD difficult. Thus, we reduced the teacher institute to one week with a few follow-up sessions planned into September and October. We also had multiple inquiries about how we might support principals to understand the work, something we had not included in the original proposal. Principals are responsible for evaluating teachers, so this was an oversight that we needed to further consider.

The project team selected three books that were written about the community within which the district was located and team members formed book clubs to read, discuss and share the historical and contemporary issues and events reported in the books. Because the community is experiencing an ongoing and longstanding water crisis, we also gathered relevant media reports and some project members attended community and university meetings related to the ongoing crisis. These meetings involved many community leaders and discussions emphasized the role of systemic racism in government and policy decisions leading up to the crisis. Notes from these meetings were shared with the project team to help inform some of the decisions we made. For example, these and additional resources were used to create a "Wall of History" activity that identified some key events in the community. Teachers spent time reading the events and writing comments about them during the teacher institute. They also added events to the timeline from their collective memory of the community, because many of the teachers had grown up in the community and/or had lived and worked in the district for decades. Finally, the research we proposed involved interviewing all of the teachers before the teacher institute. One of our team members met with most of the teachers immediately prior to the PD, and we were able to draw on some of the information she learned, but to a lesser extent due to timing.

Informing the process based on our previous work and current literature

Each member of the PD design and facilitation team brought diverse expertise and perspectives to the five strands. After designating the work of each strand to smaller sub-groups of the team, we leveraged our diverse expertise as we both planned a teacher institute curriculum that strategically emphasized individual strands and coherently integrated the strands across the week. The process involved each sub-group concentrating on designing a set of tasks, activities, or experiences that highlighted the particular strand. For example, the mathematical task group identified key goals for teachers' learning about algebraic thinking and selected tasks that would help teachers engage with those goals. The tasks were all cognitively challenging (Smith, Silver, & Stein 2005a, b), and some were modified to focus on a relevant social justice topic or to be groupworthy (i.e., draw effectively on group resources; Horn, 2005). One requirement described by the superintendent was that we had to work with the adopted curriculum, so this sub-group also found related tasks in the district's curriculum materials and designed activities for analyzing these tasks and for supporting their modification. Each sub-group entered the set of activities they hoped to do in a table and we met intermittently to share our individual progress on these activities.

Once each sub-group had multiple activities, we had two lengthy meetings in which we discussed: a) which activities we wanted to prioritize; b) what order and organization might make sense, given the individual sub-group's activities; and c) how the set of tasks, activities, and experiences might cohere and flow. As we had these discussions, we realized that sometimes a strand needed to stand alone in order for us to address it well, but other times, we wanted to merge various strands together to address multiple strands at a time. Figure 1 below is a visual data display of how we ultimately planned to weave the strands together and the emphasis that we planned to place on each strand during the teacher institute. For example, we designed and used specific activities that required everyone to identify and talk about their experiences and identities to make positionality a central focus in and of itself (represented by standalone orange in Figure 1). When we merged the mathematics and discourse strands on Day 3 (represented by green alongside purple in Figure 1), for instance, we created a script that modeled both students' mathematical engagement with a task that required algebraic thinking and illustrated a teacher's intentional use of the teacher discourse moves. Teachers engaged with the mathematics task themselves, but rather than debriefing their solution strategies, teachers acted out the script and considered the mathematical ideas highlighted during the hypothetical classroom discussion. Then, they analyzed the script more closely to consider the ways in which teacher and student discourse created opportunities for engagement with those mathematical ideas.

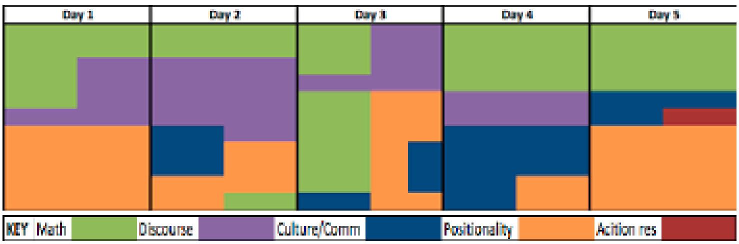

Figure 1: Visual display of planned five strands. Colors proportionally (not sequentially) represent time planned for the focus on each strand by day.

At a whole-group project meeting, the PD design team went through the overview of the week and solicited feedback. This discussion was crucial for our realizing that we had lost some of the focus on systems of privilege and oppression in our process of working in sub-groups. In particular, we inadvertently reduced the emphasis on racism and classism, two systems of privilege and oppression that were particularly relevant in a context in which the majority of students are people of color with approximately 41.5% of the families living in poverty. This discussion was a crucial turning point that prompted the team to stop and further reflect on our position as researchers and our role in this district partnership. One particular point of conversation circulated around our presence as it related to the water crisis that the community is facing, and what authority, if any, we had to come in to facilitate working toward equitable systems in a space that is publically experiencing inequality. Although this was not the first time the project group engaged in similar conversation, this forced us to reconsider our planning decisions and make shifts to ensure the PD addressed topics like racism and classism within this unique context. Consequently, we increased the prevalence of this strand across all five days of the teacher institute.

One shift that resulted from the increase of the positionality strand came with the placement and emphasis on the “target/non-target” activity. In this activity, participants explore various avenues of privilege and oppression by identifying and reflecting on what target (oppressed) and non-target (privileged) groups they identify with and their corresponding systems of oppression. Initially, this activity had been placed later in the week, but with the resurgence of the importance of addressing systems of privilege and oppression it became important to reconsider that decision. Through the target/non-target discussion the team connected back to advice from the consultants with whom we worked -they use a similar activity first thing in their institutes that focus on examining systems of privilege and oppression. In engaging with the activity early on they found that it sets the stage for the types of conversations to come and forefronts engagement around systematic oppression in their institutes. With this in mind, we decided to give the activity more prominence at the beginning of the PD, resulting in the configuration shown in Figure 1.

Questions to discuss

Individual and collaborative reflections have been an influential part of the PD design process. As PD plans and materials continue to be designed and refined, the team has committed to engaging in discussions that promote meaningful incorporation of each strand, and consider the implications of our decisions. Some of these questions include: How might further revisions to the planned sequence be necessary to emphasize an individual strand and/or to weave the strands more coherently? What implications might these strands have for your own work with mathematics teachers towards developing equitable systems for mathematics education? How might future approaches to developing PD toward equitable systems compare to this design? What other considerations are necessary to address moving forward?

CONCLUSION

The purpose of this paper was to describe the process of designing PD that focuses on situating mathematics teaching and learning within broader structures to move toward more equitable systems. The PD was designed to weave together five strands of an equitable mathematics system that incorporated mathematics, classroom discourse, community and culture, positionality, and action research. In designing the teacher institute, decisions had to be made. These decisions were informed by many things including literature on mathematics teaching and learning, our positions as individuals and researchers, our interactions with consultants and district stakeholders, the historical context of the location, as well as the contemporary context involving a water crisis. The project team's process highlights the important role collaboration, discussion, and reflection played in the design process, which prompted shifts in content and organization that helped the team to prioritize the goal of promoting equitable systems by fostering development in all of the five strands. Moving forward with the project, we consider this type of collaborative reflection necessary to promote and enhance our ability to continue to weave together these pieces of an equitable mathematics system in meaningful ways that support teacher professional development.

ACKNOWLEDGMENTS

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REFLECTIONS ON PEDAGOGY IN A REMOTE INDIGENOUS COMMUNITY

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Commonly used definitions of pedagogy refer to both the art and science of teaching, but art and science are culturally bound as indeed is mathematics and mathematics curriculum. This paper describes two experiences in a remote Indigenous community that challenge universalist notions of pedagogy and support a case for re-assessing what is meant by culturally-responsive mathematics education in this context.

MATHEMATICS AT THE INTERFACE

The reflection was prompted by two events experienced in the context of the *Building Community Capacity* (BCC) project[1] that investigated an alternative approach to Indigenous teacher education in remote communities. The approach was aimed at supporting Indigenous teacher assistants use performance-based tasks in first language (L1) to identify where the Indigenous students were in their mathematics learning and work alongside the non-Indigenous classroom teachers to progress that learning.

The driving force behind this and many similar projects (e.g., Lipka, 1998; Kiska, Lipka, Adams, Rickard, Andrew-Ihrke, Yanez & Millard, 2012; Nicol, Archibald & Baker, 2010) is that Indigenous students are not experiencing the same level of success in school mathematics as their non-Indigenous peers (e.g., Cowley, Easton, & Thomas, 2011; Thomas, De Bortoli & Buckley, 2013). This situation severely restricts Indigenous students' opportunities to participate in further education and find meaningful employment. There have been many attempts to address this situation both in Australia (e.g, Harris, 1990; Jorgensen, Sullivan & Grootenboer, 2013; Yirrkala Community Education Centre, 1994) and overseas (e.g., Greer, Mukhopadhyay, Nelson-Barber & Powell, 2009;) Wong, Lipka & Andrew-Ihrke, 2014). These vary in their scope and focus but their common aim was (is) to explore culturally-appropriate ways of teaching *school mathematics*.

1. The Building Community Capital to Support Sustainable Numeracy Education in Remote Locations Project 2006-2009 was funded by the Australian Research Council and the NT Government. The views expressed are the author's and not necessarily those of the funding bodies.

Some years back, Bishop made a case for distinguishing between *Mathematics* (with an upper- case M) and *mathematics* (with a lower-case m). This distinction was made not to privilege one form of mathematics over another or to assert that these are necessarily discrete, but to emphasise that mathematics is a cultural phenomenon, a *way of knowing* (Bishop, 1991). This raises the questions of what (M/m) athematics should be taught in what ways in particular educational settings. There is growing body of evidence to suggest that where cultural knowledge is valued and employed in the pursuit of Mathematics education, Indigenous participants are more likely to succeed (e.g., Kisker et al 2012; Lipka, 1998; Yirrkala Community Education Centre, 1994). This suggests that one way of understanding the challenges of teaching and learning Mathematics in a remote Indigenous school is to view these through the lens of intersecting communities of practice (Wenger, 1998). In this case, the shared setting of the community school at Galiwin'ku on Elcho Island in the Northern Territory of Australia.

METHODOLOGY

Galiwin'ku is the major community on Elcho Island and one of the largest in North East Arnhem Land. It is a traditional Aboriginal community with restricted access. It has a population of approximately 2200 people, although this varies seasonally with many homeland residents migrating to the community during the wet season due to inaccessibility. The school has an enrolment of about 550 students, but attendance also varies quite considerably with the seasons and cultural commitments.

The BCC research project that enabled the research team to visit Elcho Island on a regular basis, was based on the premise that Indigenous student numeracy (Mathematics) outcomes are more likely to be improved where what student's know in relation to a small number of key Mathematical ideas is identified and responded to in first language (L1) supported by knowledgeable community members.

The research was underpinned by a sociocultural, interactionist view of learning that acknowledges the importance of discourse in the shared construction of meaning (e.g., Lerman, 1998; Rogoff, 1995). This approach has its origins in a *situative perspective* that views learning and development in terms of transformation where

> the central question becomes how people participate in sociocultural activity and how their participation changes from relatively peripheral, observing and carrying out secondary roles, to sometimes being responsible for managing activities (Rogoff, 1995, p.157)

This underpinning perspective lead to the recognition of three intersecting communities of practice, the Yolngu school community, the school Mathematics community and the Study Group community that was created to explore (M/m)athematics at the interface of these communities (see Figure 1). The study group, comprised of Indigenous teacher assistants (community teachers), non-Indigenous teachers and research team members, met at least once or twice per school term over a three-year period. An experienced linguist and/or community elders who had a past association with the school also participated in the study group sessions from time to time. The study groups were intended to operate both as a space where different communities of practice could meet to negotiate shared meanings about key aspects of Mathematics and as a research tool to explore the processes involved in building community capital. A small number of performance-based tasks (probe tasks) designed to explore student's understanding of key number ideas were used as boundary objects in this process (Wenger, 1998).

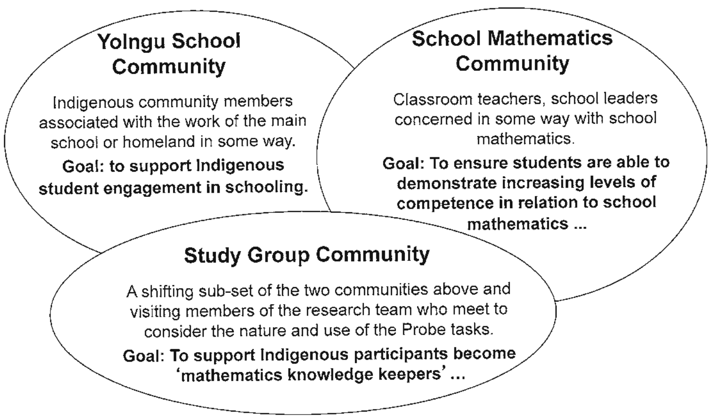

Figure 1: Three communities of practice recognised in the BCC project

While a range of factors variously impacted the project, a number of issues emerged to challenge the initial design and pose new questions (Christie, 2007; Siemon, 2009). For example, the relative positioning of Indigenous teacher assistants in the school, hereinafter referred to as community or Yolngu teachers in recognition of their critical role in the classroom, and their propensity to appropriate diagnostic tasks for teaching purposes. This questioned the use of formative assessment in this context and motivated me to learn more about the nature of the pedagogical practices Yolngu use to enculturate their children into Yolngu knowledge systems and ways of knowing.

The purpose of this paper is to illustrate what I have come to see as the *cultured* nature of pedagogy and comment briefly on the implications of this for school (M/m)athematics education in remote Indigenous community settings. In doing so, I am acutely aware that my observations are coloured by a particular worldview that is different to the Yolngu[2] world that I have had the privilege of coming to know in a very small and naïve way over recent years. In view of this, and wishing to be respectful of Yolngu mathematical practices, I shared my observations with three highly regarded senior members of the Yolngu community, Maratja Dhamarrandji, a bi-cultural consultant, Rose Guywanga, the first Yolngu Principal of the community school, and Dorothy Gapany, a teacher who had also taught at the school. On two separate occasions approximately 6 months apart, they heard these stories and conferred with me on the meanings I was to draw from them. I am indebted to them for helping me see through different eyes and I acknowledge their contribution to the interpretations below, which are offered in a spirit of respect and genuine interest in learning *from* Yolngu.

REFLECTIONS ON PEDAGOGY

In the interests of facilitating a conversation about pedagogy that parallels Bishop's (1991) M/mathematics distinction, I will use *Pedagogies* (with an upper-case P) to refer to the plurality of pedagogies used in school Mathematics (inclusive of 'traditional' and 'reform' practices (e.g., Boaler, 2002), and *pedagogies* (with a lower-case p) to refer to the possibility that different mathematics, stemming from different value systems and world views, may well entail different pedagogies. In doing this, it is important to note that I do not see these as dualities, but I do want to distinguish between *Pedagogical* practices that are advocated for use with Indigenous students by largely non-indigenous teachers (e.g., Martin, 2008; Mathews, Howard, & Perry, 2003; Jorgensen, Sullivan & Grootenboer, 2103; Zevenbergen, Mousley, & Sullivan, 2004) and *pedagogical* practices that are integral to the production and reproduction of Indigenous knowledge systems (e.g., Christie, 1985; Marika, 1999; Rennie, 2006; Yunkaporta & McGinty, 2009).

Advocated Pedagogies: Most of the practices advocated for use with Aboriginal students living in traditional communities are derived from the work on Aboriginal learning styles or preferences by Harris (e.g., Harris, 1978; Christie, 1985) and Graham's (1988) related work on the interfaces between school Mathematics and Aboriginal knowledge systems. Harris (1978) suggests that Aboriginal[3] learning is characterised

2. Yolngu is the term the Aboriginal people of Northeast Arnhem Land use to describe themselves.
3. 'Aboriginal' is used to refer to the original inhabitants of Australia – 'Indigenous' is inclusive of Aboriginal and Torres Strait Islander peoples.

by observation, imitation, and personal trial and error rather than verbal instruction or demonstration; real-life performance rather than practice in contrived settings; mastery of context-specific skills rather than abstract, context-free principles; an orientation to persons rather than information; and to present time rather than the future. According to Harris, Aboriginal learning is informal and unconscious and persistence and repetition are used as problem solving strategies rather than 'analysis-before-action'. It occurs in settings that are inherently conservative (i.e., there is little expectation that the Yolngu cosmos will change), highly respectful of authority, and discourage unsanctioned questioning; where listening rather than speaking is privileged in interactions, and embarrassment is strongly avoided. Despite considerable criticism on the grounds of cultural relativism (e.g., McConaghy, 2000; Reid, 2004), these observations are evident in the advice provided to practitioners (e.g., Martin, 2008; Warren, Baturo, & Cooper, 2005) and in the practices adopted by researchers as they work with Indigenous communities to improve student outcomes in Mathematics (e.g., Zevenbergen et al, 2004) and literacy (e.g., Martin, 2008; Rennie, 2006). Researchers have also recognised the relevance of reform practices such as collaborative group work, contextualisation, problem solving, and an emphasis on social interaction. For example, Zevenbergen et al (2008) described their intention to blend reform pedagogies with the "literature on Indigenous learning preferences" (p. 1).

Graham (1988) observed that where individuals "grow up in a society in which the system that controls the economic realities of life are based on relationships between people rather than relationships between quantities", they are more likely to be better at "talking to establish personal relationships with their teachers than they are at talking to transact knowledge inside the classroom" (p.128). Graham also makes a case for building on students' visual and spatial skills, including negotiated elements of ethnomathematics (mathematics) in school curricula, and for teaching Mathematics in L1 wherever possible. Connections can be seen between these observations, aimed at valuing and connecting home culture to schooling, and the practices advocated in the more recent literature. For instance, Martin (2008) suggests considering Indigenous education in terms of 'ways of knowing', 'ways of being' and 'ways of doing'. Zevenbergen et al (2004) illustrates the efficacy of linking home to school languages and using a more orally focussed mode of communication. Mathews et al (2003) emphasise the importance of warm, positive relationships with teachers, contextualisation, and building on the knowledge and skills that children bring with them to school. They also acknowledge the link between identity and culture and the value of having high expectations as do Warren, Cooper, and Baturo (2008), who argue that to improve Mathematics outcomes "it is essential that Indigenous students experience practices that acknowledge their indigeneity, that are based on expectations of success, and that are better suited to their learning style" (p. 44).

While it is clear that many of these practices coincide with reform-oriented Pedagogies aimed at reducing inequalities based on language, ethnicity, or class (Boaler, 2002), they are nonetheless interpreted and implemented through the lens of the dominant Mathematics culture. Whether these are described as culturally appropriate, culturally congruent, or culturally responsive (see Ladson-Billings, 1995), without L1 and a preparedness to genuinely engage with local communities, such practices run the very real risk of essentialising Aboriginal learning or drawing on Indigenous knowledges in ways that are educationally inappropriate or culturally disrespectful (Graham, 1988; McConaghy, 2000; Ladson-Billings, 1995).

Yolngu pedagogies: By contrast, the practices that are integral to the production and reproduction of Indigenous knowledge systems are much harder to identify, possibly because Yolngu see no point in separating knowledge from the method of acquiring it. There are numerous examples of situations where Balanda[4] have probed Yolngu perceptions of *how* they learn (e.g., see Christie, 1985, 2007; Harris, 1978; Rennie, 2006) which are answered by recounts of *what* was learnt (e.g., "which moon and which tide to go collecting *diyamu*", a species of shell-fish) suggesting that this is not a sensible question to consider. Not because Yolngu do not have rich pedagogical practices but because the activity of learning/teaching is largely unconscious, it happens over a long period of time and in ways that are entirely integrated with everyday practices. In the words of one of Harris' correspondents, "I just do it" (p. 20), was as close as one young man could get to explaining how to teach tracking. Learning in this sense is a function of *being Yolngu*, it is not something that is subject to scrutiny in and of itself. A further example was provided in a two-day workshop[5] prompted in part by the BCC project. Ten senior Yolngu consultants were brought together with Balanda researchers to discuss issues around (M/m)aths and (M/m)aths education (my emphases). After a lengthy discussion on various aspects of Yolngu knowledge, one of the researchers asked, "the one who doesn't understand, how will he learn?" Waymamba, a Yolngu university lecturer

4. Balanda is the term Yolngu use to describe non-Aboriginal people.

5. A report of the Mathematics as a Cultural Practice workshop, including translated transcripts, can be found at www.cdu.edu.au/centres/macp/whatemerged.html

replied, “From a knowledgeable Yolngu. From *following* the words of a clever Yolngu” (emphasis added). This points to learning/teaching practices intimately connected to Yolngu epistemology, personal relationships and social responsibilities that are exercised in and through activity, stories and song (Marika, 1999). Learning is not separated from being and being is not separated from knowing. This might help explain why formal schooling is often viewed as a practice against culture (e.g., Christie, 1985; Zevenbergen et al, 2004, 2008) and the belief that attendance is sufficient to learn (Christie, 1985).

Story 1 (May 2009): During one of my many visits I was invited to attend one of the many ceremonies associated with a funeral. This experience left a lasting impression on me and was a catalyst in helping me see school Mathematics and all that that entailed in a remote Indigenous community through different eyes. It would be inappropriate for me to attempt to describe the rituals and the deep spiritual significance of funerals to Yolngu people in any great detail –planning takes place over a considerable amount of time and many people are involved– it is important that this is done ‘properly’ (see Watson, 1989) –certain events need to take place in a certain order at agreed locations, and certain songs and dances need to be performed and/or observed by particular groups of people. The ceremonies last for about two to three hours generally from late afternoon to dark but continue beyond that if necessary to complete what was planned. The ceremony continues with seemingly little break or overt orchestration, although it was clearly proceeding in an organised and coherent way from a Yolngu perspective. Sitting with an Indigenous woman that I call *yapa* (sister) I observed as she explained the significance and meaning of some of the songs and dances. I sat there for a long time, absorbed in the collective intensity and sense of purpose. Children of all ages participated; some of the older children were invited to participate by a gesture or by name, while toddlers joined in of their own volition. Older teenagers and adults were clearly concerned to perform as well as they possibly could. Those who were seen to have performed well were acknowledged and dancers would relocate to position themselves closer to someone who was seen to be doing the dance well. Older children might be mildly scolded for not performing as well as might be expected. No particular attention was paid to younger children, although a good rendition of a particular movement was noted and rewarded with a smile or a gesture.

Reflecting on this experience both during and after the event, I was struck by the power of the learning environment dancers and spectators alike. While observation, participation, and imitation were all evident, there was something more. Apprentices become tailors through a similar process of activity-mediated enculturation and tailoring contributes to their on-going sense of identity and agency, but it does not define or explain their existence. The something more seems to have something to do with value, what Maratja refers to as *mingurrmirr* (to hold precious) and purpose.

Story 2 (5 August 2009): A young Yolngu mother, one of about 5 or 6 mothers or aunties invited to come into the Year 1 classroom at the community school, was sitting by herself at one of the small tables while the children, mostly 6 year-olds, were seated on the mat. The children were listening attentively to the classroom teacher (non-Indigenous) and the Yolngu teacher who were describing what they would be doing next. The class had just finished a subitising activity in which they were invited to say how many dots there were on a series of flash cards that were shown for about 2 seconds each. The cards depicted collections of 0 to 10 dots in different ways (e.g., dice or card patterns, ten frames and random collections). This was done as a whole class activity with the children responding orally. For numbers larger than 3, the teacher would repeat the correct number (e.g., "6" for an arrangement showing a collection of 4 and a collection of 2) and ask "6 billi?" To which the children would respond as a group or individually, "4 ga 2" or "2 ga 4". It was evident that nearly all of the children could recognise composite collections up to 7 irrespective of representation but had some difficulty with larger numbers. As a consequence, the children were asked to draw a picture that told a story for 8 in terms of its parts. An example, involving 2 birds and 6 birds was shown to the class and briefly referred to in L1 by the community teacher. The children went back to their tables where paper and coloured pencils were available. At some point, the young woman decided that she would draw a picture too and by the time three of the children joined her at one of the tables, she was deep in concentration. Instead of starting their own drawings, they immediately and without instruction, moved closer to the young woman to observe what she was doing. There was complete silence at the table (not the case elsewhere in the room), while she very carefully drew 4 trees to one side of the page and 4 trees on the other. When this was completed she said "4 ga 4" quietly to herself. She continued, detailing the trees in a consistent and patterned way completely oblivious to the little band of highly attentive observers. Still the children did not pick up a pencil or move to do a drawing despite the fact that other children in the room were moving around and showing their drawings to the teachers and other adults. When the young woman had completed the detail on three of the trees, she very carefully, indeed reverentially, picked up the paper in both hands and placed in front of the child to her left with a nod and gesture for him to complete it in the same manner. She then drew an identical picture without the detail and again reverentially placed this in front of the child to her right, all without speaking. While she proceeded to engage in a different drawing, the remaining child, without any instruction to do so, started detailing the trees on the other side of the second child's drawing. At no point did she make eye contact or speak to the children.

To a conventionally trained teacher, this episode could be regarded as poor pedagogical practice –it was highly teacher centred, involved little or no social interaction of any sort (at least to the observer's eye), there was no room for error or risk taking on the part of the children, and they were engaged in a repetitive process not of their own creation and yet they were utterly involved in the enterprise even to the extent of quietly saying "4 ga 4" to themselves as they worked. This experience challenged my taken-for-granted views about Pedagogy (reform or otherwise) and made me think again in terms of value and purpose.

DISCUSSION

One can only begin to speculate on the implications of this for schooling and Mathematics education in remote communities but at the very least it suggests that researchers and educators working in this context should question the applicability of 'best practice' Pedagogies, including those culturally relevant practices that claim to accommodate Aboriginal learning preferences. Observation and imitation may just involve something much more profound than we have understood to date. Indeed, as Lave and Wenger (1991) have observed, the impact of legitimate peripheral participation in a community of practice offers "more than an observational lookout post: it crucially involves *participation* as a way of learning –of both absorbing and being absorbed in- the culture of practice" (p. 95). This seriously questions the simplistic use of observation and imitation in Mathematics education where the cultural practices of schooling are often opaque and disconnected from the day-to-day aspects of life and death in remote communities.

Working with cultural artefacts and contextualising Mathematics might also be much more problematic than we envisage, particularly if these practices serve to disrupt or threaten Indigenous knowledge systems. This includes the very vexed question of the use of L1 in community schools for which there are two responses: learn Mathematics *through* L1 wherever possible using culturally relevant Pedagogies, or seriously question the 'elephant in the room', that is, Mathematics and all that comes with it, at least in the early years of schooling. Perhaps it is time we took seriously a recurring theme raised by Yolngu at the workshop referred to earlier, that "children can not learn to do balanda maths (Mathematics) unless they have received a firm *foundation* in their own Yolngu culture" (Christie, 2007, p. 4).

We need to recognise that all (P/p)edagogy is *cultured* -this speaks to something other than culturally relevant Pedagogies and challenges us to work as equals with Yolngu to better understand Indigenous knowledge systems and the values and purposes underpinning the pedagogies that are intricately associated with these. This requires extensive consultation and collaboration with community members, not to displace one way of knowing and being with another, but to work on ways in which we can, for different purposes at different times, agree to foreground one over the other. In my view, until we clarify our shared values and purpose, there is little to be gained from culturally relevant, appropriate or responsive Pedagogies alone.

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OLD AND NEW NATURALIZED TRUTHS IN MATHEMATICS EDUCATION

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"Where now? Who now? When now? Unquestioning. I, say I. Unbelieving. Questions, hypotheses, call them that. Keep going, going on, call that going, call that on".

Samuel Beckett, The Unnamable

The present paper is an attempt to describe the condition of the mathematical education in the contemporary political and economic environment. We initially present the general terms of the political situation, derived primarily by Jean Francois Lyotard's prophetic thesis (1979). At that time, globalization, under the influence of the internet, had not yet revealed the profound effects of neoliberalism. The "cynics" verifications demonstrate the new world that has risen. As an aftermath, we display some immediate episodes of the recent years' educational reality, deduced by current politics. Recent research in the context towards the political turn, supports these observations.

COMMENTING UP THE TRADITION

In this report we attempt a description of the current situation of the mathematical education in the political and economic environment; concepts involved are those of the function of naturalized truths, of the naturalized meaning of progress and of the significance and status of mathematics. The political and social turn that was formulated by Lerman (2000) and discussed in the MECT Conferences in Manchester (2011, 2013), had a significant impact on the theoretical perspective in Didactics of Mathematics. Theories in use shift the focus from cognitive descriptions of mathematical understanding to interpretations of political and social conditions, under which, the participating subjects in the mathematical education construct their identities. Theories of discourses are very useful in these investigations. The economic and political authorities impose truths, which use *naturalized realities* as a tool. The term *naturalization* has been introduced in phenomenology and refers to truths which appear as self-evident and necessary, whereas they are only contingent. These phenomena occur and recur in the ascertainment of researchers of the political and social current. We discuss the naturalized truths implicated in Didactics of Mathematics and firstly the alleged hierarchy of mathematics in education. Consequently, we point out the fundamental question emerging from the research in discussion “From Questions of *How* in Mathematics to Questions of *Why* in Mathematics ”, which in our opinion establishes a new naturalized truth.

As mentioned above, the political situation is here approached by employing Lyotard’s thesis, as presented in *The postmodern Condition* (1979). At that time, globalization under the influence of internet had not yet revealed the full extent of the tactics of neoliberalism. The “cynics” verifications demonstrate the new world that has risen. Additionally, we present some immediate episodes which took place in the educational programs of recent years, and originated from the educational politics in question.

THE OLD NATURALIZED TRUTH

According to the storytelling of tradition,

> “mathematics has been the advocate, essence and embodiment of rationalism. Rationalism is not a whim. It is a spirit which stimulates, invigorates, challenges, and drives human minds to function in the highest mental level, in exploring and establishing the deepest implication of knowledge already at hand.... Rationalism and exact thought patterned after mathematics can be applied to many fields... has penetrated almost all domains of inquiry and has served as the model of all intellectual enterprises”. (Kline, 1962, p. 673)

We find the above extract indicative of the endless relevant literature invoking the importance of mathematics in culture as well as the worship of science in general. (see Renyi, 1967). The assertions however haven’t always been that optimistic.

Ever since 1938, Husserl had already challenged the significance of the instrumental perception of science for the human being as well as of optimism for mathematical formalism and for the scientific progress in paragraph two, entitled: *The positivistic reduction of the idea of science to mere factual science. The "crisis" of science as the loss of its meaning for life*. (Husserl, 1970, p. 5). He does this in an era in which the worship of scientific progress becomes the pivot of development. Even the nazi movement conducts its design of a future of racial purity in the name of science, alongside the rapid development of utilitarian technocracy. Meanwhile, technocratic effectiveness –which was named *instrumentalism of science* by the school of Frankfurt– is isolated (Schecter, 2010). A parallel *futurism* evolves within the economistic interpretations of Marxism in the Soviet Union (but in spite of the existing tendencies, it will never deteriorate to racism). Husserl notices the significance of formalism as an instrument of the efficacy of science and indicates how much mathematics contributes to this:

> "Mathematics and mathematical science, as a *garb of ideas*, or the garb of symbols of the symbolic mathematical theories, encompasses everything which, for scientists and the educated generally, *represents* the life-world, *dresses it up* as 'objectively actual and true' nature. It is through the garb of ideas that we take for *true being* what is actually a method– a *method* which is designed for the purpose of progressively improving, *in infinitum*, through 'scientific' predictions, those rough predictions which are the only ones originally possible within the sphere of what is actually experienced and experienceable in the lifeworld. It is because of the disguise of ideas that the true meaning of the method, the formulae, the "theories" remained unintelligible and, in the naive formation of the method, was never understood". (Husserl, 1970, p. 51)

Husserl denounces the technocratic course of sciences and calls on the naturalized truths which science imposes as natural and self-evident. He has referred to naturalized truths, establishing them as stereotype descriptions which conceal the appearances. "The naturalist teaches, preaches, moralized reforms" (Husserl, 1965 p.81).

Bearing in mind the basic naturalized truth which we attempt to deconstruct, we encounter concerns about the history of education. Many assumptions of the philosophy of education's sociology, with Robinson's being the most distinctive, indicate that the current educational system is conceived for the purposes of a different era, the one of industrial development. It is well known that after World War II, there was a need for university degree holders of technocratic orientation. Nowadays, when university and polytechnic school graduates cannot obtain a job, or, when after hard efforts, they do acquire a precarious and poorly rewarded one, the question emerging is, why *such obsession with mathematics education overwhelmed their adolescence?* They tired themselves out in order to learn mathematics at school, a cognitive skill in suspension, so that they could be accepted in relevant scientific fields. How many of them would be accepted in technocratic faculties and for how many this specific knowledge would be actually useful? "How much" mathematics does an average scientist need during his lifetime? "Elementary", we would reply. Why, then, did he delve into an abundance of mathematical knowledge? So that he *might* be able to deal with higher knowledge in higher faculties, and *might* actually employ it, when and if, he gets involved with a specific technical object or with an assignment that requires special knowledge. As a consequence, in this educational trajectory extra credit is given to the very few who will end in very specific jobs; simultaneously, a crowd of acculturated experts finds itself at the disposal of companies, very few of which will be employed, at any price, by the system. Hand in hand with this realization, in terms of emancipation we may also take into consideration other fields of knowledge which were neglected or excluded due to this unilateral education. With the platonic arrogance of «Let no one ignorant of geometry enter», a separation takes place. It is a separation from another tradition, which even though exists simultaneously, is being ignored. In its core, this tradition highlights in equal terms the spiritual value of culture and human, of poetry and theater, of Homer, Aeschylus, Sophocles, Euripides, to name but a few. (see Naddaff, 2002).

In addition, the type of mathematics taught is of great importance, since it establishes a highly deterministic and abductive way of thinking. It is the hereditary of Descartes. As an alternative of a commutative subject, the subject is taught to think in completely subject-centered ways, according to Habermas' theory. (Habermas, 1985, p. 294). The meaning of the world becomes a private construction of the isolated individual, instead of the result of social interaction and contingency.

We may notice that interactionism attempted to point out this theoretical signalization in the field of Didactics of Mathematics. Its arguments are in the same direction (Voight, 1994; Yackel & Cobb 1996; Steinbring, 2006). Still, the above mentioned remain restricted in the cognitive field, aiming at the better understanding of mathematics without questioning the content of the mathematics subject per se.

For example, the concepts of probability theory taught in sixteen years old pupils emerge from a background already deductive; thus, the contingency of human situations is not comprehended while the structuralist input appears normative up to fundamentalism. The impression created is that everything can be organized and resolved. However, the anticipated normative world is never actualized, since it is a world of human decisions and complicated correlations. As a result, combinatorics and theory of games is by far more directly connected with real life.

THE NEW NATURALIZED TRUTH

Following the failure of the demand «Mathematics for all» posited by the program of Education of Mathematics (Lerman, 2000, Straehler-Pohl & Pais 2014), serious questions were posed. As far as mathematical education is concerned, in the reality of the crisis, the most significant ones, being the ones asked by Pais, Stentoft and Valero (2012); more specifically, the shift "From Questions of *How* in Mathematics to Questions of *Why* in Mathematics Education Research".

In order for the question not to be merely rhetorical, the present should be comprehended in its historical as well as in its contingent evolution. Thus, regarding the above question, the paradigm of Lyotard is employed, at a time when neoliberalism's results were not so obvious. Lyotard prophetically poses the paradox meta question, the question of legitimacy: "What is your "what is worth" worth"? (Lyotard, 1979, p 54).

The problem remains. Lyotard, (1979) phenomenologically discerning the new political situations, wonders: "*Who* decides what knowledge is, and who knows what needs to be decided? In the computer age the question of knowledge is now more than ever a question of government" (Lyotard, 1979, p. 9). The term *governmentality* has been introduced by Foucault (1977, 1978). Foucault argues that people are governed into practices of normalization with their consent, as opposed to being governed simply by authoritarian principles (Walkerdine, 1990).

Lyotard does not hesitate to resort to cynical ascertainment of the new postmodern world.

> "A process or a set of conditions either 'contributes' to the maintenance (or development) of the system or it is 'dysfunctional' in that it detracts from the integration, effectiveness, etc., of the system. The technocrats also subscribe to this idea. Whence its credibility has to become a reality, and that is all the proof it needs. This is what Horkheimer called the 'paranoia' of reason". (ibid., p.12).

Thus he ascertains: "The games of scientific language become the games of the rich, in which whoever is wealthiest has the best chance of being right. An equation between wealth, efficiency, and truth is thus established" (ibid, p.45). He carries on the same pattern: "The State and/or company must abandon the idealist and humanist narratives of legitimation in order to justify the new goal: in the discourse of today's financial backers of research, the only credible goal is power". (ibid, p. 46).

DISCUSSION

All of the above (20 years before the Bologna Process for education) have already been formulated in the context of philosophy and sociology and relatively recently this discourse has been reverberated in the discussion field of mathematics' education. This infinitely dynamic postmodern approach to knowledge, including knowledge created by mathematics education researchers, takes criticism a step further by examining mathematics education research itself, investigating how researchers are complicit in the (re)production of the power relations inherent in the discursive practices of mathematics and mathematics education research. In short, postmodern theory provides a vehicle through which mathematics education researchers can unmask both the superficial neutrality of mathematics education reform and their own subsequent complicity in (re)producing a certain régime of truth according to which the teaching and learning of mathematics deserves a privileged place in the universal educational system. Ultimately, this privileged treatment of mathematics per se might be complicit in the degradation of the human spirit (In Stinson, & Bullock. (2013, p. 15) As a consequence, we are confronted with the hidden meaning of naturalized truths.

> "In fact, educational policies are increasingly influenced by an expert-discourse. This movement of experts creates and circulates concepts and ideas without structural roots or social locations. These new ways of governing have proved to be extremely attractive. They are very sophisticated in naturalizing policies, in raising a sense of inevitability. It is as if they "only" construct data, or identify good practices, or compare best methods, whereas, in truth, these data, practices and methods are in themselves powerful political tools. In this sense, the process of "learning from one another" is a way of thinking and acting which establishes an educational policy without specifically formulating it. That is why I have been using the expression "governing without governing" to describe the process of elaborating policies through "statistics" while constantly giving the impression that no policy is being implemented". (Novoa, 2013 p.146).

Foucault (1972, p. 151) observes insightfully how discourses and practices "obey that which it hides" and become "the path from one contradiction to another". He argues that "to analyse discourse is to hide and reveal contradictions; it is to show the play that they set up within it; it is to manifest how it can express them, embody them, or give them a temporary appearance".

The Greek word poria means direction, and a – poria (query, in English, see Derrida in Moran, 2000, p. 436) can be interpreted as "without direction" (Mendick, 2011). Modernity assumes that the very directedness of progress is pointed out by scientific rationality. This rationality represents a direction, a-poria. This leads to the intellectual optimist, which we find in modernism. When we give up [this] assumption, we find ourselves in quite a different epistemic situation. We are without direction, an image which shows the European but in particular the Greek experience.

EPISODES

Let us watch some episodes of this embrace of curricula with politics.

Episode 1

Due to misguided politics, reforming the national curriculum could become an exceptionally vulnerable venture, notably in Greece. In spite of the efforts of the teams in charge of the curricula reform, and the fact that innovative resources were produced and there was a positive evaluation of the pilot implementation of the curriculum (including the mathematics), its generalization to all schools, five years later, is still under discussion. Changes in educational policy (due to new governments, ministers of education, centers of educational policy) actually constrained this effort, confirming Novoa's (2013) assumption that knowledge is a question of government.

Episode 2

Part of neoliberism's favorite toolkits is the Program for International Student Assessment, known as PISA. PISA assesses students' practical knowledge in literacy, science and mathematics, across various countries.

According to the Greek Institute of Educational Policy, PISA's aim "is not to assess the national curricula but to assess the knowledge and skills which are considered –by the specialists of the countries which take part- the most important in order for the students to be integrated successfully in contemporary society" (Institute of Educational Policy, Announcement regarding the results of the latest PISA examinations, 6/12/2016)

As Popkewitz has pointed out "the making of numbers as "facts" (is) a presumption that makes the comparisons of PISA possible…. The comparing that inscribes a seeming naturalness to answers in different national setting" (Popkewitz, 2011)

Thus quantification, comparison and prioritization impose a "reality" based on numbers ("objectivity" and "necessity") concealing the fact that political choices need to be made in order to resolve problems. Similar to the world wide strategy of identities and salaries equalization.

Episode 3

Educational policies throughout Europe perpetuate social inequities. In Bray's study (2011) on behalf of the European Committee, it is indicated that in many European countries, social competition, school performance rankings, examination-based learning and anxiety of families and children have been a leading force in the expansion of private tutoring, designated as 'shadow education'. Also, financial cuts have reduced the ability of educational institutions to provide individual learning support within school. (Bray, 2011)

What is surprising though is Bray's concern: "much tutoring is of low pedagogic value: in some countries, anybody can become a tutor without professional qualifications or a business license" (Bray, 2011), as well as his suggestion that policy makers should consider ways to regulate and guide the shadow education system, whereas one would expect policy makers to find ways to render private tutoring unnecessary.

Episode 4

Curricula do not take into account what Bourdieu (1986) called "educational capital", which is inherited by the families to their children. The ones belonging to higher social classes dispose specific educational privileges, knowledge, skills, "good taste", values equivalent to the ones promoted by school, positive attitude towards learning and high expectations! These privileges are not a perquisite for school, nevertheless, they are acknowledged and rewarded. In fact, they are assessed as inherent properties of the students, as "intelligence" or "gift" (ibid.).

Much of the above apply for Greece. Neoliberalism's "measures" for overcoming the crisis, leave little space for learning support financed by the state; mathematics is one of the most common subjects for private tutoring, since it is involved in high stakes examinations in order to be accepted in university faculties.

The current government in Greece is taking baby steps in educational reform. Students no longer need to take mathematics exams in order to attend medicine faculties, which was a great relief for both students and their families, and a serious decrease of mathematics subjects taught in secondary education is considered. Both measures appear to take into consideration the basic question: "Why mathematics and why this concept?" or, as Gutierrez has put it: "The rush to move on to the next mathematical concept (or response to intervention procedure) almost ensures we will not ask why this concept? Who benefits from students learning this concept?" (Gutierrez, 2013, p. 37).

In addition, vocational and technological secondary education, which has been neglected for ages, is being reformed taking into account that graduates need not be acculturated and students do possess skills that are useful both to them and to society.

Still, a lot needs to be done. We need research studies as to how the last six years of financial crisis have affected the learning of mathematics. We can no longer ignore the presence of media in our classrooms. They promote the need for certainty and for normalized truths and thus create chaos in our students' minds, once they face reality. Poverty is also present in our classrooms. With the rate of unemployment being approximately 49,8% in the ages of 20-30 years, our students do not worship scientific progress anymore, nor do they wish to be "enlightened" citizens. They prefer to be working, *happy citizens*.

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EQUITY IN A COLLEGE READINESS MATH MODELLING PROGRAM: LIMITATIONS AND OPPORTUNITIES

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In the United States, secondary school students can complete a university class taught by a qualifying secondary school teacher at their secondary school, a program known as concurrent enrolment, and at our university as College in the Schools (CIS). This practitioner reflection reports on the first five years of an equity-focused CIS course in algebraic math modelling. We use several artifacts to highlight tensions and affordances in the achievement of equity goals: analysis of enrolment data by race/ethnicity; teacher political commentary at a moment of crisis for the course; and reflections by the course coordinator.

INTRODUCTION

Young adults seeking a higher education in the United States experience crisis in terms of access, preparation, and enduring financial risk. Students from non-dominant racial groups and from low income backgrounds tend to enter less-prestigious universities and they tend to take –frequently using student loans– more remedial classes that do not earn credits towards graduation compared to White and higher income students (Complete College America, 2012; Venezia & Jaeger, 2013). Across all races and income levels, few students complete both remedial classes and the subsequent credit-bearing university classes (Complete College America, 2012). The debt incurred while pursuing U.S. higher education is extremely high and may contribute to increasing racial economic inequality (Jackson & Reynolds, 2013). In the U.S., algebra courses are among the most highly enrolled undergraduate courses and many students fail, withdraw or receive poor grades in these classes, requiring re-enrolment (Haver, et al, 2007). It is plausible, therefore, that early undergraduate algebra classes contribute to the student loan debt problem and difficulties completing a university degree.

United States' federal legislation allows secondary students, under certain conditions, to earn university credit while in secondary school. Through this College in the Schools (CIS) program, students can complete various university courses taught by qualifying secondary teachers at their secondary school location. Students earn university credit that is free to them. The secondary school pays a nominal fee per student registration. This dual or concurrent enrolment option is an increasingly important strategy for students and their families to access higher education and to make it affordable.

This practitioner-based discussion essay uses several modes of reflection to report on an equity-focused CIS course in algebraic math modelling. First, we analyse enrolment data to describe the degree to which the equity objective was achieved during the first five years of the CIS math modelling class. We also offer teachers' perspectives –positive and critical ones– to provide a sense of the opportunities and tensions that they experience within the program. We use Harouni's 2015 commentary on the political economy of mathematics to understand these dilemmas in terms of the interaction of secondary and tertiary educational institutions. Though this project is grounded in U.S. educational structures, it is hoped that this paper will foster conversation on equity issues in the transition to higher education in other countries.

THEORETICAL FOUNDATION

Harouni describes the historical and economic processes that led to the procedural and instrumental approach to mathematics education –commercial and administrative mathematics as he calls it– that is common in some European primary schools (2015). Over the 16th to late17th centuries, reckoning school curricula was incorporated into primary schools to support the growing mercantile class, and primary school curricula incorporated elements of university curricula –especially Euclidean geometry– in order to smooth this transition. Based on this case study, Harouni proposes four principles of a political economy of mathematics (2015, p. 71):

1. The economic purpose of math defines its most basic characteristics.
2. The economic characteristics of math impact how it can be taught.
3. The institutional setting within which math is taught also modifies the character of its practice.
4. All of the foregoing aspects impact one another in relation to the socioeconomic forces that shape them.

In this paper, we will focus largely on Harouni's third and fourth points, to understand how different educational institutions interact through the CIS math modelling course to produce and limit teachers' experiences and educational access for students.

EQUITY GOALS OF CIS MATH MODELLING

The first author, Staats, has taught the algebra-based math modelling class in the College of Education and Human Development, University of Minnesota for eight years. For seven years, she also worked within the university CIS program to coordinate secondary school offerings of the class. Using the modelling perspective, students create mathematical methods for solving realistic problems such as how to design a public rent-based bike-sharing program, or how to divide student athletes into “fair” teams based on their performance data. The course perspective on modelling differs from standard modelling approaches in that students can generalize and improve their solution to the teacher-posed problem, or they can pose and answer a new, related but slightly more challenging problem that appeals to them (Staats & Robertson, 2014).

The CIS math modelling classes are offered through an equity-focused CIS sub-program called the Entry-Point Program. Entry-Point Program CIS teachers are asked to reserve 60% of their seats for students who are racially underrepresented in higher education, English-language learners, of low-income, in their families’ first generation to attend college, or who are in the middle percentile rankings of their secondary school class. Traditional CIS programs have targeted the highest-scoring students in a secondary school; the Entry-Point Program is thus an innovative effort to offer university experiences to students who could benefit the most from them. Teachers are expected to manage their class, including selecting students for participation.

The course cohort had grown dramatically from two schools in 2009/2010 serving 24 students to 28 schools in 2015/2016 serving over 700 students. Teachers receive substantial face-to-face professional development through three annual cohort workshops and through classroom observations.

EQUITY AND ACCESS AS FRAMED BY DATA

We used an opportunity ratio to compare the racial and ethnic composition of CIS math modelling classes to the composition of 11th and 12th graders in each school for academic years 2009/10; 2010/11; 2011/12; 2012/13; 2013/2014. Data are based on College in the Schools office enrolment records and Minnesota Department of Education school population records (n.d.). This theme was chosen because data on race and ethnicity are available for both CIS participation and school composition; other data of interest, such as ELL status or class rank, are not available for CIS participation. The opportunity ratio compares the percent of students of a particular category in a CIS class to the percent of students in the same category in the last two years of secondary school, the most likely to enroll in the class. The following is a sample opportunity ratio calculation:

$$\frac{Percent\ of\ Asian\ students\ in\ a\ school's\ CIS\ class}{Percent\ of\ Asian\ 11th\ and\ 12th\ graders\ at\ the\ school\ overall} = \frac{0.65}{0.47} \approx 1.37$$

This opportunity ratio is greater than 1, suggesting that in this year, Asian students were over-enrolled in the algebra class compared to potential Asian students at the school. In the Twin Cities, most Asian students belong to Southeast Asian immigrant families, particularly Hmong. This opportunity ratio is therefore consistent with the equity mission of the math modelling course.

To reduce potential for identifying schools, we offer broadly summarized results. Table 1 shows the maximum opportunity ratio for each ethnicity in each year across three regions: schools in the inner metropolitan area, which tend to be ethnically diverse; the outer or suburban metropolitan area, which tend to have variable but increasing ethnic diversity; and greater Minnesota, which are more rural areas that tend to be less ethnically diverse. Table 1 shows the maximum opportunity ratio among CIS schools for each region for the first five years of the program. The minimum opportunity ratio for academic years 10/11 to 13/14 was nearly always 0, meaning that there was a school in the region that had at least one available student of a particular ethnicity but didn't enrol the student in the math modelling class. *NS* means that this grouping did not have participating schools in that year. *NA* means that schools in this grouping did not have any 11th or 12th graders of a particular ethnicity in that year.

Interpretation of Table 1 is limited by several factors. Data are sensitive to small class sizes and small numbers of non-White students available in some schools. Several inner metro schools that serve students in the target population have very small class sizes, which is an advantage for the students, but they do not influence the "combined" results much. Further, if a student enrols in a class at a higher level than the algebra-based math modelling class, it's a good thing, but it reduces the opportunity ratio.

Table 1: Opportunity ratio by region and race/ethnicity for academic years beginning 2009 to 2013.

	09-10	10-11	11-12	12-13	13-14	09-10	10-11	11-12	12-13	13-14
	Native American					Asian				
Inner	3.5	0.0	7.1	14.4	4.3	0.0	2.5	1.8	6.8	1.2
Outer	NS	NS	NS	0.0	6.3	NS	NS	NS	4.2	1.1
Greater	NS	NA	0.0	2.8	0.0	NS	NA	15.6	10.5	10.5
Combined	3.5	0.0	1.2	1.8	0.5	0.0	1.8	1.4	1.5	1.8

	Latino					African American				
Inner	1.4	1.3	3.3	4.0	1.7	4.5	0.6	1.1	1.2	1.6
Outer	NS	NS	NS	1.7	1.9	NS	NS	NS	0.0	1.7
Greater	NS	0.3	0.8	0.0	2.1	NS	NA	0.0	0.0	0.0
Combined	1.4	0.7	1.2	1.4	1.4	4.5	0.3	0.8	0.5	0.6

	White				
Inner	0.9	15.5	2.7	1.1	1.2
Outer	NS	NS	NS	0.9	1.0
Greater	NS	1.1	1.1	1.1	1.0
Combined	0.9	1.0	0.8	0.8	0.8

Despite the ambiguity of the results, we believe it is important to monitor ethnic and racial participation in the CIS algebra program. A positive result is that the combined course cohort has never over-enrolled White students, providing opportunities for students of other ethnicities. In many cases, the program has met its mission by over-enrolling Asian, Latino and sometimes Native American students. In a few cases, schools in greater Minnesota have used the class to serve ethnically diverse students in their mostly White schools. A negative result is that the course cohort has almost always under-enrolled African-American students. It is possible that the course cohort has not met its mission for African American students. Lower access for African-American students could be explained by small samples, teacher bias, lower student preparation, and students self-selecting out of this concurrent enrolment class. At a broader level, we know that Minnesota has one of the highest secondary graduation rate disparities in the country, as well as one of the highest employment disparities between African-Americans and Whites (Minnesota Advisory Committee, 2013). Our programmatic concerns reflect broader structural inequalities in our state.

EQUITY AND ACCESS AS FRAMED BY TEACHER COMMENTARY

In September of 2014, a collegiate reorganization put many classes for first-year undergraduates in the College of Education and Human Development at risk of ending. If the on-campus math modelling course were to be terminated, then the secondary school offerings would end also. At this moment of crisis for the CIS math modelling program, the CIS administrative office emailed a request for support letters to all participating teachers and their principals. About a third of the secondary schools submitted support letters to the college dean requesting continuation of the class, which the dean subsequently supported. We can take these commentaries not as objective representations of the program, but the respondents' most positive perspectives of the program (reproduced here with their permission). Most support letters came from inner metro schools, but some came from outer metro and greater Minnesota schools.

Several urban educators commented on the way school goals are facilitated by the class. "PSTL" is the course designator that students use to register for the course. Summaries or parenthetical comments are in brackets.

> *Inner metro* teacher 1. This class has had a very positive effect on our school and students in a very short period of time. Our student population is completely English Language Learners and mostly recent immigrants living in poverty. The CIS program has allowed us to offer higher level academic classes within our sheltered instruction framework to meet the needs of these students in ways traditional high schools cannot.
>
> Inner metro teacher 2. [In addition to filling gaps in the math curriculum, the] class offers another opportunity of helping a student start to see themselves as college students. As an urban educator many students have aspirations but don't understand too much what needs to be done to transition from a high school student to a college student. Hope is an important offering of PSTL 1006 beyond the content.
>
> Inner metro principal 1. This class has been an excellent addition to our math curriculum as it gives students a taste for what college courses are like and gives them a sense of hope that they can indeed succeed in the post-secondary world… Most of the students here at [our school] who take PSTL are a little afraid of college level classes, but after attending UMN class work days [CIS students visit the university for on-campus math activities] and succeeding in class, they start to realize that their future has a lot more potential than they originally thought.

Another urban educator from a school that serves many Hmong students valued the writing that accompanies modelling projects, along with the free university credits.

> *Inner metro teacher 3.* College Algebra through Modelling is one of the most significant courses I have taught in my 19 years as a math instructor… it was incredible to see the students beaming with pride the day they registered as a University of Minnesota student. The course is significant for our students because not only is it mathematically challenging for them, but also because of the modelling nature of it-- they learn important technical writing skills as well…The incredible carrot that I get to dangle in front of them? 3 semester credits from the University of Minnesota!

A suburban educator who works in an alternative school for students who were not successfully served by a standard school valued her students' change in self-concept.

> *Outer metro teacher.* Most of my students have had things happen in their lives that have made progressing in high school difficult. Many times it is due to family issues. [The coordinator told them] that they are talented and would fit into the course being taught on campus with no problem. From that moment forward my students were convinced that they had what it takes to be successful in the academic world. They could see that reasoning and learning through this type of process was not just something they could do, but something they could enjoy.

A teacher from greater Minnesota became very involved with modelling pedagogy through the program, and made the point that equity and access to higher education can be an issue for rural White students:

> Greater Minnesota teacher. Have you ever felt stuck in your job, like you had reached the end of your creative energy but saw no way out? That's the way I felt in 2012. [He loved middle school math teaching but wanted something new]...Now in my second year, this course truly changed my outlook... It has given me new energy for my teaching and my students... This course is reaching students that would not normally be able to earn college credit. [Our county] is one of the top 3 poorest counties in the state. This class has given [our] students $19481.70 worth of college credits. What a huge blessing for them!

TEACHER CONCERNS ABOUT THE CIS CLASS

To balance the positive commentary above, we summarize negative perspectives about the CIS math modelling course that teachers raise consistently in evaluation surveys and in our three-times yearly professional development workshops. Based on a review of teachers' program evaluations and workshop agendas, teacher concerns generally fit into three categories: alignment of grading, assessment and policy with the on-campus classes; pedagogical approaches; and whether the University of Minnesota math modelling credit is honored at other universities. As reflective practice research, these finding are somewhat limited through the use of artifacts like program evaluation surveys that were not distributed for research purposes. We hope to conduct interviews with teachers in 2017 to give voice to their summative perspectives on the program.

Teachers tend to worry about whether their class practices and policies are sufficiently aligned with those of the university class. This is a very important concern, because the national accreditation of concurrent enrolment programs depends on equivalence of content, pedagogy and practices between secondary and on-campus classes. Concerns are about attendance, extra credit and late assignment policies, and grading practices in math modelling assignments.

Staats considers some of these concerns to be irresolvable due to the on-campus context of the class. The home program of the math modelling course has a long history of interest in educational equity for early undergraduate students across many disciplines. The curriculum is be student-centered and tends to be experimental, and often involves students in critical social commentary. As a result, on-campus classes may vary substantially in specific assignments, grading practices, and regimentation of student participation. In short, on-campus classes do not emphasize standardization across class sections. The directive of “equivalence” with on-campus classes therefore means that secondary teachers have similar choice, within boundaries, to establish their own assignments and classroom practices as do university faculty. Staats choses to keep this flexibility open for secondary teachers, even though it may cause them some anxiety, so that they can choose a student-centered approach that fits their school context.

DISCUSSION: INSTITUTIONAL TENSIONS AND AFFORDANCES

In the first five years of the CIS math modelling class, its equity mission has been fulfilled in ways that are partial and patchy, in response to the interaction of several institutional conditions. Latino, Asian and sometimes Native American students have generally had opportunities to enrol in the class, but African-American students have had fewer opportunities. The first schools to participate were in the inner city, where most schools enrol many students in the target population. However, for several subsequent years, most of the new schools added were in greater Minnesota and in suburbs of the Twin Cities. These schools enrol students with some of the target characteristics such as low-income or first generation status, but are not always racially or ethnically diverse. Some teachers enjoy and value the program greatly, but the conditions of the program require a great deal of effort and decision-making of them.

In the 17th century, the curricula of reckoning schools, grammar schools, and to some extent, university mathematics, blended to serve the economic needs of the middle and upper classes. Similarly today, concurrent enrolment programs like CIS provide a conduit between secondary and tertiary educational institutions. Redefining CIS student participation guidelines have made this pathway somewhat more accessible to diverse students. As Harouni suggests, the “economic purpose” of mathematics –or of a mathematics class– interacts with the affordances of teaching and the institutional setting of the mathematics class (2015, p. 71). Our current reflections highlight two broad tensions in the program, and an affordance, which derive from the interaction of secondary and postsecondary educational institutions.

Tension: CIS math modelling cannot ensure the fulfilment of its equity mission.

Because the University of Minnesota is a publicly-funded institution, it cannot restrict participation on a racial or ethnic basis. While racial equity is an important component of the Entry-Point CIS mission, it cannot reject schools with low racial diversity, if the teacher qualifies. Reasons for the early expansion of the math modelling class in rural and suburban areas are not clear, but critical race theorists have observed that programs intended to improve racial access to a public good can inadvertently support the status quo because dominant groups may access the program more successfully than marginalized ones. This effect has been documented for higher education access programs in the U.S., India and Brazil (Pazich & Teranishi, 2014; powell, 2009). For the CIS program, observing institutional interaction suggests that secondary schools have significant power to access university curriculum regardless of whether their school serves diverse, low-income or first generation students. The qualifications of the mathematics teacher matter more than the school's population of diverse students.

Tension: CIS math modelling can't ensure an accelerated first year at university.

CIS classes offer the possibility of reducing student time at university and thereby student debt, a motivating factor mentioned by two teacher support letters. This economic utility explains much of the dramatic growth of the program. Students hope that the CIS math modelling class will replace a mathematics class that their intended academic program requires. A CIS alumni has earned University of Minnesota math credit, but we have little control over the course replacement choices of other institutions of higher education. Some universities accept math modelling as an equivalent for an algebra class, and some accept the class only as graduation credit, so that the student must pass through the gauntlet of early undergraduate algebra classes, usually with a highly procedural pedagogy; sometimes with low success rates. The institutional independence of universities across the U.S. affects the equity mission because there is no way to ensure that diverse students who complete the class will be able to parlay it into an accelerated experience in their first university year.

Affordance: CIS math modelling is a protected space for teaching and learning.

The most important requirement for U.S. accredited concurrent enrolment programs is that the secondary class conforms to the curricular content and pedagogical practices at the sponsoring university. In the math modelling program, this means that secondary teachers enjoy substantial flexibility to create new models and to experiment with inquiry pedagogies. The curriculum is determined by the university and not by the secondary school or school district. Students, too, enjoy some flexibility in their work because they can either generalize the teacher's model or they can pose and answer a related mathematical question for the model extension. In this way, the interaction of tertiary and secondary institutions produces a space where teachers and students have opportunities to experiment with personal forms of mathematical expression.

This protected space is best realized during students' model extensions and in CIS class projects that are presented at on-campus Field Days. We feel that the majority of these fit within Harouni's commercial-administrative category of mathematics education. However, sometimes students use this part of the assignment to create problems that fit into other types of mathematics that Harouni defines. An on-campus student, for example, extended a model about a math game (admittedly, not a game that people would play naturally) by asking, "If (an American) football game could go on forever, would every number be a possible score?" The abstract nature of this question perhaps qualifies it as philosophical mathematics.

CIS classes present models at on-campus Field Days. Some are conceived by individual students, some selected as a full-class project, and some are suggested by the secondary teacher. Fairly often, these models are social-analytical. For example, a group of inner metro students created a data-based argument that their day-to-day classroom performance measured by grade point average was a better predictor of success at college than college entrance exams. A greater Minnesota group experimentally measured the power output of hunting rifles to determine which were compliant with wolf hunting regulations in both Minnesota and the neighboring Canadian province of Manitoba. An inner metro student expressed her enjoyment of K-Pop (Korean popular music) by developing a method to predict the current K-Pop song of the year (she got the group right but not the song). This last example does not have utilitarian, social justice, or even artisanal dimensions, and so it suggests an addition to Harouni's purposes of school mathematics: recreational math. Some of these are highly-developed examples, but each modelling activity allows students to deviate from the teacher's posed problem. This is a positive outcome of the interaction of higher education and secondary institutions through concurrent enrolment.

CONCLUSION

Experiencing university classes while in secondary school has a strong economic and intellectual value for many students and families in the United States. The College in the Schools program has the potential to reduce student debt, shorten time in college, and familiarize students with the increased challenge of university curriculum while in a more familiar secondary school setting. The Entry-Point subprogram prioritizes enrolment for students who are poorly represented on university campuses.

In many cases, these equity aspirations are fulfilled. Traditionally underserved students of color and low-income students enjoy challenging curriculum. Teachers extend pedagogical knowledge and students reduce the cost of their first year at university. Still, differential levels of power in the interaction of various educational institutions –the University of Minnesota, secondary schools, and other universities– create contradictions. Secondary schools enjoy more power than one might expect in gaining access to university programs. Teachers gain substantial control over their mathematics class, even when they wish it was more standardized. The highest status public university in Minnesota is not always recognized as a producer of exchangeable university mathematics credits. Despite the equity mission, African-American students are not able to enrol in the class in proportion to their school population. The political economy of concurrent secondary and university enrolment is fraught with tensions.

The institutional discussion in this paper requires it to have a very local purview. We would therefore like to close with discussion questions that may support readers in making comparisons to international settings:

- What curricular or programmatic choices are available to secondary schools in other countries that might smooth the transition from secondary to tertiary education?
- How is inequality in access to tertiary education addressed in other countries?
- How can equity-focused programs housed in larger institutions ensure their stability and effectiveness?

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THE CONCEPT OF THE TANGENT IN THE TRANSITION FROM EUCLIDEAN GEOMETRY TO ANALYSIS: A VISUALIZATION VIA TOUCH

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Tangent lines appear in different contexts depending on grade [“class”/”year of high school”] leading to confusion among students during the transition from Euclidean geometry to analysis. Biza, Christou, and Zachariades (2008) study this transition during the last year of high school, as experienced by Greek students. Building on this study, we attempt a comparative study of the strategies students with visual impairments use in forming conceptual images of tangents to determine if visualization by touch aids students’ understanding of tangents. We look at the roles tactile perception, gestures, and language have in the formation of conceptual images and definitions for visually impaired students. Finally, we examine these two surveys in relation to misunderstandings of tangents, as well as approaches to understanding tangents.

INTRODUCTION

This paper discusses the comprehension strategies that students with visual impairments use while learning the meaning of tangents and how visualization via touch may aid in acquiring this understanding. Currently, there are not many researchers working with students that have a complete loss of vision. With modern technology, this field acquires new dimensions and may allow for enhanced techniques to teach mathematics and other subjects. The conceptual image of the tangent for the majority of students aged 17 to 18 years is based on previous experience with geometry, specifically, the tangent of a circle. However, according to Vinner (1982, 1991), they may consider that the tangent is defined by a single point of intersection between the tangent and curve, which creates problems when dealing with tangents of functions. Based on this fact, we undertook a study based on the research of Biza, Christou, and Zachariades (2008) titled “Student perspectives on the relationship between a curve and its tangent in the transition from Euclidean Geometry to Analysis.” We compared the results of our survey with Biza, Christou and Zachariades (2008) to ascertain whether visualization can help reconstruct (Harel & Tall, 1991) a suitable conceptual image of tangents for students with visual impairments.

THEORETICAL FRAMEWORK

TANGENT

The concept of the tangent is one that a student may encounter in both school and everyday life. Moreover, tangents are encountered in a variety of mathematical fields, and this variety affects student perceptions. The initial concept of a tangent is introduced early in greek high school within Euclidean geometry. Later, the standard definition of a tangent is given. Students first construct a conceptual image of a tangent based on the fact that the tangent of a circle has only one shared point with the circle. Students may apply this statement to functions and assume that a tangent can only intersect or coincide with the function elsewhere (Vinner 1982, 1991). Tall and Vinner (1981) define the conceptual image as an overall cognitive structure associated with the concept, which includes all the mental images, relevant properties and processes that a student carries with him or her and which affect cognitive processes. On the other hand, the conceptual definition is a sequence of words used to give meaning to the concept. When considering tangents, the defining property of a tangent of a circle, that it intersects the circle only once, is transformed into a tacit definition of tangents in general. This is incompatible with the definition of a tangent when dealing with functions where the tangent may intersect a curve multiple times or even coincides with a function (Castela 1995; Tall 1987; Vinner 1982, 1991). The conceptual image of a tangent acquired while learning geometry is a generalized trajectory, and changing this requires a difficult reconstruction of the conceptual image (Seldon, 2006). Harel and Tall (1991) define reconstructive generalization as the reconstruction of "an existing schema in order to widen its applicability range". It is important not to have parallel schemas, thus it is important that the schema of a tangent introduced while learning geometry is later reconstructed to allow use with other curves beyond those of conic sections.

VISUALIZATION

Visualization, in mathematics, is directly related to the notion of mental image, meaning the mental representation of a mathematical concept or a mathematical object. Moreover, visualization is application of a conceptual image to mathematical problems to create mental images. Presmeg (2006) illustrates the mental image as a conceptual schema which depicts visual and/or spatial information. Essentially, visualization in mathematics is a process that must include manufacturing and transformation processes for both visual and mental images for any spatial representations that may be involved in a mathematical problem (Presmeg 2006). Arcavi (2003) defines visualization as the ability, process, and product of creating, interpreting, using, and reflecting on figures, pictures, and diagrams in both mental and paper environments. The purpose of visualization for Arcavi is to illustrate information, interactions, and ideas which allow development of understanding. The complex role of visualization is vital in relation to the understanding of mathematical concepts (Tall & Vinner, 1981), including tangents. Naturally, this does not mean visualization cannot lead to misunderstandings. Lastly, Bishop (1989) notes that visualization capability supports inductive reasoning processes and also can lead to translation of abstract relations. Specifically in relation to visually impaired students, visualization relies more heavily on tactile perception. Moreover, intuition through images created with the eyes of the mind takes a greater role than usual (Miller, 1987). A greater importance is placed on mental visualization in learning mathematical concepts as visual visualization is not possible. Thus visualization is not merely a visual product, but the method of achieving a more complete understanding of mathematical concepts for all students those with or without visual impairment.

TACTILE PERCEPTION

Tactile perception is a complex process that includes more than the sense of touch (Millar 1994, 1997). The sources and structures of tactile perception fall into two categories:

1. Primary information sources such as touch, movement, and posture
2. Secondary information sources such as language, prior knowledge, object type, and conditions in which an activity is performed

These two sources of information build a multidimensional conceptual image that is a powerful tool for collecting information that is clearly much more important for visually impaired students than for others.

LANGUAGE

The foundation of mathematical reasoning is the mathematical language that includes symbols, terms, notations, definitions, logical rules, and syntax. Mathematical language also requires effective use of these components to justify, represent, understand, and communicate concepts, relationships, and information. Disagreements can arise from different uses of the same terminology, while others are rooted in substantive and conflicting mathematical assertions (Crumbaugh, 1998; Lampert, 1998). The term "language" is used to refer to the whole linguistic infrastructure that supports mathematical communication and which meets requirements for accuracy, clarity, and economy of communication. Thus verbal communication in the context of education should be based on terms and concepts for which meaning will be understandable to students.

Communication must be effective, as proof and language are closely related concepts, especially in mathematics. To visually impaired students, verbal communication is a more complex and important issue. For example, common verbs such as "see" or "feel" carry different meanings for visually impaired students. Also, visually impaired students can create a non-standard vocabulary of mathematical terms (Argyropoulous, 2008). For example, what is called an angle in standard mathematical vocabulary may be called a section or top in non-standard vocabulary. These factors must be considered by the instructor in order to ensure the highest efficacy in communication with a blind student.

GESTURES

In addition to verbal communication, nonverbal communication conveys a large portion of one's message. Generally, posture, gestures and facial expressions convey a large amount of information. Particularly in mathematics, gestures consist of a way for a speaker to access inaccessible verbal objects, facilitating easier speech. According to Iverson and Goldin-Meadow(1998) gestures are part of speech; they highlight spoken terms or replace terms that remain unspoken. McNeill (1992) supports the idea that gestures and speech flow from the same cognitive source.

Visually impaired/blind students' gestures and general posture are a primary speech source that helps an instructor build an understanding of deeper cognitive processes and thoughts. Sometimes, gestures can lead to thoughts or thought processes/patterns as Radford (2009) notes. Based on the above, in relation to tangents, when transitioning from Euclidean geometry to analysis, students may be working with a conceptual image that includes the notion that the tangent should not intersect a curve more than once. Students may also believe the tangent should always leave the curve to a half-plane. Verbal and nonverbal communication have important roles in the process of understanding and remodeling of the concept, particularly when addressing blind student's gestures and generally tactile perception. The mathematical cognition mediates not only through written symbols, but through actions, gestures and meanings of other species (Radford, 2009). In this case we will try to understanding the concept of tangent by blind children and we will rely on visualization via touch. The research question that arose through this survey is whether the visualization via touch facilitate the transition from the geometrical to analytical thinking on the concept of tangent to students with visual impairments.

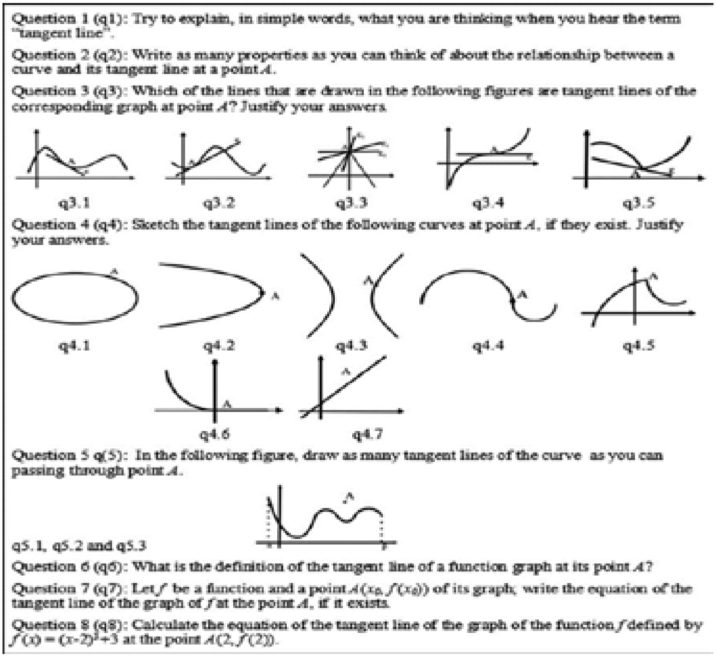
RESEARCH METHODOLOGY

The original research presented in this paper was carried out in a school for the visually impaired in Athens. Four students participated, M1, M2, M3, and M4. Three of them M2, M3 and M4 are in the last year of high school in the age of seventeen years old. M1 is a third year bachelor student studying at the Mathematics Department of the University of Athens. Two of the students are male while two are female. All four participants have received instruction in calculus, mostly in relation to functions, limits, derivatives, tangents of functions, monotonic functions, extremes, convexity, and points of inflection. This qualitative research was conducted through observation, interviews, conversation, and video recording of gestures and tactile examination. All required authorizations were acquired prior to the commencement of any research activities from both parents and school. The data gathered from the four participants was compared with a study conducted by Biza, Christou and Zachariades titled "Student perspectives on the relationship between a curve and its tangent in the transition from Euclidean Geometry to Analysis." This article was published in Mathematics Education in 2008. For that study, a questionnaire was submitted to 196 students in their last year of high school. The study examines students' conceptual images of tangents and groups them based on differences in conceptual image in relation to the progression from Euclidean geometry to analysis. The results of Biza, Christou and Zachariades were in three groups, which are discussed further below.

DESCRIPTION OF RESEARCH

In the Greek education system, students encounter the concept of the tangent in different contexts. Initially, tangents are introduced in relation to circles in Euclidean geometry to first year high school students. During the second year of high school, students are introduced to the concept of a tangent line in relation to conic sections in analytical geometry. Finally, in the third year of high school, students are introduced to the concept of a tangent in relation to the graph or curve of a function. At this time the tangent is defined as a line that intersects a function at a given point with the slope of the function at that point. Knowing the slope of the function also requires that the student has an understanding of derivatives. These three contexts allow us the opportunity to study the effect that each context has on the comprehension of tangents. More importantly, they allows us to reproduce the conceptual image of tangents that students have at each stage in this progression. The questionnaire in Biza, Christou, and Zachariades (2008) has eight questions (referred to as question 1 through question 8 in this article), some of which are subdivided. Question 3 asks students to determine if presented lines are tangent lines and to explain their answers. This is an identification question. Questions 4 and 5 require the student to construct tangent lines based on presented curves. Question 5 was trickier than question 4 in that there are three solutions.

Questions 1, 2, 6, and 7 ask for written explanations of students' understanding of tangents. For questions 1, 2, and 6 responses were not required to include mathematical language or terms. Question 7, conversely, required mathematical language and terms. Thus, questions 1, 2, and 6 ask for spontaneous answers that seek to expose the conceptual image held by the student, while question 7 refers to symbolic representation of tangents. The questionnaire is illustrated in figure 1.



Question 1 (q1): Try to explain, in simple words, what you are thinking when you hear the term "tangent line".

Question 2 (q2): Write as many properties as you can think of about the relationship between a curve and its tangent line at a point A.

Question 3 (q3): Which of the lines that are drawn in the following figures are tangent lines of the corresponding graph at point A? Justify your answers.

q3.1 q3.2 q3.3 q3.4 q3.5

Question 4 (q4): Sketch the tangent lines of the following curves at point A, if they exist. Justify your answers.

q4.1 q4.2 q4.3 q4.4 q4.5

q4.6 q4.7

Question 5 q(5): In the following figure, draw as many tangent lines of the curve as you can passing through point A.

q5.1, q5.2 and q5.3

Question 6 (q6): What is the definition of the tangent line of a function graph at its point A?

Question 7 (q7): Let f be a function and a point $A(x_0, f(x_0))$ of its graph; write the equation of the tangent line of the graph of f at the point A, if it exists.

Question 8 (q8): Calculate the equation of the tangent line of the graph of the function f defined by $f(x) = (x-2)^2+3$ at the point $A(2, f(2))$.

Figure 1: The questionnaire.

RESULTS OF BIZA, CHRISTOU AND ZACHARIADES (2008)

Biza, Christou and Zachariades (2008) demonstrated that students form a conceptual image of tangents when the tangential concept is introduced during instruction in Euclidean geometry. This initial conceptual image relies heavily on the idea that a tangent intersects the curve only once and does not pass through the curve. This conceptualization is particularly evident when students are presented with tangents that do not conform to these assumptions. Biza, Christou and Zachariades (2008) also identified a hierarchy of conceptual images that were grouped into three different groups (referred to in that article and this one as A, B, and C). In group A, students succeeded in reconstructing the conceptual image of a tangent to allow for a local, rather than global, understanding of tangents. In group C, however, students "approached the tangent as a line with a specific relation to the entire curve" as tangents are contextualized in Euclidean geometry. In between these two groups was group B. While students in this group had not reconstructed the conceptual image of a tangent, they tried to transfer the existing conceptual image to new scenarios, with result a partial abandonment of a global approach to tangents. In relation to specific questions, 93 percent of students identified the same tangent in question 5. The tangent identified in this question has only one common point with the curve or graph, thus it aligns with each group's conceptual image of a tangent. Students in group B outperformed students in group C for questions 3.1, 3.2, and the two remaining tangents in question 5. These questions required a tangent to intersect the curve or graph at some other point. While groups A and B had comparable performance in 3.1 and 5.2, students in group A had greater success with questions 3.2 and the last remaining tangent in 5. For these latter two questions, the tangents had more than one point in common and there was confusion around the contact point. Only group A had notable success in questions 4.6 and 4.7. For these two questions, the tangents shared an infinite number of points with the curve, requiring students to have completely reconstructed the initial conceptual image. Group B faired similarly to group C for these questions. Likewise, group B and C faired similarly on 3.4 and 4.4 where the tangent occurs at a point of inflection and thus passes through the curve or graph. Again, here a correct response requires a reconstruction of the conceptual image of a tangent. Only those in group C fared poorly on questions 3.3, 3.5, and 4.5 where tangents were not possible due to the curve being unable to be differentiated at the point of intersection. This group also performed worse than the other two in symbolically representing the concept of a tangent in questions 7 and 8. All questions that used conic sections were largely correctly answered by all groups.

RESULTS OF RESEARCH ON STUDENTS M1, M2, M3, M4

The responses of all four students differed for most questions; however, they were similar in one aspect: they did not take into account whether the curve was differentiable where the tangent intersects it. Students M1 and M4 appeared to have a clearer conceptual image, as defined by Vinner (1982, 1991), than M2 and M3 of the tangent based on their experiences in learning Euclidean geometry. Based on the answers of M1 and M4 to question 1 both reply that the tangent is the line that shares only a single common point, or "touches" a circle at only one point. The most important criterion for these students in correctly determining tangency is whether or not there is a second common point of the tangent with the function. For example, in response to questions 3.1 and 3.2, both stated that the line is not tangential. Conversely, in response to questions 3.3 and 3.5, M1 and M4 applied the same criterion and state that the presented lines are tangential, despite the fact that the functions shown are not differentiable at the point of intersection. Interestingly, they answer question 3.4 correctly without rationalizing their answer. They continued, in question 3.4, to apply the definition of a tangent as having only one point in common with the graph or curve. Subject M2, however, answers 3.1 as being tangential, perhaps as the tangent line is not extended to a second point of intersection, but in 3.2 wonders if a tangent a second point of contact means a line cannot be tangential:

M2: Is it important that the line cuts the function in a second point?
Q: What do you think? Does the fact that it cross the graph at a second point mean that it is not tangent?
M2: I would say that at point A it is tangential while at the other common point it is not.
Q: So it is tangent to a region of A?
M2: Yes, to one region

In question 3.3 M2 answers that only the horizontal line is a “normal” tangent, while in question 3.5 she repeats that the line is tangent because it has only one common point with the function. In response to question 3.4 she explains:

M2: It is not [tangential], because the tangent intersects the function, it is not tangential.
Q: [...] because the function is in both planes separated by the line?
M2: Yes.

For questions 4 and 5, the students were asked to draw a tangent to a curve that intersects, point A. The responses given were captured on video. The figures below show the responses in the clearest possible way. Answers to both question 4.1 and question 4.3 were correct, which fits the findings of with Biza, Christou and Zachariades. M1 defended her response to 4.3 arguing that there is no second common point (see Figure 2).

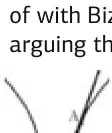


Figure 2

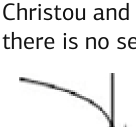


Figure 3

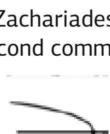


Figure 4

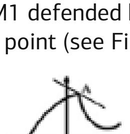


Figure 5

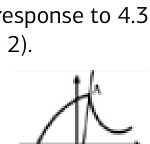


Figure 6

For question 4.2, students answered that the tangent will be as shown in Figure 3. M3 justified his response by saying that the function is located entirely in a half plain relative to the tangent line while M1 stated:

M1: We could draw the tangent like this (Figure 3).
Q: So could we draw it another way?
M1: Yes we could draw it like that (Figure 4).
Q: Could we draw it another way beside the horizontal and vertical?
M1: No, because it would intersect the function elsewhere

For question 4.5, the responses of both students indicated that they believed the line is tangential at point A without considering that the graph is not differentiable at point A (Figures 5 & 6).

M4 – with questions 4.4, 4.6, 4.7 – answers that we cannot draw a tangent, rationalizing his answer for question 4.4 by stating “the line divides the function into two planes” while he justifies his answers for question 4.6 and 4.7 by stating “that the tangent cannot have infinite common points with the function”. M2 and M3 responded to these questions based on their misconception that it is sufficient to have only a single common point with the function. Figures 7, 8 and 9 illustrate their responses.

Figure 7

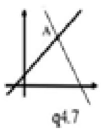


Figure 8

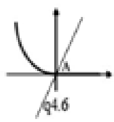


Figure 9

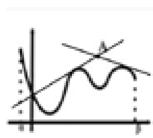


Figure 10

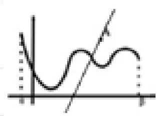


Figure 11

For question 5, M3 provides two tangents, seen in Figure 10, while M4 states:

M4: I can draw this tangent (Figure 11) [after examining the sculpted figure and indicating a line] ... we can draw infinite tangents.
Q: Is there a criterion all the lines that pass through point A are tangent?
M4: No! The lines should not “meet” the curve more than once!

Lastly, surprisingly all the students made a successful transition from the conceptual definition of a tangent to the standard definition by correctly answering questions 6, 7, and 8.

SUMMARY OF RESULTS

In summarizing the results of the research presented here and in Biza, Christou and Zachariades (2008), the first impression is that all students (M1, M2, M3, and M4) answered questions 4.1, 4.2 and 4.3 correctly, which matches the findings of Biza, Christou, and Zachariades (2008). In contrast, only one of the four subjects in this study correctly identified the tangent in question 5 that has only one common point with the curve. This is despite the fact that 93 percent of respondents in Biza, Christou and Zachariades succeeded in constructing this tangent. For question 3.2 and the two remaining tangents in question 5, group A students did better than any other group. Of the four students examined here, none successfully answered these questions, though M2 successfully identified one additional tangent in question 5. Group A did well with questions 4.6 and 4.7, while M1, M2, M3 and M4 each responded incorrectly for different reasons. Likewise, while M1, M3 and M4 correctly answered 3.4 and 4.4, like most of group A, they did so with incorrect rationale. M2 in contrast mistakenly answered these questions considering that a tangent line in these cases intersects the functions. Biza, Christou, and Zachariades (2008) concluded that group C held a conceptual image of the tangent as including the notion that tangent line has a relationship with the entire curve or graph which was learned when studying Euclidean geometry. In contrast, students in Group A had adopted a comprehensive way of approaching the relation between the curve and tangent. Students in this group have reconstructed a conceptual image of tangent, in accordance with the terms of reconstructed generalization as proposed by Harel and Tall (1991). In addition to groups A and C, there are a group of students in between, group B. These students attempted to expand the application of the conceptual image to new scenarios, often by taking a less global approach to tangents. They largely expanded on the concepts that were well known to them, such as one common point with the curve remaining in the same semi-plane and applied these concepts in the neighborhood of the point of tangency. Based on these observations, we can group M1 and M4 in group C, while M3 and M2 fall into group B.

CONCLUSIONS - DISCUSSION

Based on the original research presented in this paper and previous studies, we can conclude that visualization via touch can help blind students, in certain scenarios, form a mental image (Presmeg 2006) of the tangent and use that mental image to reconstruct the conceptual image of tangents in the transition from Euclidean geometry to analysis. Comparing our work with that of Biza, Christou, and Zachariades (2008) shows that visually impaired students make the same mistakes as other students. These mistakes are rooted in maintaining the conceptual image acquired when first taught the concept of tangents in Euclidean geometry. For students with visual impairments, tactile perception and verbal communication have central roles in the reconstruction of the conceptual image (Harel and Tall 1991); because of this, it is important to question if visualization via touch enables better understandings of other mathematical concepts. If it does, how? Further research is required in this field as the quality of mathematical education for those with visual impairments ought to be the same as for others as their rights to a quality education are the same as those of others. In researching educational methods for those with visual impairments we may also uncover methods that are applicable to the wider student base while acquiring a greater understanding of cognitive approaches that students take in comprehending mathematical concepts.

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RESEARCHING "RACE" WITHOUT RESEARCHING WHITE SUPREMACY IN MATHEMATICS EDUCATION RESEARCH: A STRATEGIC DISCURSIVE PRACTICE

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In this essay, through reviewing three "equity" articles over the span of nearly 30 years, the author argues that researching race in mathematics education research has become a strategic discursive practice. But what about racism? What happens when racism is opened up –theoretically and methodologically– as an object of inquiry in mathematics teaching and learning? Doesn't researching racism require an examination of the pervasiveness of White supremacy? That is to say, can we (ethically) examine racism without examining White supremacy? After all, aren't racism and White supremacy two sides of the same coin?

INTRODUCTION

A few years ago, I wrote an editorial titled "'Race' in Mathematics Education: Are *We* a Community of Cowards?" (Stinson, 2011) The purpose of the editorial was to bring to light that the percentage of (Anglophone) peer-reviewed journal articles which address race and mathematics teaching and learning had stayed pretty much constant throughout the 1980s to 2000s, roughly 4%. Using the work of Lubienski and Bowen (2000) and Parks and Schmeichel (2012), I provided numerical evidence that there had *not* been a proliferation of "race talk" (or gender talk, or culture talk, etc.) within the mathematics education literature[1]. In building my argument to the provocative question *are we a community of cowards*, I made reference to some of the earlier research and scholarship that began explicitly attending to issues of race in mathematics teaching and learning, and then briefly highlighted current research and scholarship. In this essay, I revisit the editorial to do two things: (a) review and contextualize three journal articles on race and mathematics education; and (b) bring to the fore, for discussion, a vital aspect that continues to

1. The argument was counter to the collective sentiments of the mainstream or "White-stream" (Gutiérrez, 2011) mathematics education community at the time; see Martin, Gholson, and Leonard (2010) for a critical response to the assumptive question *Where's the math in mathematics education research?* (Heid, 2010)

be absent in research (and in conversations generally) on race and mathematics education.

A couple of caveats are necessary before I begin, however. First, the discussion is centered with and in a USA perspective; that is the sociohistorical and geopolitical context that I know. The discussion, however, is neither reflective of only the United States of America nor should it remain only in the USA context. Valoyes-Chávez and Martin (2016), building on the work of race theorists such as Omi, Winant, and Bonilla-Silva, recently argued:

> The meanings of *race* and *racial categories* are created, politically contested, and re-created in any given sociohistorical and geopolitical context as a way to maintain boundaries of difference related to domination and oppression.... No matter what country (e.g., the USA, South Africa, Brazil, and throughout the European Union), these meanings emerge to shape all social structures and institutions in a given society..., including mathematics education. (p. 1)

Second, the reviewing of the three articles on race and mathematics education over a span of nearly there decades is done cautiously. Given the limitation of space here, I attempt to capture only a few of the big ideas of the past and present. This essay and talks delivered at other conferences (Stinson, 2014, 2016) are an introduction, if you will, to a larger project of conducting a Foucauldian archaeology/genealogy (cf. Foucault, 1966/1994, 1975/1995; see, e.g., Bullock, 2013) of race discourses and discursive practices found in the USA mathematics education enterprise. Through the larger project, my intent will be to clarify, with respect to issues of race and mathematics teaching and learning, not only what we have been researching about (and how and why) but also, and perhaps more importantly, what we have not been researching about (and how and why).

RACE AND MATHEMATICS EDUCATION RESEARCH

In this section, I briefly review three articles that span nearly thirty years –1984 to 2013. Contextually, all three articles are from "equity" (broadly defined) special issues of the *Journal for Research in Mathematics Education* (*JRME*), the leading mathematics education research journal in the United States of America. Each of the three issues was guest edited by recognized leaders of the larger mathematics education community:

- Minorities and Mathematics – 1984 (Vol. 15, No. 2): Guest Editor: Westina Matthews
- Equity, Mathematics Reform, and Research: Crossing Boundaries in Search of Understanding – 1997 (Vol. 28, No. 6): Guest Editors: William F. Tate and Beatriz S. D'Ambrosio
- Special Equity Issue – 2013 (Vol. 44, No. 1): Guest Editor: Rochelle Gutiérrez

Westina Matthews – 1984

The first article reviewed is "Influences on the Learning and Participation of Minorities in Mathematics," written by Westina Matthews (1984b). This article was the introductory article, so to speak, to the first *JRME* equity issue. The special issue aimed to bring to the attention of *JRME* readers "various aspects of research into the learning of mathematics by minorities" (Kilpatrick & Reyes, 1984, p. 82). The *JRME* Editorial Board hoped to "provide a continuing forum in *JRME* so that reliable knowledge of the learning of mathematics by minorities is shared as widely as possible with people who can put that knowledge into practice" (p. 82).

Matthews (1984a), in her introduction, noted that the authors who contributed to the special issue represented a "rainbow coalition of researchers with a history of involvement and interest in the topic of minorities and mathematics" (p. 83). Many of the contributing authors had attended, in February 1981, the National Council of Teachers of Mathematics' (NCTM) Core Conference on Equity in Mathematics. In total, including the editorial, there were 16 mathematics educators and researchers who contributed to the 96-page special issue.

In her lead article, Matthews (1984b) marked 1975 as the starting point in researching "minorities" in mathematics education, but noted several problems that limited the "usefulness and appropriateness" of these early studies:

> One problem is that most reports of the studies are either unpublished papers or final reports to funding agencies and therefore are relatively inaccessible. Another problem is that some of the findings could be fortuitous in that neither the original nor the primary focus of the study was on minorities. More often than not, the study concerned sex-related differences, and race was included as a background variable. Inadequate reasons are then given to explain any race effects. (p. 84)

With the limitations of the existing research noted, Matthews (1984b) proceeded to provide a summative review of 24 studies, which although flawed, collectively, did identify some stable patterns. The data (largely quantitative) of the studies reviewed varied from single- to multi-year collection periods, including the years from 1960 to 1981; published report dates ranged from 1976 to 1982. Neither the instruments used nor the classifications made of "minority populations" were consistent across the reviewed studies. Nonetheless, there were two outcomes examined that were somewhat consistent throughout the 24 studies: participation and performance.

Matthews (1984b) noted three clusters of variables that influenced minority students' participation and performance in mathematics: parent, student, and school. Parent variables found to have an influence on participation and performance included cognitive (e.g., parents' education level and occupation), affective (e.g., parents' attitudes toward mathematics), and cultural (e.g., parents' native language). Student variables included ascribed (e.g., students' belief about who is or is not "good" in mathematics), cognitive (e.g., students' enrollment patterns in advanced mathematics courses), and affective (e.g., students' attitudes toward mathematics and its perceived utility). School variables included climate (e.g., school discipline and attendance), organization (e.g., class size and academic tracking), resources (e.g., adequate or inadequate facilities and materials), racial composition (e.g., course offerings correlated to racial demographics), and personnel (e.g., student–teacher relationships).

In concluding her review, Matthews (1984b) highlighted three findings. First, collectively, school variables have important influences on minority students' participation and performance in mathematics, yet little research has been conducted. Second, there is limited research with respect to course-taking patterns and minority students. Third, additional research is needed with respect to the parents' (especially the mother's) cognitive, affective, and cultural influences on minority students' participation and performance in mathematics. She also expressed significant concern that research on minority students had over emphasized students who had been unsuccessful. Matthews made a direct call for more studies that explored both mathematically successful and unsuccessful minority students. In the end, she claimed, "If energy and resources could be directed toward minorities and mathematics as effectively as we have seen done with women and mathematics another step would have been taken toward ensuring equal access and equal opportunity for all students" (p. 93).

William F. Tate – 1997

The second article reviewed is "Race-Ethnicity, SES, Gender, and Language Proficiency Trends in Mathematics Achievement: An Update," written by William F. Tate (1997). This article, like the Matthews (1984b) article, was somewhat of an introduction to a *JRME* equity special issue. Twelve mathematics educators and researchers contributed to the 134-page special issue. Tate and D'Ambrosio (1997), in the guest editorial of the second equity issue, noted that the larger political movement devoted to social justice that seemed possible in the 1980s had all but disappeared in the 1990s "because of a period of political retrenchment" (p. 650). They contended that questions around how race, class, gender, and language matter in mathematics teaching and learning were no longer mere educational questions but also (polarizing) political questions. In short, the "Rainbow Coalition [had] stalled" (p. 650).

In his lead article, Tate (1997) documented the changes in USA mathematics achievement at the elementary and secondary levels during the 1980s and 1990s. Specifically, he reviewed the quantitative literature on national achievement trends, college admission examinations, and Advanced Placement tests of “various social groups defined along lines of race, class, gender, ethnicity, and language proficiency” (p. 652). The review, nearly 30 pages long, was painstakingly detailed and provided a clear picture of the current mathematics achievement (based on standardized measures) in the United States of America. Some of the key findings included: (a) race, class, and language proficiency differences in mathematics achievement were more pervasive than gender differences; (b) mathematics achievement differences between race and ethnic groups had narrowed but African American and Hispanic students continued to perform at significantly lower levels than their White and Asian American peers; (c) all students across the different demographic groups benefited from additional mathematics courses in high school; and (d) male students tended to outperform female students on standardized measures of mathematics achievement but the differences were not statistically significant.

After discussing, in detail, the findings of his review, Tate (1997) outlined some limitations of the mathematics education literature. He noted two specific limitations found in many of the quantitative studies reviewed: (a) the data were not organized in such a way that the examination of two or more demographic variables was possible, and (b) the complexity inherent within demographic groups called for more integrative statistical analyses than those conducted. Tate then provided a pivotal critique of the mathematics education research in general:

> The paradigmatic boundaries of most mathematics education research –mathematics and psychology– have constrained the nature and scope of scholarship to the development and testing of new methods and materials.... Thus the scope of recommendations to administrators and policymakers responsible for urban and rural schools has been limited to suggestions that inform decisions on curriculum, student assessment, and teachers' professional development.... These recommendations are important. However, they do not completely address the realities of many students of color and low-SES students in urban and rural communities. Thus the need to borrow from scholarship in which the political and cultural dimensions of low-SES students and students of color have been explicated.... (pp. 673–674)

Tate (1997) concluded by recommending both fiscal and cultural policy options in search of *equitable* responses to the rhetoric of “high standards for all” found in the federally mandated, standards-based movement of the late 1990s. In making his recommendations, Tate was compelled to cross "epistemological boundaries" (p. 675) because of the restrictive paradigmatic boundaries. Briefly, his fiscal policy option recommended changes to the allocation of educational funds moving from fiscal equity to fiscal adequacy. He noted, "an equity strategy that fails to include an appropriate fiscal adequacy component cannot fully support the adoption and implementation of high-level mathematics standards for all" (p. 675). Tate's cultural policy option recommended future equity-related policies be informed by the *Professional Standards for Teaching Mathematics* (see NCTM, 1991)–

> which calls for mathematics pedagogy to build on (a) how students' linguistic, ethnic, racial, gender, and socioeconomic backgrounds influence their learning; (b) the role of mathematics in society and culture; (c) the contribution of various culture to the advancement of mathematics; (d) the relationship of school mathematics to other subjects; and (e) the realistic application of mathematics to authentic contexts. (p. 676)

In the end, Tate (1997) argued, "The importance of the mathematics standards movement for traditionally underserved students is obvious: previous reforms efforts have not met their needs. ... The challenge is before us" (p. 676).

Danny Bernard Martin – 2013

The third and final article reviewed is "Race, Racial Projects, and Mathematics Education," written by Danny Bernard Martin (2013). Unlike the Matthews (1984b) and Tate (1997) articles, it was not an introduction per se but rather a closing of a *JRME* equity special issue. Twenty-five mathematics educators and researchers, including an eight member Special Issue Editorial Panel (Martin was a member of the panel) and the *JRME* editor in chief, contributed to the 334-page third equity issue. In the introduction, the members of the Special Issue Editorial Panel (D'Ambrosio et al., 2013b) noted that the equity issue arose out of interest from the NCTM Board of Directors "to understand how issues of equity play out in today's mathematics classrooms" (p. 5). With an initial targeted focus on identity and power, contributing authors explored how, as a field, mathematics education influences the ways in which individuals are constructed in schools and in society, who is seen as intelligent or not, and whose "voices" are heard or silenced. Within this targeted focus, issues around racism, classism, and the politics of language were revisited throughout, illustrating "that mathematics education is always social and political" (p. 6).

In his closing article, Martin (2013) conducted a critical structural analysis of the internal dynamics of the USA mathematics education enterprise. He noted that many critical scholars are making powerful arguments about the dangers of mathematics education becoming increasingly influenced by and aligned with neoliberal and neoconservative market-driven projects and agendas. Martin, however, believed that many of these critical scholars' responses to issues of race and racism were often problematic. In particular, Martin characterized their responses as an unfortunate backgrounding of race and racism in some analyses or a conceptually flawed foregrounding in others, which, in the end, obscured the evidence that mathematics education all the while has been influenced by and aligned with neoliberal and neoconservative racial agendas (p. 316). Martin organized his critical structural analysis around three questions: What kind of project is mathematics education? What about racism? Is mathematics education itself a racial project? Each question is discussed in turn.

What kind of project? In response to this question, Martin (2013) provided a review of critical mathematics education research and scholarship over the past 30 years or so. The review included the work of mathematics education researchers and scholars who are credited with critical mathematics, ethnomathematics, social justice mathematics, and mathematics as a civil right, to name just a few. The review was impressive; it illustrated what kind of project mathematics could be or should be. So what kind of project is mathematics education? In the end –mathematics education is a political project.

What about racism? Here, Martin clarified what he meant by *unfortunate backgrounding* and *conceptually flawed foregrounding* responses to race and racism. Unfortunate backgrounding occurs simply when race and racism are inadequately conceptualized in mathematics education research, which, unfortunately, has been the norm not the exception. Specifically, Martin argued, "racism –especially white supremacy…– rarely has been centered in the analyses, rarely theorized for conceptual clarity, and rarely theorized in relation to the market-driven goals of globalization that mathematics education increasingly is said to serve" (p. 319). Conceptually flawed foregrounding occurs when race and racism are framed primarily historically, which disallows an "accounting for the contemporary, political expedient forms of everyday, institutional, and structural racism in the post-Civil Rights era, including neoliberal and neoconservative color-blind racism" (p. 321). Martin noted that these responses to race and racism are particularly troubling given the attention that these issues receive in scholarly arenas outside mathematics education.

Is mathematics education a racial project? Yes. Martin's (2013) response was intended to be provocative. He began here by first "turning the gaze inward" (p. 322). In so doing, he positioned mathematics as a *white institutional space*, borrowing the term from sociologists. Such spaces are characterized by:

> (a) numerical domination by Whites and the exclusion of people of color from positions of power in institutional contexts, (b) the development of a White frame that organizes the logic of the institution or discipline, (c) the historical construction of curricular models based upon the thinking of White elites, and (d) the assertion of knowledge production as neutral and impartial, unconnected to power relations. (p. 322)

Martin then proceeded to provide a historical sketch of mathematics education reform efforts over the past 50 years. Each reform effort, as Martin illustrated, "had not been disconnected from the racial projects that have continued to shape [USA] racial dynamics and social policy" (p. 325).

Martin (2013) concluded by contending that the "critical structural analysis of the internal dynamics of the mathematics education enterprise show that it is a racialized space, an instantiation of White institutional space" (p. 328). In the end, Martin called mathematics educators to continue to ask:

- What kind of project is mathematics education?
- Whose interests are served by this project?

WHITE SUPREMACY AND MATHEMATICS EDUCATION RESEARCH

> As I write, I try to remember when the word racism ceased to be the term which best expressed for me exploitation of black people and other people of color in this society and when I began to understand that the most useful term was *white supremacy.*
>
> – bell hooks (as cited in Gillborn, 2005, p. 485; emphasis added)

> **~~WHITE SUPREMACY~~** could just as easily be crossed out in the heading above. Unlike the previous heading **RACE AND MATHEMATICS EDUCATION**, it just doesn't apply. Does it? Let's see. A Google Scholar search of "race" and "mathematics education" returns nearly 24,700 results[2]; a search of "White supremacy" and "mathematics education" returns 282. So roughly 1.1% of the scholarly discussions that mention race in mathematics education also mention White supremacy. Correct? What about "racism"? Let's see. A Google Scholar search of "racism" and "mathematics education" returns about 4,180 results. So then, roughly 17% of the scholarly discussions that mention race in mathematics education also mention racism.

Staying with Google Scholar analytics, how many scholarly discussions mention just "mathematics education"? The search results –about 456,000. So using the previous search of "race" and "mathematics education" (about 24,700) roughly 5.4% of the scholarly discussions that mention mathematics

2. Google scholar searchers are not an exact science; they can, however, provide a sketch of the discourses that frame topics. The search reported here was conducted on January. 3, 2017; it is important to note that the search results included scholarly publications written in English, not just those originating from USA sources.

education also mention race. Nearly 1.5 percentage points higher than the 4% noted in the introduction of this essay. But Google Scholar searches also capture scholarly books and other scholarly publications (e.g., conference proceedings); the roughly 4% calculated independently by Lubienski and Bowen (2000) and Parks and Schmeichel (2012) included only peer-reviewed journal articles in the percentage of mathematics education articles that contained descriptors of race and/or ethnicity.

Let's do some more math; again, staying with Google Scholar analytics. Using 456,000 as the denominator (the search return of "mathematics education"), what percentages of scholarly discussions that mention mathematics education also mention racism? White supremacy? Roughly, 0.9% and 0.06%, respectively.

Need something more precise? The nature of Google Scholar analytics are that they are somewhat imprecise, providing algorithmic-determined estimates of word and phrase searches. For more precession, let's explore the three *JRME* special equity issues that included the three articles previously reviewed. Although not intending to provide an exacting picture of each equity issue, I did intend to capture at least the spirit of each issue through the three reviews. The 1984 special issue contained 12 contributions (contributions counts include editorials, introductions, and articles): six mentioned race (or racial), one mentioned racism, and zero mentioned White supremacy. The 1997 special issue contained seven contributions: six mentioned race (or racial), three mentioned racism, and zero mentioned White supremacy. The 2013 special issue contained 15 contributions: 12 mentioned race (or racial), nine mentioned racism, and two mentioned White supremacy.

But a mentioned is just that, a mere mention. So how are race, racism, and White supremacy being addressed (or not) in mathematics education research across nearly 30 years –as least as depicted in USA-based *JRME* special equity issues? Through the 96 pages of the 1984 issue, race was mentioned 25 times (racial 38 times). In each case, it was used "primarily [as] an easily defined category to which one belongs and to which particular traits or outcomes can be assigned" (Parks & Schmeichel, 2012, p. 244). Johnson (1984) provided the single mention of racism: "These factors [for black students' lack of interest in taking mathematics] are related to one another and are rooted in centuries of institutionalized *racism* that perpetuated unequal education for black people" (p. 149, emphasis added). White supremacy was never mentioned throughout the 96 pages.

Through the 134 pages of the 1997 issue, the word race is mentioned 26 times (racial 33 times); again, most often as a category. Different from the first special issue, however, the contributors to this special issue cross the paradigmatic and epistemological boundaries "to address the realities of many students of color and low-SES students in urban and rural communities" (Tate, 1997, p. 674; see, e.g., Ladson-Billings, 1997; Gutstein, Lipman, Hernandez, & de los Reys, 1997). Nonetheless, the word racism is rarely used (only seven times); and again, White supremacy is never spoken.

The analysis of the 2013 *JRME* special issue is much different; this difference is clearly visible in the Martin (2013) contribution previously reviewed. The attempt to address issues of race and racism (and White supremacy) head on, so to speak, is communicate through a published dialogue among the Special Issue Editorial Panel so titled "Addressing Racism" (D'Ambrosio et al., 2013a). The purpose of the dialogue is to–

> highlight how teachers and researchers are often more comfortable talking about race, but not racism; how the field of mathematics education is implicated in the construction of race; and how we still have insufficient knowledge about the contexts and experiences of Latin@, African American, and American Indian students to inform policies and practices that will be in their best interest. (D'Ambrosio et al., 2013b, pp. 7–8).

Race, racial, and *racialized* are mentioned over 200 times in the 334-page special issue, but here, race (and its derivatives) is used and understood not only as a socially constructed category but also as a category that can be and is contested. Racism is mentioned about 130 times, but still dangerously absent in many of the discussions. The panel defined racism as–

> both individual practices and institutional structures that support whites maintaining a position of privilege and superiority in society. Racism is not an inherent quality of people, but rather something into which we are socialized. Through the practice of racism, students, teachers, and others are given their roles in society. (D'Ambrosio et al, 2013a, p. 36)

White supremacy is mentioned eight times in only two of the contributions. Stinson (2013) merely mentions it, once. But Martin (2013) places it side by side with racism, returning to it and racism often throughout his argument that substantiates mathematics education itself as a racial project.

CONCLUDING THOUGHT

In the end, researching race in mathematics education requires researching racism. But opening up racism as an object of inquiry in mathematics teaching and learning requires an examination of the pervasiveness of White supremacy. After all, racism and White supremacy are two sides of the same coin. But do we have the theoretical frames to research White supremacy? [3] Do we have the methodological tools? And, more importantly, do we have the will?

3. See Battey and Leyva (2016); they offer a "(*developing*) framework to support mathematics education scholars in general, and White scholars specifically, in examining the racist internal structure of mathematics education" (p. 50).

> White supremacy is the unnamed political system that has made the modern world what it is today.
>
> Charles W. Mills (1997, p. 1)

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EXTENDING THE LANDSCAPES OF MATHEMATICAL INVESTIGATION THROUGH PHILOSOPHICAL INQUIRY

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In "Landscapes of Investigation" Ole Skovsmose (2001) examines six different milieux for learning. Each one is associated with one of two types of mathematical activity, which fall into the broad categories either of "exercise" or inquiry. Each refers to one of three types of reference: reference to mathematical notions, to invented situations, or to real situations. In this paper I propose to extend these landscapes of learning through opening a further dimension to be explored through critical dialogical philosophical inquiry. The paper argues that philosophical dialogue has a place in the math classroom by way of its capacity to facilitate understandings that may serve to complement and critically judge the inferences acquired in and with mathematics per se. In other words, a philosophical dimension may contribute to the opening of a "wider horizon of interpretations" that includes a critical dimension.

In his article, "Literacy, Matheracy and Technocracy –the New Trivium for the Era of Technology," Ubiritan d'Ambrosio (1999) offers the concept of "matheracy," which he describes as "the capability of inferring, proposing hypotheses and drawing conclusions," and is the basis for developing what he calls an "intellectual posture" that is, he says, "closer to the way Mathematics was present both in classical Greece and in indigenous cultures. The concern was not with counting and measuring, but [a] deeper reflection about man and society. . . ." This implies that when mathematical thinking is understood as something that only happens in the time and space of a math classroom, it reduces our capacity to understand its role and significance in our culture and civilization, and to evaluate its uses. In a civilization in which mathematics plays a part in most of the decisions we make –whether technological development, social engineering, financial regulation, disease-control, space exploration, weapons development or urban design– cultivating matheracy offers a way for all students to become mathematicians in their own right, with enough understanding and enough cognitive fluency to be able not only to decide when a mathematical approach to a problem is appropriate and when it is not, and if so, how to go about it; but also to reason about consequences and make value judgments about possible outcomes, with special sensitivity to public issues and in particular, social justice.

Toward this end, I have previously argued that utilizing philosophical inquiry in the mathematics classroom promises to help students acquire a critical stance towards mathematics and its uses in the society; to provide bridges for establishing richer and more meaningful connections and interactions between mathematics and students' personal experience. This includes connections, not only with other disciplines, but with the broader culture. All are necessary for the promotion and development of the critical posture and perspective that is essential for cultivating genuine matheracy. (Kennedy, 2016; Kennedy, 2012a).

This paper will reflect on the pursuit of matheracy through, among other things, the incorporation of philosophical inquiry into the mathematics classroom. It extends Skovsmose's notion of landscapes of mathematical investigation into landscapes of philosophical inquiry. By "landscapes of philosophical inquiry" I mean the multiple existential, social, scientific, and political life world contexts with their various mathematical references that may trigger philosophical inquiry. I will discuss the following landscapes of philosophical inquiry: a) ones that are associated with mathematical notions per se, b) ones that refer to invented reality (e.g. a reality that cannot be observed or experienced but has been constructed implicitly by the author of a textbook, for example) c) ones that refer to real-life situations, and d) ones referring to the role of mathematics in society and the economical, aesthetic, and political use value of mathematics (e.g. with the implicit understanding that the latter are held in place by broad philosophical assumptions –epistemological, ontological, axiological, ethical and metaphysical, whether hidden or open) that act to legitimize mathematics as a primary form of knowledge.

EXTENDING THE LANDSCAPES OF MATHEMATICAL INVESTIGATION

In "Landscapes of Investigation" Ole Skovsmose (2001) examines six different milieus for learning. Each one is associated with types of mathematical activities that can be broadly assigned to one of the two following categories –"exercise" or exploratory activity– each associated with either the "exercise" paradigm or the inquiry approach. Additionally, each task in Skovsmose's categorization refers to one of three types of reference: reference to mathematical notions, to invented situations, or real situations. I this paper I propose to extend the landscapes of learning through opening a further landscape to be explored through philosophical investigation.

Seldom do mathematical investigations cross established epistemological boundaries, and encounter questions like, "Do numbers organize our experience?" "How do numbers influence our thinking?" or "Can my body count?"–questions that promise the creation of new meanings, and, in the long run, new ways of thinking with and about mathematics. I want to argue then, that in their daily practice, mathematics teachers would profit from opening and exploring other epistemological spaces than the traditional, and take opportunities to engage with their students in a continuous "worldmaking" with mathematics, which as Goodman (1988) suggests, represents a hermeneutical task of ongoing reconstruction of meaning that I interpret to include critical reflection that moves beyond disciplinary boundaries *as well as* meaning-making while actually practicing mathematics.

Skovsmose comments on two paradigms for mathematical engagement: exercise and inquiry (Skovsmose, 2001). Both paradigms occasion at least six different milieux for learning and meaning making. Discursively, these milieux do position students primarily as "insiders" to the realm of mathematics. There they are expected to engage in "*doing and talking mathematics*" –from doing more contrived exercises to performing more complex problem solving or mathematization that might involve defining problems, interpreting, selecting an appropriate method or design or model, arriving at solutions, reflecting on possible alternative methods or models, verifying solutions, and drawing conclusions.

In addition, I propose extending the landscapes of learning discussed above by introducing a form of philosophical inquiry that goes beyond inquiry within the mathematical system, and that more often than not does not impose prefabricated questions, but invites children to pose questions of their own about mathematics, both in its internal relations and its relation to the world –and, by implication, across the disciplines. It involves a discursive positioning that travels outside of the mathematical system, and thus leads us to *talk about mathematics*. As we go forward, I will discuss each landscape separately, and the affordances of each for philosophical investigation. Before I do that I will briefly discuss the practice of philosophical inquiry in the classroom, and community of philosophical inquiry as a pedagogical format for conducting such inquiry.

PHILOSOPHICAL INQUIRY IN THE CLASSROOM

By philosophical inquiry I understand, the process of arriving at *critical judgments* regarding philosophical questions or issues that have become a focusing point of a given group dialogue (Lipman, 2003). These judgments are necessitated by an urge to resolve problematic experiences with philosophical dimensions, such as –to follow the traditional categorization– ethical questions (e.g. What is fair to do?), ontological questions (e.g. What is the relation between a mathematical model and the "real world"?), epistemological questions (e.g. What does it mean to learn something? What can we know through mathematics?), and aesthetic questions (e.g. What is an elegant solution to a math problem?). For a philosophical judgment to be reasonable it must be well-reasoned, rely on sound arguments and good evidence, and be well-informed and reflective of multiple and diverse perspectives. It must be capable of surviving the scrutiny of critical, communal dialogue, and be relevant to one's personal experience (Lipman, 2003; Gregory, 2006). As such, its most important assumption is that philosophical inquiry is carried out in the context of a community of philosophical inquiry (CPI) through a process of collaborative and dialogical deliberation.

The primary objective of its deliberations is the construction of meaningful arguments, not through transmission, individual reflection or debate, but through building on each other's ideas –that is through distributed thinking in a dialogical context. The ideal inquiry proceeds through a form of argumentation which, because it is inherently dialogical, is thus by implication a dialectical process, which is to say a process which moves forward through encountering and attempting to resolve tensions, ambiguities, or contradictions. The chief pedagogical significance of the constructive process of community of philosophical inquiry is that it operates in the collective zone of proximal development or ZPD (Vygotsky, 1978), which acts to scaffold concepts, skills and dispositions for each individual. The scaffolding process functions through subprocesses such as clarification, reformulation, summarization, and explanation, as well as through challenge and disagreement (Kennedy & Kennedy, 2011). Uncovering and analyzing assumptions is a form of fundamental work done in the process of inquiry that is oriented towards arriving at a collective judgment. It could be argued that such a dialogic space represents the ideal situation for the intrapersonal appropriation of the interpersonal –or "internalization"– not only on the conceptual but on the behavioral level, i.e. in the development of habits of both cognitive and behavioral self-control and self-regulation, all of which emerge within the groups' collective ZPD.

ORGANIZATION OF PHILOSOPHICAL INQUIRY

We use an approach to philosophical inquiry whose key point is the notion, taken from John Dewey (1933) and Matthew Lipman (2003), that inquiry should begin with a particular experience –in this case a mathematical exercise, a mathematical investigation, or a list of questions, or other stimuli, such as a short narrative– in order to provoke a unified cognitive event that is impregnated with conflicting ideas, that can prompt students to encounter uncertainty and perplexity, and that motivates them to inquire into the problematics of a situation and to search for its resolution. The agenda of the group discussion is guided by students' interests, which

Dewey (1933) insisted was necessary for meaning making.

The methodology is founded on the primacy of the students' questions, induced by the problematic of a presented stimulus that guides the agenda of inquiry, and the goal is that students' questions and interventions guide the inquiry process itself. In this case, what is seen and felt as problematic and perplexing in the situation presented by the stimulus must reflect the experiences of the group of students, or as Dewey puts it, the situation must "occasion" the inquiry. Finally, the communal deliberative discussions are natural outgrowths of previous discussions, strategically guided by the teacher-facilitator. The stimuli are selected or designed to provoke philosophical questions, but only insofar as the students can relate to them personally, i.e. through aspects of their own experience, which is based on the proposition that ethical, aesthetic, political, and other philosophical dimensions underlie most people's ordinary experience.

A philosophical-mathematical inquiry can occur in at least two ways: a) it can be "staged" –meaning planned in detail by the facilitator, who has prepared a stimulus and intiates the inquiry with one of those questions, chosen by the group. Similarly, it may follow the tradition of Lipman's *Philosophy for Children* program, and use a narrative text that offers or suggests one or more implicit philosophical questions with reference to mathematics, and ask of the group to generate its own questions and choose one from them for a start; b) the philosophical-mathematical discussion emerges from a mathematical discussion. In this case the facilitator has a choice whether to embrace the emergent philosophical impulse and allow the discussion to unfold, or to forestall it by adhering strictly to the mathematical inquiry as framed –although it is quite difficult to draw a line between philosophy and mathematics proper.

LANDSCAPES OF PHILOSOPHICAL INQUIRY

Now, following Skovsmose's format, I discuss landscapes of philosophical inquiry with four different references. They may be seen as complementary to the more concrete mathematical investigations in that they allow one's own construction of meanings and understandings of concepts and connections to be applied –constructions and understandings that are not readily available or encouraged if they have not been a focus of reflection.

A LANDSCAPE WITH REFERENCE TO MATHEMATICAL NOTIONS

Group philosophical inquiry focused on mathematical notions might encourage discussions and a search for meanings that have not traditionally been offered for discussion in school mathematics –e.g. number, infinity, axioms, algorithm, or mathematical models. Any individual's concepts are grounded in his or her lived experience, and their growth is shaped to a great extent by personal belief and personalized processes of justification. It is to be expected that there will be differences in the relationships between each student's naïve and scientific concepts, between each student's ability to navigate between descriptive and normative thinking, and in each student's capacity to "break set" and think analogically, metaphorically, and across conventional categories. In an environment of communal, dialogical deliberation, these differences tend to emerge spontaneously in the course of discussions connected to a particular concept, and if the teacher allows and even encourages their expression, mathematical inquiry often merges seamlessly into philosophical inquiry.

For example, elsewhere (Kennedy, 2012b) I describe a situation in which a group of middle school students worked on comparing the infinite set of all natural numbers to the infinite set of even natural numbers, in order to answer the question whether those sets have equal numbers of elements or not. As the group struggled to work through several conflicting hypotheses, one student asked, "Is infinity a number?" The teacher seized the moment and reoriented the mathematical inquiry towards a philosophical one by paraphrasing the question as "What is infinity?" This sudden shift in inquiry created a space for students to agree and to disagree, to collectively take apart and reconstruct their conceptions of infinity while they reflected on their own experiences and evaluated the propositions put forth by their peers, and enabled them to return to the mathematical inquiry with enriched meanings. Such a landscape for philosophical investigation is a direct extension of the mathematical landscape, following up on the question that emerges from the mathematical exploration. Not always, however, will there be such spontaneous emergence of a space in which mathematical and philosophical inquiry can interact and, so to speak, cross-pollinate. A framework is needed for a more permanent space, an "in-between" where mathematical and philosophical inquiry can interact on a more regular basis.

Towards such an end, teachers might search out opportunities in the curriculum for philosophical inquiry. For example, aesthetic inquiry into mathematical notions like symmetry, fractals, patterns, and more offers the possibility of expanding student appreciation of mathematical beauty, but also of developing criteria for aesthetic judgment in general. Questions such as "Is there a connection between symmetry and beauty?" and "When is a pattern beautiful?" may be examples of bridges from mathematical explorations of symmetry or patterns to a philosophical one. Similarly, questions like the following may lead to the process of de- and reconstruction of our understanding of the ontology of number: What is number? Is mathematics a language? If so what role do numbers play in it? How are numbers and letters the same or different? What can or cannot be expressed in numbers? Such landscapes for philosophical investigation may not only prompt students to think about math concepts in new ways, but also to undergo new experiences of "relatedness" that go beyond current utilitarian concerns of the curriculum and its characteristic texts.

A LANDSCAPE WITH REFERENCE TO INVENTED SITUATIONS

Typically mathematical word problems refer to situations that have been invented for the purpose of a mathematical exercise (Skovsmose's milieux #3 and #4). Usually such exercises do not invite students to bring their own practical understanding to the classroom. The nature of the tasks based on invented situations or "virtual reality" (Christiansen, 1997) assumes that the tasks fully describe the situation. The students have been conditioned to treat it as a simple exercise void of making any sense of the context, and as having one answer only (Verschaffel, Greer, DeCorte, 2000). However, I would like to suggest that there is much to gain from mathematics tasks that are ambiguous, open to interpretation, and which might call for definitions of certain concepts or terms. The following is an example of such a task, which was designated for use as a mathematical exercise, set in the context of an invented situation. Treated as a "text" that is to be interpreted, such a problem may open a whole new landscape for philosophical investigation. The task was used in a group discussion with middle school students, and reads as follows: *A frog finds itself at the bottom of a 30-foot well. Each hour, it climbs 3 feet and slips back 2 feet. How many hours would it take the frog to get out?* (Kennedy, 2012b)

Students were asked to collectively interpret the task, and identify implicit assumptions. To that end, the group spent considerable time clarifying and interpreting ambiguities. They discussed what was meant by "the frog to get out" and whether the problem implies that once out, could the frog slide back into the well. It was unclear when the getting out was going to occur. After the interpretation of the text and the reformulation of the question in the task as "When is the frog *first* going to be out of the well?" the discussion moved to an exploration of possible solutions. Several plausible ones were presented, each reflecting a different assumption about the frog's mode of climbing. After further negotiation, the students agreed on two legitimate ways for the frog to climb out of the well: a strict up three feet/down two feet plan, as well as an irregular pattern that amounted to the same distances. Finally, the students were asked to think about possible answers for the frog's time in each situation.

A major goal of this form of inquiry is not so much finding the "right answer," but the "residue" that is a result of the collective experience of such a group deliberation. This "residue" could be anything from an understanding that a mathematical task could be often "read" and interpreted in different ways, to the realization that assumptions buried deep in the statement of a problem can make a distinct difference to its final solution. It is also about helping students gain understanding that doing mathematics is a sense-making process; that mathematical tasks are matters of interpretation and require careful examination of the data given; and that any inferences made are based upon implicit assumptions, which also call for examination. Other facets have to do with understanding the relationship between mathematics and uncertainly, and the role that the one who poses or solves a mathematical problem plays in defining and interpreting the problem. Yet another facet has to do with understanding the role of the group as an interlocutor, as a generator of ideas, and as a reflector and corrector of each member's reasoning and perspective. As with the first landscape discussed above, this one not only allows but encourages students to problematize mathematics, a learned disposition that may help develop an attitude of healthy skepticism towards claims about the world made by mathematics, and a propensity for critical evaluation of its methods, assumptions, and conclusions.

A LANDSCAPE WITH REFERENCE TO REAL SITUATIONS

Below, I describe a mathematical task that in this case activated a landscape of mathematical investigation (in Skovsmose's sense), which then opened onto a landscape of philosophical inquiry. The task was assigned by a colleague and myself to a group of middle school students, and it was based on the following prompt[1]:

Choose to describe mathematically one of the following:

1) *Draw a bar graph that represents your activities during one day chosen by you in relation to the time spent.*
2) *Draw a diagram that represents the arrangement of furniture in your room.*
3) *On a coordinates system, draw a sequence of connected line segments to represent your route from home to school*
4) *Choose something else and describe it mathematically.*

The descriptions of their own rooms volunteered by students were varied: some made drawings portraying the color and the patterns of their wallpapers, flowers on their desks, etc. Others drew rectangles with smaller rectangles inside them, showing the placements of their bed and desk; yet others offered a list of measurements of the length, width, and height of the room. These representations were then used to initiate a comparison and pose the question: What is a mathematical description? How is it different from another –say an artist's or a poet's descriptions?

1. The projects described here were carried in collaboration with David Kennedy from Montclair State University & IAPC (The Institute for the Advancement of Philosophy for Children).

Often one philosophical deliberation opens the door to more questions than we started with. We often end a discussion by jotting down new questions that have arisen in the course of this one. For example, an initial inquiry into the question: What is a mathematical description? which was prompted by the various kinds of descriptions that the students brought up in class, yielded the following list of new questions, voiced by the students and collected at the end of the session:

1. Why can't math descriptions have all the information in them?
2. Are math descriptions useful if they don't have all the information in them?
3. How do we know which is the best math description?
4. Why do people use math descriptions?
5. Are mathematical descriptions always helpful?
6. What can we gain by using them?
7. What can be lost in using them?
8. Can a math description be harmful?

Similarly, when discussing specific modeling tasks with high school students, we might consider opening landscapes for philosophical investigation through generating and discussing general questions about mathematical models, such as: What is a mathematical model? What is a non-mathematical model? How is the former different from the latter? How does a math model describe the 'real-world'? What would a good metaphor for math model be: a map, a mirror, or something else? What happens during the translation process from the "real world" to the production of a math model? How does a math model represent a "real world" situation? These questions and students' dialogical reflection on them might help to develop a better understanding of math models as products of mathematical abstraction. Moreover it is crucial to examine –although limited space does not allow me to do so here– how further philosophical inquiry can mediate the critical analysis of developed mathematical models in relation to different criteria for judging the success of such models, and for helping identify and critiquing value judgments that may prioritize one model over another.

A LANDSCAPE WITH REFERENCE TO THE ROLE OF MATHEMATICS IN SOCIETY

Finally, I will focus on some prompts that might instigate philosophical inquiry with reference to the role of mathematics in society. Here I offer the following exercise[2]:

2. The projects described here were carried in collaboration with David Kennedy from Montclair State University & IAPC (The Institute for the Advancement of Philosophy for Children).

Choose one thing from the list below and try to answer the two questions: 1) Can these things be described mathematically? 2) If so, how would one go about it?

A barn; A dance; A soccer game; A card game; The game of chess; A human life; A conversation; A rabbit warren; The economy; An election campaign; A thunderstorm; A dream; Personal "coolness"; Happiness; Anxiety.

The purpose of this exercise is to expose the differences between mathematizing phenomena from the physical world, like a barn, or a dance, and social constructs like "coolness" and even happiness or anxiety. Its effect is to help us see that mathematics can be an appropriate tool for describing and studying the former, but not as good for studying the internal human world –human emotions, beliefs, knowledge, understanding, hopes, etc. The discussions generated by such an exercise can be a way of encouraging students to critically examine viewpoints absorbed from parents, media and school, whose basic message is often "Mathematics is everywhere and it is powerful. It can be used successfully in every single aspect of our lives." Participating in discussions that untangle multiple examples, perspectives and ideas voiced by peers can help students develop more personal views about mathematics, help them understand that mathematics has limits like any other tool we use, and give them an opportunity to explore some of those limits.

One could go further. Does mathematical knowledge of the world and the use of a mathematical approach to study and understand the world actually inhibit or distract us from other, equally (or in some cases more) powerful forms of knowledge? Is mathematics always the one best way to understand the world? Is it a way to understand ourselves? Is there anything that it might miss? Mathematics students commonly question its usefulness, which is related, in turn, to its evident relation to the lived world that we all inhabit in more or less the same spatio-temporal way. Our goal is to encourage questions that go deeper than the utilitarian uses of mathematics –questions that stimulate critical inquiry into our culturally constructed and transmitted beliefs and assumptions about mathematics. We believe that such inquiry acts to heighten awareness of the power and limitations of mathematics.

CONCLUSION

I have discussed the potential that lies in extending the landscapes of learning as formulated by Skovsmose (2001) through opening a further, philosophical landscape, to be explored through dialogical group investigation. My goal has been to show that such a landscape has a place in the math classroom, by helping to facilitate understandings that serve to complement and critically judge the inferences acquired in and with mathematics per se. In other words, the philosophical dimension promises to aid us in the opening of a "wider horizon of interpretations" that includes a critical dimension. The introduction of a philosophical dimension through landscapes for philosophical investigation offers the possibility of a more spacious epistemological approach to mathematics, where students can engage mathematics in the lifeworld, where personal meanings and perspectives can enter, shape and enrich students' learning experiences, and where the relationships between mathematics and other fields of meaning and practice are examined and clarified. Matheracy is best developed in diverse contexts, where multiple modes of thinking and investigation are allowed and encouraged, and the connections between these forms of inquiry and thinking –their interrelationships and correspondences, as well as their conflicts– are made fully conscious.

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BEYOND POVERTY AND DEVELOPMENT: CASTE DYNAMICS AND ACCESS TO MATHEMATICS EDUCATION IN INDIA

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In the literature on Mathematics Education, India belongs with other developing countries and in the low-income bracket. There are other narratives about India (some academic and some popular) focusing either on the history of mathematics in the ancient times and or on the fact that like other Asians, Indians are good at mathematics. However, caste as a major determinant of access to mathematics has received practically no attention in the literature or popular accounts. This paper seeks to foreground the centrality of caste in the life of a person in India and argues that there is no way to engage with mathematics education in India without factoring caste in the study.

INTRODUCTION

On 17th January 2016, Rohith Vemula a PhD student at University of Hyderabad committed suicide, leaving behind a moving letter that said: "*I always wanted to be a writer. A writer of science, like Carl Sagan. At last, this is the only letter I am getting to write" (Indian Express, 2016; Harvard SAI, 2016). A bright young man, politically sensitive, academically good, a student of science who chose to pursue science studies for his doctoral thesis, Rohith's suicide shook the country like a powerful earthquake, the tremors of which can still be heard. Rohith was not the first student to commit suicide on the University of Hyderabad campus. In the year 1993, Sunita, a student pursuing her master's degree in Mathematics committed suicide because of love failure. Sunita was not known to be a good student at the university, though her neighbours in the village thought she was very good in mathematics and lamented the fact that she went to faraway Hyderabad to pursue higher studies only to return as a dead body. Between Sunita and Rohith, there were several suicides in many institutions of higher learning (The Citizen, 2016).*

Unrelated to these accounts of suicides on university campuses and at a different space-time co-ordinate, young Nikita, a fifth grade student in a government school in a remote village in the state of Madhya Pradesh, and was part of a longitudinal study that I conducted on alternative approach to teaching fractions during the years 2008-2011 explains that though 2/3 and ½ are both just one piece short of 1, 2/3 is bigger than ½ because 2/3 has 4 pieces of 1/6 while ½ has only 3 pieces of 1/6. To visualize these equivalences, she does not require any material support. Elsewhere, we have discussed how she compares the fractions 4/5 and 7/8 (Subramanian et al, 2015). Though she was not the only one in her class competent to provide valid supporting arguments, Nikita's ability at abstraction and logical thinking was, far superior to her classmates. We do not know how Nikita coped with mathematics at the middle school and high school level, whether she can factorize polynomials, compute square roots, understand the difference between rational and irrational numbers, and prove theorems in geometry; we do not know how well she faired in mathematics in the school final examination and if she will pursue higher education. Why this uncertainty about the academic future of a student who demonstrates high calibre in mathematics?

What connects Rohith Vemula and Nikita is not just their brilliance and poverty; what connects Nikita and Sunita is not just their gender and rural government schools where they studied; another crucial factor, a strong identity that connects all the three of them is their caste. All three of them belong to the scheduled castes. Dreams of becoming a writer like Carl Sagan or hopes to finish school or access higher education are rarely realized for the likes of Rohith, Sunita and Nikita. In this paper I propose to discuss caste as a social category that plays a crucial role in the life of a person in contemporary India and its implication for access to mathematics education.

THEORETICAL POSITION AND STRUCTURE OF THE PAPER

The paper seeks to foreground the centrality of caste as a social category that plays a significant role in determining access to mathematics education. There is, to the best of my knowledge, very little research in mathematics education in India, that explicitly focuses on caste to understand failure and exclusion. Often failure in mathematics is attributed to poverty or the absence educated adults to provide additional support to the learner. For anyone who has studied caste and the caste-based struggles in India, it is clear that caste is integral to one's identity in India and like other social categories such as gender and race, it operates, sometimes in very blatant forms and sometimes in extremely subtle ways, to order the social space, marginalising some sections of the population and privileging others. This paper makes a small attempt to problematize access to mathematics education in India by posting caste as a crucial factor in determining who gets to learn mathematics. The paper is largely descriptive in nature and draws upon several disjoint narratives to illustrate the point.

The major objective of the paper is two fold. At a local level, the paper calls for the need to carry out empirical studies to understand the complex ways in which caste operates to limit opportunities for learning mathematics, for those coming from marginalised caste background, across diverse regional, class and linguistic background. At the larger level, the paper calls for a need for mathematics educators to go beyond race and recognize caste as an equivalent, if not a stronger factor in determining who gets to learn mathematics.

The structure of the paper is as follows: first, the paper gives a brief description of the caste system and its status in the colonial and post-colonial India. Then it moves on to describe some of the policy measures to address under representation marginalised castes in higher education and the resistance faced by students from marginalised castes in the institutions of higher learning. Finally, the paper discusses at a very broad level, how caste operates at the school level.

A BRIEF DESCRIPTION OF CASTE SYSTEM

There is a large body of literature focusing on the historical, socio-cultural, and political dimensions of caste, its link to various religions and on how caste reconfigures itself to have profound impact on the lives of the people in modern India as well as the Indian diaspora. The account of caste and caste system presented here will be very brief, largely drawing from Anand Teltumbe's book 'The Persistence of Caste: The Khairlanji Murders & India s Hidden Apartheid' (Teltumbde, 2010).

It is believed that the word caste itself was used by the Portuguese (who colonised some part of India) to refer to the form of social stratification that operates in India. The equivalent term used in large part of India is *jati*. Another term that is frequently confused with caste is *varna*. The varna system is believed to have evolved and stabilized between 600 to 200 BCE. Castes may have arisen out of subdivisions among the varnas, but it is caste that determines the rules and regulations of life. Both caste (jati) and varna are hierarchical. While there are only four *varnas* consisting of the priestly class (*brahmins*) at the top, the warrior class (*kshatriyas*) below them, the trading class (*vaishyas*) in the third position, and the working class (*shudras*) at the bottom of the hierarchy, there are thousands of castes with a lot of regional variation. And hence it is difficult to place the castes in a linear order, even though within a geographical location one may experience it as a clear hierarchical social order.

Apart from the four varnas, and several castes, there are people who are placed outside the caste and varna system variously referred to as panchamas (the fifth ones), out-castes (pariahs), or untouchables. The untouchables faced and continue to face in several parts of the country, extreme forms of exclusion that deny them right to proximity to caste Hindu, to drink water from the same source as the caste Hindus do, or to enter temple. The exclusion is referred to as 'apartheid' by several thinkers from this category. Castes are closed endogamous units, the membership to which is ascribed by birth and hence unlike one's religious identity, one cannot change one's caste identity. Castes that are higher in the hierarchy are more powerful and have control over those who are placed lower in the hierarchy. Also, contact between any two castes is restricted. Traditionally, each caste group may be engaged in one form of labour. Even now in many parts of rural India, caste-based segregation exists in place of residence, with dominants castes owning the fertile and high lands. Historically, the out-castes have largely been engaged in work that those within the caste system refuse to do, such as cleaning and carrying away the night soil, removing and skinning the dead animals, scavenging, and so on; they live in ghettos in the outskirts of the village. There was and there continues to be in some parts even now, caste practices that prohibited the untouchables from drawing water from the same source as other castes, or entering into places where the caste Hindus live. It should be noted, however, that large variations exist in these practices. Apart from these castes, there are also indigenous communities referred to as *adivasis* who live in forest areas.

Caste in the colonial and contemporary India

During the colonial rule, the period of social reform movements and in the post-independent India significant changes took place in the both the understanding of caste and in the practices (Aloysius, 1997). In the colonial and post-colonial India, there have been strong anti-caste movements across the country; Jyothiba Phule and Savirthribai Phule, Iyothee Thass, Dr. B. R. Ambedkar, E. V. Ramasamy Naiker, and Narayan Guru are some of the leading figures in the anti-caste movement in India. Many of these leaders themselves came from untouchable castes. Dr. B. R Ambedkar, from the untouchable mahar caste, lead many struggles against caste discrimination, wrote and spoke extensively calling for annihilation of caste. He was also the principal architect of the constitution of India and made special provisions for reservation in the legislative assembly and has written extensively about caste and its implication for the life of the 'depressed classes' as the untouchables were referred to in the pre-independent India.

In the post-independent India, in the official records, castes have been grouped into caste categories based on the extent their of socio-cultural marginalisation or privilege. The category Scheduled Castes (SC) consists of the so called out-castes, placed castes outside the caste system, the category Scheduled Tribes (ST) consists of adivasi people across the country and the category Other Backward Classes (OBC) consists of the labour classes who are at different levels at the bottom of the caste system. The castes that do not come under any of the above categories and have enjoyed varied levels of power and privilege are grouped under the category Forward Castes (FC) also referred to as general or open category as they do not qualify for any kind of affirmative action. The term

Dalit refers to the political awakening among the untouchable caste communities, though sometimes it is also used interchangeably to refer to someone from the SC category. Dalit writing, Dalit struggles, Dalit feminism, and Dalit studies, are some examples of how the term Dalit carries a political connotation. The terms dominant castes and increasingly the term *savarnas* are used by Dalits to referred to the castes that marginalised them. Both in the pre-independent and post-independent India, some form of positive discrimination or reservation system has been introduced by the government to address the socio-economic and political marginalisation of the historically marginalised castes and tribes.

According to the highlights of the census data (2011), at present SC&ST together constitute a little more than 25% of the population. It is believed that OBC's constitute another 41% of the population of India. It is well known that these categories are poorly represented in higher education and in employment in the formal sector. In order to increase the representation of these communities in higher education and employment 15%, 7.5%, and 27% of the seats in government funded higher education institutions and in government jobs are reserved for those belonging to SC, ST and OBC categories, respectively. This means, in the event that suitable candidates from the particular category are not found, these seats cannot be filled by those from other categories.

It is important to note that caste hierarchies and prejudices continue to operate not only in rural India but in newer and subtler ways in the towns and metropolitan cities in spite of modernisation, urbanization, and globalization. The documentary film '*India Untouched: Stories of a People Apart*' (Stalin, 2007) documents how caste operates across the country and across religion. Even though people from marginalised caste backgrounds converted to Islam, Christianity, Buddhism, and Sikhism to escape caste discrimination, they continue to face discrimination from dominant castes who converted into these religions. Moreover, nearly 80% of SC's and more than 90% of ST live in rural India. Among those who migrated to cities in search of better jobs and to escape caste discrimination, a large section lives in urban slums. A significant percentage of SC&ST are poor and are engaged in daily wage labour. A small minority of SCs and even smaller percentage of STs have managed to access higher education and overcome economic deprivation.

CASTE AND ACCESS TO HIGHER EDUCATION

The discourse on caste and education has largely hovered around the question of merit and that of reservation for SC, ST and, OBC categories. The years 1990 and 2006 saw large-scale protests from the dominant castes against reservation in higher education and government jobs, (particularly for OBCs) saying reservation would result in promoting mediocrity. These protests lead to the emergence of Dalit voices across the country arguing in favour of reservation and challenging prevalent notions of merit. Those who entered higher education institutions under reservation have had to face several forms of abuse and marginalisation. Not being able to cope with the situation of academic isolation and social exclusion, a significant number of Dalit students who entered higher education institutions committed suicide and many dropped out over the years. A series of documentary films under the title 'The Death of Merit' engage with individual cases of suicide These unfortunate events drew attention to the socio economic background of these students, their struggle to overcome the situation, and their high academic achievements at the entry level. They have also brought to the fore the fact that students in higher education institutions dropped out or ended their lives because they were harassed, humiliated, and left with no hope of realising their dreams by the faculty members and fellow students. Caste based discrimination, therefore, could not be dismissed as something of the past; it is an everyday reality even in the institutions of higher learning, in the metropolitan cities of the country (Thorat, 2007).

Rohith Vemula himself was a meritorious student who did not need reservation to join the university. This is because if a student from SC, ST or OBC category earns a high grade that will enable the student get admission automatically, he or she will have to be admitted in the open quota and not under reserved quota. An emerging intellectual, Rohith was active on the campus politics for democracy and social justice. He along with four others were expelled from the hostel on a false allegation by a student affiliated to a right wing student body. The expelled students set up what they called a *veliwada,* the local term where historically Dalits are supposed to live. Rohith tragic suicide signifies the struggle faced by Dalit students to claim a democratic space on the campuses and it led to a large scale agitation across campuses for several months, opening up space for Dalit students in the mainstream media to talk about caste discrimination they face in the academic institutions (The Hindu, 2016; The Citizen, 2016).

CASTE AND ACCESS TO MATHEMATICS

Indian has a long tradition of engaging with mathematics. Indian contribution to mathematics in ancient and pre colonial India has received much attention in recent times (Plofker, 2012). Apart from Srinivasa Ramanujam, there have also been several other Indian mathematicians whose work has received international attention. To name a just few, one could consider the contributions of Harish Chandra, Subramaniam Chandrasekar, P. C. Mahalanobis, C. R. Rao, R. C. Bose, S. Abyankar M. S. Raghunathan, M. S. Narasimhan, all men and at least half of them Brahmins. A few women like Bhama Srinivasan, Parimala Raman and

Sujatha Ramadorai also have received recognition across the world. It would be of interest to note that all the three women mentioned are Brahmins. India has not done well in the International Olympiads, but over the years India has won 11 gold medals. Indian engineers are employed in the software industry all over the world.

From a certain location mathematics education in India would seem at a par with some of the most developed countries. A number of International schools that follow the IB curriculum have emerged in the last decade. Children seeking admission in grade 1 in these schools are expected to know numbers up to 100 at the time of entry. And these are several alternative schools such as Waldorf schools, Krishnamoorthy Foundation schools catering to the elite of the country where mathematics learning is integrated with learning in other subjects. Apart from these, there are top ranking schools (all private or private aided schools) and coaching centres that train a large number of students to get admission in the prestigious Indian Institutes of Technology or other engineering colleges. And in the recent past, some students are even choosing to go abroad for higher education. Most of the children who have been though any of these programs, would be comparable to children from anywhere in the world.

But if we step out of this location and take a look at education in the country as a whole, then, the scene presented above almost disappears from one's vision. Only 10% of the youth of college going age access higher education in India and among them, those who can get world-class education would be a tiny minority. In the larger domain of mathematics education, India figures alongside other developing countries in the low-income bracket with poor literacy (72.1% according to the UNESCO report for the year 2015) and numeracy levels.

In the post liberalization and globalization era, school education in India is getting more and more privatised. Among the schools affiliated to the central and state boards of education in India there is a fine gradation in fee structure reflecting the variation in the economic status. Apart from these, there are also free schools run by the state governments, referred to as 'government schools'.

The government schools cater to the poorest of the poor in the urban locations. Even in rural India, education is slowly getting privatised. From a study conducted in the year 2012 to understand the caste dynamics in six states of the country, we find that in the government schools, the total percentage of children belonging to SC, ST, and OBC categories in the primary grades is between 73.6 (in Assam) to 97 (Orissa) and in the secondary grades it is between 57 (Assam) to 97.6 (Orrisa). Among these, SC's constitute about 20 to 28 %. In some states ST's constitute nearly 40% and in others about 8%. Except for Rajasthan, in all the other state, the savarnas or dominant caste form only about 7%. In Rajasthan the dominant castes are about 15% of the total enrolment. Moreover, nearly 70% of the children from SC category study in government schools. It is not clear what percentage study in private schools and what percentage have dropped out. From these statistics and the fact that about 80% of the SC population lives in rural India, it is evident that school choice depends on the caste category to which a student belongs and that a child from SC category is most likely in a government school in rural India. Equally, a child from dominant caste background is in a good private school in an urban location. In fact, even in villages where there are no private schools, a family belonging to a dominant caste would move temporarily to a nearby area where there is a private school. It is important to note that everyone wants to put one's children in a private school the moment one has a little more money to pay the monthly school fees. Thus, private schools catering to low-income background have a significant number of OBCs and a few SC, and ST children.

Caste as a living reality in the schools

Caste is a living reality that figures in the school in many ways. Ramchandran and Naorem (2013, p. 48) list in detail the ways in which caste figures in government schools. Dalit children do not have access to drinking water from the same pot or tap. Opportunity to go to the board and write does not always exist for Dalit children. Often Dalit children are asked to clean the classrooms and toilets in the school. Caste determines directly or indirectly who gets to sit where in the class: children from dominant caste and OBC categories sit in the front while Dalit and tribal children sit at the back (Ramachandran & Naorem, 2013). Casteist practices from teachers also reinforce what the students bring in from their everyday life, resulting in the formation of caste groups in the classrooms. As a result, the scope for children from marginalised caste background to see themselves as equal in the school and learn from their peers regardless of the caste background gets limited.

Teachers' knowledge and attitude in teaching mathematics

In our experience with schools and teachers in several parts of the country we found that there are many possible situations because of which Dalit and tribal children miss out their opportunity to learn mathematics unlike their counterparts from dominant castes.

Many government primary schools have just one or two teachers teaching all the five classes. In such situations, teachers may not be able to devote the time required for teaching even if they wanted to do so. On the other hand, in private schools catering to low-income families, teachers who come in to teach mathematics at the primary school may not like mathematics or feel comfortable teaching mathematics.

In schools where the teachers do have the subject knowledge, they are reluctant to teach these children because they do not believe these children have the ability to learn and they need to learn what is prescribed for them. In our work with government schools and private schools catering to low-income background in Madhya Pradesh, we found that the schools were poorly resourced, with blackboards not writing and classrooms crammed (Subramanian et al, 2015). In the school where we experimented an alternate approach to teach fractions, we faced unwanted sympathy from the teachers saying "You are working so hard with these children" the implication being, "Unless one worked very hard it would not be possible to teach these children and in any case it is not worth it". In the longitudinal work spread over a period of three years when we worked with the same set of children, moving with them from grade 3 to grade 5, none of the teachers from the school showed any interest in knowing what we were trying to do or what the students have accomplished. The achievement of these children did not attract their attention. That the same students who otherwise would have struggled to compare ½ and 1/3, can compare fractions using meaningful approaches, provide argument to support their claims, place fractions on the number line, did not surprise the teachers because, they never really found out that the students can do these. Their only response to the alternate approach was to taunt students by asking "Oh, will you always share rotis whenever you have to answer a question on fraction".

We found the same response from teachers in other schools where we experimented with teaching children the notion of angle and how to measure angles. In several instances, teachers found one excuse or the other not to teach. When we visited classrooms, we found that children were either copying pages and pages of mathematics from a guide book in which answers to the problems were worked out or they were reciting the tables. Teachers also told us that the kind of children who are coming to government school these days are not interested in learning. Teacher absenteeism is known to be a common phenomenon in these schools. In short, even at the primary level, teachers who could teach mathematics chose not to teach, showed no interest in learning new approaches, ignored the evidence that when taught children can reason very well, because they did not think that there is any value in these children learning mathematics. Nikita who is a Dalit girl and Bharati who is a tribal girl, both very keen and bright studied in these schools. If only they were not from the castes to which they belonged and if they studied in schools where the dominant caste, middle class children studied, there would have been no doubt that they would excel in mathematics.

In very remote locations, it is difficult to find teachers who know mathematics enough to teach what is required of them at the upper primary and secondary level. This is because, no one qualified enough wants to work in these locations and anyone from the same location who has managed to do better in schools leaves the place for better opportunities in life. Families belonging to dominant castes have migrated from these locations leaving behind only those from SC, and ST and a few from OBC categories. Even private schools catering to low-income background find it difficult to attract competent teachers because neither the pay is good nor is it a matter of pride to teach the socio-economically marginalised children.

We also found some very motivated and competent mathematics teachers when we were involved in mathematics curriculum and textbook development in the state of Rajasthan and during textbook revision in the state of Andra Pradesh. The teachers from government schools in Andra Pradesh were adept at using digital resources for teaching mathematics. However, their knowledge does not always reach the Dalit and tribal children in the government schools where they are employed. Rather, they put their knowledge to use outside the school, in the private tuition centres where they teach those children who have the money to pay and deserve to learn. For children in government school, they teach just that much mathematics that will help them scrape through the final examination.

Dalit Learners experience of mathematics in school

Murali Krishna, a Dalit academic who says he succeeded in not dropping out of school because he managed to barely pass in mathematics in the school final examination talks about the kind of discrimination and exclusion that Dalit children face in school (Murali Krishna, 2012). In a personal conversation, I told him that if he had had the opportunity to study in a good school, where the teachers were knowledgeable and competent he would have probably excelled in mathematics; he responded by saying that studying in a good school is not a sufficient condition, because, often teachers do not want to teach mathematics to Dalit children. Echoing his voice, another Dalit educator who managed to study mathematics up to graduation says, many Dalit children who get very good grades in the school final examination are refused opportunity to opt for mathematics and science at the intermediate level saying they will not be able to cope with mathematics at that level. Two Dalit women who are currently pursuing higher education say they hardly got an opportunity to learn any mathematics in school. They say, that they managed to pass the school final examination with whatever oral mathematics they learnt from their illiterate parents. One of them says in a classroom with 120 students, Dalits students never had the courage to talk to teachers or ask questions.

Teachers only addressed the ‘good’ students seated in the front. Another says, dominant caste children had opted to learn mathematics outside the school paying additional tuition fees, which the poor Dalit students could not afford. So they were mostly ignored in the classroom except when the teacher was angry and ask them a question with the intention to punish.

CONCLUSION

For Nikita and Bharati, it is not their economic status or lack of ability in mathematics that is responsible for poor access to mathematics. It is the caste background that determines their scope for learning mathematics (Sadafule & Bernstein, paper submitted for this conference). They have managed to pass the school final examination on their own resources and perhaps they will enrol in college to major in mathematics. It is easy to see that Sunita, from a similar background enrolled to graduate in mathematics in a prestigious university on her own efforts. Ill prepared as she was and marked as a dull student and ignored by the university, she committed suicide when even her love life failed.

Given that 80% of the Dalits and 90% tribals live in rural India and a large percentage of Dalits and tribals are poor, a typical Dalit learner goes to government or low-income school, and faces caste-based exclusion in the school. Typically, he or she sits in classrooms where no teaching happens, and so he or she struggles to cope with school education and gets to learn very little compared to her cohorts from dominant castes. As a result, for every Murali Krishna and others who have miraculously managed to pass the mathematics paper school in final examination and succeeded in accessing higher education, there are several Dalit and tribal children who fail in school mathematics which forces them more often than not to return to the kind of demeaning occupation that they desperately want to escape. The minority among the Dalit learners who did not drop-out of school at some stage, fail in mathematics in the school final examination, and were not counselled not to opt for mathematics at the intermediate level, enter college wanting to major in mathematics with limited knowledge of mathematics irrespective of their ability and interest in the subject. And they will be pushed to the back benches in the college. For female Dalit and tribal students it is even more difficult to access mathematics because they suffer both caste-and gender-based discrimination in school. It is a miracle that some of them managed to reach a university to pursue a masters degree in mathematics as Sunita did.

Segregation in education in India may appear to be based on the economic background with a negligibly small minority accessing quality education in mathematics. This picture does not reveal the fact those who can afford good education in India belong to the dominant castes. The bulk of students who have very poor access to quality education in mathematics do so because they belong to marginalised castes. Poverty alone therefore is not adequate to understand issues in mathematics education in India. Caste needs to figure as a central factor to understand access and exclusion in mathematics.

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CHALLENGES IN DEALING WITH SOCIAL JUSTICE CONCERNS IN MATHEMATICS CLASSROOMS

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In this paper, I discuss instances from teaching practice to problematize the nature of teacher's engagement while dealing with the social justice concerns in the teaching of mathematics. I will draw upon the data collected as part of my research study, which aimed to explore and enhance teachers' knowledge of students' mathematics as it gets manifested in their practice. The instances from teaching practice are used to discuss the negotiations made by teachers in the selection of content, what constitutes learning of mathematics, and views about learners from different backgrounds. In this discussion, I will raise some of the challenges and complexities in dealing with social justice concerns when teaching mathematics in classrooms.

INTRODUCTION

Several mathematics educators have suggested the integration of social justice concerns in the teaching of mathematics with an aim of developing a critical perspective towards teaching and learning of mathematics (Skovsmose, 2014; Penteado & Skovsmose, 2009; Gutstein, 2003). The recent National Curriculum Framework (2005) in India explicitly states the aim of education to be social transformation. The new vision of mathematics education focuses on engaging students in doing *meaningful mathematics* (NCERT, 2005). The curriculum framework analyses the fear and failure of mathematics among school children and proposes a focus on the processes of estimation, abstraction, problem solving, optimisation, generalisation, etc. to engage students with the structure of mathematics (NCERT, 2006). The revised primary school mathematics textbooks emanating from this vision have tried to address the concerns by bringing in several real-life examples of mathematics from the environment. Despite an extensive discussion on integrating social justice concerns with the mathematics curriculum, preparing teachers to deal with these remains a rather under-developed area (Gonzalez, 2009). The objective structuring practices dominant in classrooms and enacted by teachers might normalise the existing structural inequalities (Gates & Jorgensen, 2009). Proposals for pedagogy based on 'discomfort' and 'inquiry' are made in literature on integrating social justice issues in mathematics classroom. However, pedagogies need to be situated in a practice perspective and engage with the challenges that teachers face in handling these issues in classrooms. Nolan (2009) problematizes the marriage between mathematics social justice in classroom by providing an example of the 'didactic tension' faced by the teachers in directing students' attention to a powerful perspective and respecting students' agency at the same time. The linear models of teacher change are insufficient in unpacking the nature of engagement that teachers have in dealing with such tensions in classrooms (Boylan & Woolsey, 2015). The complex integration of social justice issues in mathematics classrooms requires that teachers are prepared and supported in handling the tensions and conflicts arising from dealing with these issues in classrooms. In this paper we make an attempt to engage with teachers' negotiations with social justice issues in classrooms through a deeper understanding of their practice. We find that teacher decisions when handling social justice concerns in the classroom depend on their (a) perception of what constitutes as important mathematics, (b) assumptions about learners from different backgrounds, and (c) processes involved in 'schooled' learning of mathematics. The teachers' relation with the textbook and selection of content (identified as 'relevant') for students is kept in mind when analysing each of these aspects.

The Question of Teacher Knowledge

The literature on mathematics teacher education (MTE) identifies the knowledge of a mathematics teacher as *specialised* for communicating the content to students of different age groups. Several frameworks on MTE (categories of teacher knowledge, cognitively guided instruction, knowledge quartet, mathematical knowledge *for* and *of* teaching) propose typologies of organising and understanding teacher knowledge. These frameworks have enhanced our understanding of different processes involved in planning teaching of specific mathematical topics. For instance, teacher knowledge required for teaching fractions requires an engagement with its different sub-constructs or meanings, students' prior knowledge and ways in which it interacts with their new learning, use of representations and their affordances, explanations for composition of fractions and operating on them, etc. Observing these practices of mathematics teaching - providing explanations and justifications, utilising students' prior knowledge, using representations and making connections; in actual classrooms remains complex for each of these are embedded in the context of the classroom. The orchestration of content and use of tools for teaching varies in different classrooms depending on how communities of students and teachers relate with them. While it is acknowledged that the context of teaching is important in understanding how students are receiving it, the intent of analysis focuses on inferring the decontextualized and individualised aspects of teacher knowledge (Rowland & Ruthven, 2011). The situatedness of teaching and its embeddedness in the context of the classroom spaces determined by the students' and teachers' activity needs more systematic analysis. The vast literature on MTE also seems to have missed an emphasis on the social justice concerns in conceptualising the construct of mathematics teacher knowledge. In fact, it is not unimaginable that the orientation to social justice concerns would inform the different practices of mathematics teaching. Thus, it becomes important to study how different aspects of teacher knowledge interact with the social justice concerns in the teaching of mathematics in classrooms.

In this study, an attempt is made to address the problem of studying teacher knowledge by focusing on the practice perspective. It is through an engagement in the real contexts of teaching practice that teachers' perspective on issues concerning social justice and mathematics teaching and learning are discussed. The participation of researcher (in different roles) in teacher's practices during (and after) the period of study helped in gaining an insider's perspective on teachers' negotiations with social justice concerns, and related constraints and challenges.

ABOUT THE STUDY

The larger research study aimed to explore and enhance teachers' knowledge of students' mathematical thinking and ways in which it gets manifested in their practice. Four elementary school mathematics teachers participated in the two-year long study (2012-14). All the participating teachers had a degree in mathematics or physics and in education. The teachers belonged to a school located in Mumbai and run by the central government of India. The school caters to students from mixed socio-economic backgrounds. There are five other such schools located in the vicinity. While the other schools have students with parents who are scientists, this particular school has students with working class parents or single parents who do chores like cleaning the nuclear reactor facility or are drivers, office assistants, peons, etc. The teachers seemed to be aware of the students' backgrounds and home conditions from their interactions with parents and the previous year teachers who taught the same batch of students. Each participating teacher had more than 15 years of experience in teaching school mathematics. They have taught using the textbooks which followed the earlier mathematics curriculum (prior to NCF 2005), focusing on the procedures and rote memorisation of algorithms, as well as the new curriculum which focuses on the processes of doing mathematics and brings in real life contexts into the learning of mathematics.

During the research study, the teachers were observed in their Grade 5 and Grade 6 classrooms, teaching different topics of mathematics. Although the researcher tried to interact with the teachers before and after every lesson observation, this was particularly difficult in the first few months of the study. The reasons included lack of time available with the teachers, and less interest in discussions with the researcher. Also, it was discovered that the teachers were not encouraged to talk or discuss with each other and they often asked the researcher to know more about each other's classrooms. Regular participation in several teacher related activities and presence in the school for whole day, helped the researcher in getting access to the teachers' thought processes and in establishing communication channels with these and other teachers of the school. The data was collected using classroom observations, pre- and post-lesson interviews with participating teachers, informal conversations with the teachers and students, and systematic interactions during weekly teacher-researcher meetings. For this paper, we use the data from classroom observations, and pre- and post-lesson interactions with teachers. The data has been transcribed using the audio and video records, and researchers' observation notes.

INSTANCES FROM TEACHING PRACTICE

The instances from teaching practice refer to those classroom episodes which were found routine and interesting. The researcher (mostly but not always) selected these instances for discussion with the teacher in post-lesson interviews. The purpose was to create a reflective social space for teachers and researchers to engage in a dialogue about the decisions made by teachers while teaching. While the initial emphasis was on probing teachers to encourage discussions about their practice, in the later discourse teachers were found to be challenging their own and other teachers' decisions. The changing perspectives of teachers about their own practice, informed by discussions, were reflected in their practice in the second year of the study.

Dealing with conflicts in classroom

The first instance focuses on the teaching of a real-life problem on multiplication in a Grade 5 classroom. The problem (refer Fig 1) invited students and teachers to discuss the differential treatment received by men and women workers while being paid for the same amount of work. Also, an observation was made about both the workers being paid less than the government norm.

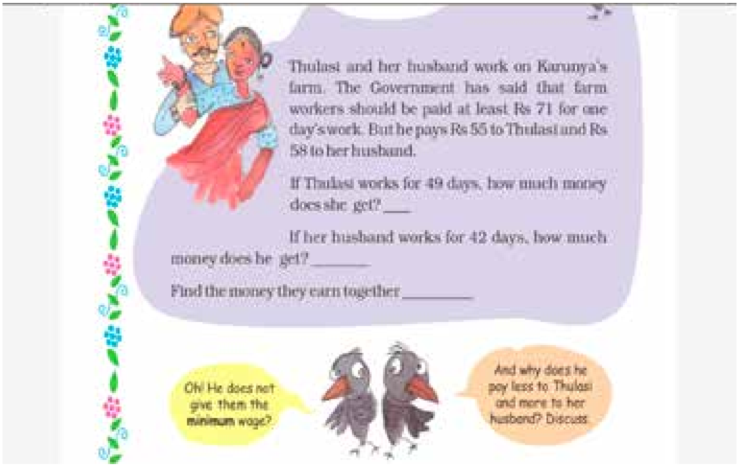

Figure 1: Salary of a worker (NCERT, 2007)

The episode below is a transcript from a Grade 5 classroom where this problem is being done.

On Board: *Teacher writes the following problem on board and students are copying.*

Thulasi and her husband work on Karunya's farm. He pays Rs. 55 to Thulasi and Rs. 58 to her husband. Thulasi works for 49 days and her husband for 42 days. How much they earned together?

Teacher P: Now all of you stop writing. Okay, read [aloud] the problem.

Students read aloud the problem in chorus

Teacher P: Kya karna hai? [What should be done?]

Students: Multiply.

Teacher P: Sab ek saath multiply karna hai? [Is everything to be multiplied together?]

Students: No teacher.

Teacher P: Give me a statement and tell me how to do.

Teacher P: Each and every part [of the question] should have a statement.

Teacher P: So there are three parts to this question - find Thulasi's salary, find her husband's salary, and find the total.

Students did the multiplication using the algorithm to find each of these parts.

To conclude the problem, the teacher did the two multiplications and an addition on the board, and asked the students to check their answers. After the lesson, the researcher probed the teacher for reasons of modifying the problem while teaching. The brief post-lesson discussion with the teacher is reproduced below.

Researcher: The question given in the textbook is different. You changed the problem.

Teacher P: Hmm… changed.

Researcher: What made you change it? Were you thinking something?

Teacher P: This is a multiplication problem.

Researcher: It [*pointing to the textbook*] has this note which says that you could discuss the salary of Thulasi and that why is it lesser than her husband, and then… the government norm.

Teacher P: Haan. [Yes.] Where, where?

Researcher: See here in the end [*pointing to the dialogue boxes*].

Teacher P: Haan but we should avoid such things in the class.

Researcher: Means?

Teacher P: I did the math. That is the important part. The whole story is just a waste of time, no? I mean they [students] will get into the story only. Then when will they multiply?

Researcher: Do you think students can discuss such things in the classroom?

Teacher P: Haan, but now they are in Class 5. They should know the algorithm. They should know the fastest way na. Actually you know I introduced the algorithm in Class 4 only. Now they need a lot of practice of it. Because I do all the textbook problems, I did this word problem also.

Teacher S handled the word problem in a similar way in the other section of Grade 5 in the same school. She mentioned that there is a difference in salaries of Thulasi and her husband and she works for more number of days (49) as compared to her husband (42). Then, she asked the students to find their salary for the given number of days, and add it. She also mentioned that both of them are getting less than the government norm. The post-lesson discussion with Teacher S follows.

Teacher S: I understand what you are asking. See, they are getting less salaries. And this is how it is na. This is outside knowledge. How much of it to bring in class, I don't know. I mean I don't know how to handle this. They are getting less. At this age, can we tell all these to children? And we did the multiplication.

We notice that in each of these classrooms the teacher selectively omitted the conflict arising from the mismatch in the salary decided by the government and given to the workers, as well as differential treatment to women and men. Both the teachers felt that they had done the multiplication, which is central to the mathematics to be taught. While Teacher P seemed confident of avoiding such conflicts in classroom, Teacher S qualified it as a reality and therefore outside school knowledge. Teacher S begins to articulate the difficulty in handling such situations in a classroom.

Analysing the teacher's experiences we propose the construct of 'knowledge demands' posed on teachers due to such situations arising in classroom. What kind of knowledge does a teacher need to discuss and handle social conflicts in a classroom? And what constitutes important mathematics in such situations? The teachers' decision indicates a choice made to keep students away from the conflict situations and do the operations. However, it is important to think what kinds of demands do the conflict situations pose on teachers. In their own experience of *doing* mathematics, teachers have not dealt with such issues and the dominant image of mathematics does not allow for such discussions as they are considered as 'deviations' from learning what is understood as 'important school mathematics' (in this case, learning to perform the algorithms). Teacher's decisions need to be understood in the context of their agency in handling such situations in a mathematics classroom.

In and out of school knowledge

As Teacher S indicates, there seems to be a clear demarcation between the out of school knowledge and school learnt mathematical knowledge. The lines drawn between outside and in school knowledge have implications for the selection and emphasis on particular aspects of content being learnt in mathematics classroom. The following instances indicate the demarcation made between the two kinds of mathematics and its implications for practice.

Teacher P and the researcher are studying different student responses to multiplication problems. Students were asked to use a paper (and not the notebook) to solve these problems as they were expected to use their own ways of solving these problems. While studying student responses, the teacher remarked the following.

Teacher P: If they use all their methods only, then what are they learning in school? In school maths, they learn the algorithms. That is very important.

The observation that solving the problems using the 'algorithm' is the schooled way and all other methods are out of school is quite common. In another lesson by teacher J (teaching in a different school) trying to teach direct proportions, the distinction was explained further.

Students of Grade 7 were given a few proportion problems designed by the researcher based on study of research literature in the field of ratio and proportions. The worksheet was given before the teacher taught the algorithm for direct and *inverse proportion. When asked to anticipate student responses to these problems, the teacher felt that it was unfair to give them such problems.*

Teacher J: I have not taught them the algorithm, then how will they solve these problems. These problems you [researcher] give them at the end of the chapter, when they have learnt and practiced the algorithm. Here [worksheet] you have also given inverse proportion, they will never be able to do it.

After the students solved all the problems on the worksheet, the teacher and researcher examined students' written work. The teacher was amazed to see that all the students had solved the inverse proportion problem correctly. However, Teacher J is unhappy with students' performance.

Teacher J: See all of their methods are out of school knowledge. But that is not what they are here to learn in school. They can learn it outside only. They come to school to learn the algorithms.

The teachers consider the knowledge of algorithms as central to learning of mathematics in school. The over-emphasis on explicit teaching of algorithms and repeated practice by students to learn them is quite common. However, the decision of whether or not to teach the algorithms can be problematized in the context of working with learners from different backgrounds as evident in the following teacher's statements.

Teacher S: Yes, there are first generation learners in my class. The teacher has the responsibility of teaching the method to them. But they have to go home and practice…Some of the children also have to work [after school], but they have to practice what is being taught in class to score [marks]. It becomes important for these children even more.

The concern of making students from different backgrounds learn the algorithm is raised from the perspective of access. While it is important to critique the practice of procedural learning and reproduction of algorithms, the access to this selected knowledge of mathematics for all is considered significant. The teacher seems to be pointing to this complexity of the learning situation.

We find several examples of the fixed use of resources (particularly board and student notebooks) in classrooms also as important evidences in an attempt to characterise the decisions made by the teachers when demands are made on them in the complex socio-cognitive environment of teaching. Some of these examples will be brought to discussion in the presentation.

DISCUSSIONS AND CONCLUSIONS

In this paper, we discuss a few instances of teaching practice where teachers are dealing with issues related to social justice in different mathematics classrooms. The rationale for particular decision-making in such situations is guided by teacher's knowledge of mathematics, which incorporates an understanding of in and out of school mathematical knowledge, and what counts as valuable mathematics. I have attempted to show that the proposal of integration of social justice issues in a mathematics classroom needs more careful attention particularly from the practice perspective. An important aspect of the proposal is to engage with teachers in dialogues on expanding their notions of mathematics teaching while supporting them in practice in handling such issues in classrooms. Further, working with teachers is incomplete without reconfiguring their identity in the discourse on mathematics teacher education. In other words, mathematics teacher education needs to envision and build teacher's professional identity, which includes strengthening their knowledge of the subject matter (as conceptualised by research on teacher knowledge) and the socio-cultural context in which mathematics teaching is situated. The integration of social justice issues in mathematics classrooms poses knowledge demands on teachers. One of the ways of pursuing this is through the development of communities of teachers and researchers by creating a social space for articulating and discussing the issues arising in mathematics teaching. A careful analysis of knowledge demands posed on teachers when integrating social justice issues in classrooms needs to be done.

Based on the insights gained from such classroom practices, we are in the process of developing a pre-service teacher education course for preparing mathematics teachers. The course engages student teachers in the critical aspects of teaching specific topics of mathematics, while contextualising the discourse of mathematics education in its historical context and familiarises them with different practices of mathematics existing in different communities. Further, case based analysis of teaching episodes and a critical evaluation of their own practice (in the sense suggested by Petreado & Skovsmose, 2009) is used to engage them in real contexts of classroom teaching. We believe that further work needs to be done to understand and engage with the complexities of integrating social justice issues in diverse mathematics classrooms.

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MATHEMATICS AND HUMAN FLOURISHING

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To interrogate the place of mathematics in schools is to question the work of humans for the past millennia. Nonetheless, I do so here, as an inquiry into the paradox that students spend roughly a decade in mathematics classrooms, yet leaving formal schooling with few skills in quantitative literacy, or the ability and disposition necessary for effectively dealing with numbers as they manifest in daily life (Jones, 2004; Steen, 2001). I examine the extent to which mathematics is necessary for students' ability to flourish upon exiting school (Brighouse, 2006). These results engender serious questions for mathematics educators and policy-makers.

A GUIDING PRINCIPLE

Let us make the familiar strange (Mills, 1959). A typical child going through her or his country's public education system will take ten to twelve years of mathematics courses. In the United States, and at the secondary level, this coursework often includes Algebra 1, Geometry, and Algebra 2 (or some permutation of the topics therein). What other disciplines have this privilege? Aside from English –sometimes referred to as language arts– no other permeates a students' schooling experience as much as mathematics does. It is a paradox, then, that students in public schools spend roughly a decade in mathematics courses, yet leave –to a large extent– with few skills in quantitative literacy, or the ability and disposition to use, interpret, and criticize numbers as they manifest in daily life (Apple, 1992; Jones, 2004; Steen, 2001; Wilkins, 2010). Of course, this is not solely a U.S. phenomenon (Venkatakrishnan and Graven, 2006; Schumer, 1999), and albeit I use the term paradox here, I doubt the reader feels any element of surprise thus far. Indeed, given the competing ideologies and politics that play a role as groups and individuals make decisions that affect mathematics education (Stanic, 1986), it should be of little wonder that we consistently bemoan the state of mathematics education across the globe. Perhaps, then, we should take a step back to examine our assumptions about the aims of mathematics education. Indeed, in focusing on the engineering of schools (e.g., the content of mathematics and how it is taught), we obscure its ends (Postman, 1996). Thus, in this paper, I examine mathematics teaching with a top-down approach, in that I first choose a guiding principle for education, then examine how mathematics might situate within such. Though there are certainly issues inherent in suggesting that such a first principle exists, it is necessary that we elect something; otherwise we run the risk of meandering aimlessly. Indeed, deciding upon goals –notwithstanding the political tension inherent in doing so– is necessary for any meaningful decision-making surrounding schooling (Labaree, 1997).

My guiding principle is that public schools should foster students' present and future flourishing (Brighouse, 2006), a concept akin to Hilton's (1984) notion of a successful life. Of the ideological camps Stanic (1986) and Ernest (2002) describe that have influenced mathematics education –notably humanists, developmentalists, social efficiency (or utilitarian) educators, and social meliorists– my view of flourishing takes pieces from all of them. Brighouse's (2006) notion of flourishing is straightforward: objectively valuable goods (e.g., a family one loves, a home, material goods), coupled with a life that is lived from the inside (i.e., is commensurate with one's desires), are together necessary and sufficient for a flourishing life (p. 16). Despite its definition's simplicity, flourishing incorporates a spectrum of needs, ranging from having a career, to spending time with friends and enjoying hobbies or other interests. To boot, flourishing is or everyone. It aligns with humanist goals insofar as it encourages one to pursue a worthwhile education, but with developmentalist goals as it necessarily accounts for one's own desires –not just those viewed by others as "intrinsically worthwhile" (Hilton, 1984, p. 2). It aligns with social efficiency goals insofar as it encourages one to pursue interests of immediate relevance to their future lives; all the while, there is no assumption that there are pre-defined roles for students to fill in society. It is perfectly acceptable if scores of students become engineers, but there is no assumption that our society has such aims, nor that we desire to compete economically with other nations. Finally –and quite importantly– flourishing aligns with social meliorist goals, as it aims for the flourishing of all. I discuss the latter goal in depth later.

If one subscribes to the notion that flourishing should be a primary aim of schooling for all students, a question that immediately arises is how schools are to cultivate such. This is a nontrivial query, and as one might anticipate, Brighouse (2006) does not have a prescription for doing so. On the contrary, any prescription would likely ignore the multiplicity of ways in which humans can flourish –counterproductive to our aim. We are left on our own, then, to situate mathematics within a curriculum that fosters flourishing. Before moving on to this quandary, I describe mathematical and quantitative literacy –distinct constructs that inform our views of how mathematics might fit into school curricula.

MATHEMATICAL AND QUANTITATIVE LITERACY

A central component of my argument is that the mathematics curriculum should foster quantitative literacy, which in turn promotes flourishing. Mathematical literacy involves, among other things, the skills necessary for doing traditional school mathematics, such as algebraic manipulation and computational dexterity; in many ways, it is encompassed by the notion of symbol sense (Arcavi, 2005). Large-scale attempts to measure mathematical achievement –such as the Programme for International Student Assessment– primarily examine students' mathematical literacy (Wilkins, 2010). On the other hand, quantitative literacy is described as:

> The ability to adequately use elementary mathematical tools to interpret and manipulate quantitative data and ideas that arise in an individual's private, civic, and work life. (SIGMAA in Quantitative Literacy, 2004).

The ability to adequately use elementary mathematical tools to interpret and manipulate quantitative data and ideas that arise in an individual's private, civic, and work life. Like reading and writing literacy, quantitative literacy is a habit of mind that is best formed by exposure in many contexts (SIGMAA in Quantitative Literacy, 2004).

This definition suggests that a quantitatively literate individual should be able to successfully work with numbers as they manifest in day-to-day living –whether on talk shows, in the doctor's office, or on a nutrition label. Not captured in the definition, but nonetheless important, is that quantitative literacy includes a tendency to use "mathematical knowledge as part of a process of social criticism and renewal" (Apple, 1992, p. 429). In many ways, this is Freire's (2005) "critical consciousness" as it applies to mathematics, or that which we evoke when we discuss criticalmathematics (Frankenstein, 1990) or mathematics for social justice (Bartell, 2013). Mathematical and quantitative literacy are related, but not equivalent, as having one does not imply having the other (Madison, 2003). Mathematical literacy develops over the span of mandatory schooling and through post-secondary course-work, while the seeds of quantitative literacy –percentages, ratios, linear equations, and basic statistics, among other topics– are sewn in early grades and grow (if at all) through interdisciplinary efforts and critical reasoning tasks at the late- and post-compulsory levels of schooling (Hughes-Hallett, 2003).

Given that we are inundated with numbers each day, quantitative literacy enables one to participate in democracy (numbers flood election cycles), to participate in other disciplines and professions (e.g., law requires logic, entrepreneurship involves financial skills), to manage one's personal finances and personal health, as well as to recognize social injustices, among other things. This is not mere rhetoric. Research has linked quantitative literacy (distinct from mathematical literacy) with better decision-making (Jasper et al., 2013), nutrition label understanding (Rothman et al. 2006), and even risk comprehension in healthcare (Fagerlin et al., 2007; Lipkus and Peters, 2009). The practical necessity of quantitative literacy calls for an examination of how current curricula serve to foster it. I use the U.S., South Africa, and England as examples, given that each of them has had major discussions surrounding notions of or related to quantitative literacy. The current U.S. mathematics curriculum –primarily driven by the Common Core State Standards for Mathematics– focuses on mathematical literacy through all of the compulsory grades K-12, with quantitative literacy (without a critical lens) as a tertiary goal, if one at all (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). In South Africa, where the last year of compulsory schooling is grade nine, mathematics is mandatory in all phases of schooling, both the compulsory and non-compulsory phases (Venkatakrishnan and Graven, 2006). The compulsory curriculum focuses on mathematical literacy (as described in this paper), whereas in the post-compulsory curriculum includes quantitative literacy (without a critical lens) as a less rigorous option for individuals interested in careers non-quantitative in nature. An apparent issue in both countries' curricula is the conflation of quantitative literacy with mathematical literacy, with no impetus to focus on the former, if at all. In England, where the last year of compulsory schooling is grade 11, there have been pilots of a curriculum where quantitative literacy (in this context, called functional mathematics) is woven throughout the compulsory program of study; in post-compulsory schooling, the two constructs are distinct and are fostered in separate courses. Functional mathematics did not continue beyond its pilot stages. In each of the three countries, mathematical literacy is central, and quantitative literacy –if included at all– does not have a critical component. As I will expand upon below, quantitative literacy permits flourishing insofar as it legitimately influences one's potential for obtaining objective goods; it also cultivates a life lived from the inside, as being quantitatively literate allows one to make informed decisions as they relate to numbers. On the other hand, mathematical literacy typically permits flourishing only insofar as its exchange value allows one to obtain objective goods –a key distinction.

FLOURISHING WITH MATHEMATICS

From its definition, it is apparent that flourishing in our world is a nontrivial endeavor. I begin by seeing how mathematics connects to the objective goods of Brighouse's (2006) framework for flourishing. One may obtain objective goods in a myriad of ways, with the most significant that connect to mathematics being the remunerated work that brings about material goods –a subset of objective goods. To boot, it is also well-known that having material goods increases the likelihood of finding a partner, and thus building a family –another type of objective good. Is this exchange of mathematics for material goods a legitimate one? Labaree (1997) argues that the most recent force in education is the goal of social mobility, wherein students compete for knowledge that they can later exchange for status and pay (i.e., objective goods). This is especially true in learning mathematics, where competency in the discipline often permits one to enter university (through one's SAT scores), exit university (through general education requirements), and even obtain a job (through one's GPA), completing significant, status-earning tasks that –in and of themselves– have no significant relation to the mathematics itself. Research readily confirms this advantage, as mathematical ability (as defined through grades or test scores) is connected to both wage increases and likelihood of fulltime employment (Eide & Grogger, 1995; Levy et al., 1995; Rivera-Batiz, 1992). Thus, as it stands, mathematics education artificially promotes the flourishing of people privileged enough to have access to a quality education. In large part, this conflation of exchange value and use value manifests from our belief that mathematics is a surrogate for rationality (e.g., Gibson, 1986), which is problematic. The conclusion I draw from this is that for careers that require no more mathematics than that demanded of quantitative literacy –the content found up to grade eight in most countries– the mathematics learned beyond such a grade is of little use for students in everyday life and work (Hughes-Hallett, 2003). The humanist tendency to continue marching through mathematics after grade eight –albeit well-intended– runs counter to the notion of flourishing if it clashes with students' interests.

Indeed, the second component of Brighouse's (2006) framework for flourishing, living a life aligned with one's inner desires, dictates that one be able to pursue their passions. On an "extreme" level, this means that an Amish child be able to choose a different lifestyle as she or he grows older (Brighouse, 2006); more mundane, but applicable here, is that students be able to take coursework in subjects of their choice. Coursework in mathematics is readily offered by most schools, though its equitable accessibility is contentious (Berry, 2008). With or without access to mathematics courses, students are less likely to be able to focus on courses in subjects such as art, geography, or sociology, among many others, should they desire to do so. I leave it to the reader to gander at what might replace four years of mandatory mathematics for students; students might engage in discovering psychology, sociology, human geography, and philosophy, among a host of other disciplines, all of which would aid in directly developing students' reasoning and communication skills, skills which many equate as an inherent reason for learning mathematics. (Whitney, 1987). Indeed, my belief is that is not necessary to learn mathematics in order to advance the aforementioned skills, nor to develop an appreciation of –as Hilton (1984) viewed mathematics– a great human activity.

IMPLICATIONS FOR CURRICULA

With my analysis using the framework of flourishing in hand, the conclusion is straightforward: students do not need very much mathematics –it is merely a tool we gaze upon the world with, useful only insofar as we create the need for its use (Dowling, 1998). What are we to do? I believe that through grade eight, students (in non-tracked classrooms) should all participate in an engaging, challenging mathematics curriculum that promotes both mathematical literacy and quantitative literacy. While some have argued that we should delay mathematics until students are older (Benezet, 1929), I believe that all students are capable of learning mathematics when taught in a developmentally appropriate way. This would prepare students for the mathematics they would encounter in higher levels, should they choose to enroll in such, as well as sow the seeds for critical thinking using numbers outside of school. Beyond grade eight, it should be up to students –not test scores or advisers– to decide whether they desire to take further courses in mathematics. At the same time, courses in other disciplines (e.g., biology, English, sociology) would begin reinforcing the skills and critical reasoning of quantitative literacy as they manifest within the discipline, following a spiraling approach akin to that suggested by Bruner (1977) or Whitehead (1929). This means the endeavor should be interdisciplinary (Steele & Kilic-Bahi, 2008), in that we integrate quantitative skills and critical thinking across the disciplines, regardless of whether the student is taking further mathematics courses. For example, though students may develop a basic conception of percentages in primary school, they would revisit the concept often in varying grades, discussing concepts like percent change, the distinction between relative and absolute change, or even how the stock market works –meaningful and challenging applications. Mathematics would never leave the curriculum –students simply would not be forced to take it directly. The freedom from mathematics courses in the upper-secondary grades would permit student flexibility in taking courses that promote their flourishing. Important to additionally note, the curriculum should be set up so that students may opt back in to mathematics courses or a STEM pathway, should they be interested in doing so at a later time.

LIMITATIONS

Readers are likely to be wary of my proposal. Given the inequities typically associated with tracking, it is certainly logical that one might view this reasoning with caution (Oakes, 1982; Phillip et al., 2016). Indeed, as it has become increasingly apparent that mathematical knowledge permits social mobility (Labaree, 1997), we are likely to view any subtraction of the discipline from the curriculum (for any student) as blasphemous, regardless of whether it remains in their best interests for flourishing. To boot, given the recent push to make STEM careers available to all students –especially those in historically marginalized groups– it would appear backward to propose any option that might hinder progress toward equitable outcomes in education. The response that some tend to have in pushing for equity in mathematics education is curricula like criticalmathematics (e.g., Frankenstein, 1990) and mathematics for social justice (e.g., Bartell, 2013). All the while, such pedagogies are useful only insofar as they involve the use of statistics and basic mathematics to heighten students' awareness of injustice. Such curricula give no advice on how to imbue, for example, trigonometry or precalculus, with any meaning for students. That is not their purpose, as Frankenstein (1990) notes "A critical mathematical literacy curriculum can be relevant to getting more people of color into mathematics and science fields" (p. 346). The implication of this is that they are not completely transformative curricula –after a certain point, students return to traditional mathematics and perpetuates its hegemony. As I have argued, quantitative literacy encompasses a critical awareness of mathematics and its role in society; to boot, includes knowledge of basic statistics and how they can be used in the real world. Thus, it appears that one can weave criticalmathematics and mathematics for social justice throughout our proposed curriculum. What remains essential is that we should not push mathematics on students simply because it has high exchange value; doing so out of a push for equity only perpetuates the problem. This, of course, does not remove the issue of students' ideas for flourishing being influenced by hegemonic discourses and institutional structures; that being said, keeping mathematics as the answer –or savior curriculum– is problematic.

CONCLUSION

It has long been suggested that in order to improve mathematics education, we need to –among other things– disrupt traditional ways of teaching (Gadanidis, 2012), make the content better connected (Woodhouse, 2012), and make available the benefits of mathematical knowledge to everyone (Apple, 1992). These are laudable aims; however, in this paper, I have argued that to a large extent, none of these changes disrupts the root of the problem –that most students do not need twelve years of mathematics in order to flourish. As Pais (2013) eloquently notes, "If the purpose is the high ideals of peace, democracy, social justice and equality, the route via mathematical thinking, in which we currently invest so much, is a dead end" (p. 5).

My thesis –that the curriculum in most countries requires too much mathematics of students– rests on the assumption that the primary aim of schools should be to help students to acquire objective goods and to lead lives on their own terms. The latter, as I have argued, suggests that students take as much or as little mathematics as they desire to –at least after grade eight. The former component –the acquisition of objective goods such as friends, material items, or love– requires mathematics only insofar as one's career does. Time spent learning mathematics is necessarily less time spent learning material of one's interest. With all of this written, I recognize the inherent complexities and issues embedded within the argument. All the while, the current practices we see are fundamentally incongruous with empowering students to flourish. I encourage educators and policy-makers to critically reflect on our complicity in the matter, as –albeit we are often well-intended– by allowing the current quantity of mathematics content in schools to remain constant, we ignore a significant component of the problems we purport to mollify.

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HOW LOCAL ARE LOCAL PEOPLE? BEYOND EXOTICISM

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In this article we shall evaluate the possibility of a meta-level analysis based on two empirical long term ethnographical investigations. This analysis is conducted by an interdisciplinary team of researchers involved in the ethnomathematics programme. Insights drawn from anthropology, mathematics, learning theory, epistemology and philosophy converge in our meta-level analysis. The main research questions at the basic level of empirical research are the analyses of local ethnomathematical practices on four levels: a descriptive level (appearance), an analytical level (mathematical rationale), an educational level (informal learning) and an interactive level (integration of local and global knowledge). The purpose of this meta-analysis is to broaden the theoretical toolkit for further empirical investigations in this area.

INTRODUCTION

Ethnomathematics is a field of research which is primarily interdisciplinary. The concept itself implies utilizing anthropology, cultural studies and mathematics. These disciplines can be aggregated by a single researcher (e.g. Paul Gerdes, etc.) or they can be spread out between members of a team or a couple of researchers (e.g. Marcia and Robert Ascher, respectively a mathematician and an anthropologist). The present article is the result of the collaborative work of four researchers from different backgrounds involved in the ethnomathematics programme. Insights culled from anthropology (Vandendriessche, 2015; Mafra & Fantinato, 2016), mathematics (Ascher, 1988), learning theory (Wenger, 1998; Lave & Wenger, 1993), epistemology (François, Mafra, Fantinato & Vandendriessche, upcoming) and philosophy (Žižek, 1997) converge in our analysis, which we will refer to as a 'meta-analysis'. This list of literature is definitely not all-inclusive. From within the article's framework, we shall concentrate on the major influences, but without ignoring any pertinent field research available. The appraisal of the potentiality of meta-level analysis is based on two long term empirical ethnographical investigations conducted in different places and with different peoples. The meta-analysis of these data is made through critical analyses from theoretical insights as mentioned above. The field research was carried out in Northern Ambrymese society (Ambrym Island, Vanuatu, South Pacific); as well as in the Aritapera region, a rural area near the city of Santarém, state of Pará, in northern Brazil. The empirical research on these groups' activities focused on string figure-making; sand drawing; and handcrafted *cuias*, respectively. The peoples at study are the Ambrymese society and the craftswomen from the city of Santarém [organized in the Santarém Riverside Craftswomen Association (ASARISAN) since 2003]. The purpose of this meta-level analysis based on both empirical investigations is to broaden the theoretical toolkit for further empirical research in the field of ethnomathematics programmes. Accordingly, we will present our two cases focussing on four reference points which will be developed in the following section dealing with the theoretical framework.

THEORETICAL DEBATES ON ETHNOMATHEMATICS

In this theoretical framework we will first describe the points of reference we used to present the two empirical cases as a basis for our meta-analysis. We make the distinction between (not discrete but somehow overlapping) categories. (i) The methodology used. As anthropologists and critical ethnographers we have to take into account the different perspectives and political consequences of studying local practices by academic researchers. Even within the theoretical framework of critical ethnographical research and long term ethnographical research we have to assess the involvement of the researcher and of the people included in the research –the people and their local practices studied. (ii) The main research questions at the basis of empirical research are the analyses of local ethnomathematical practices on (a) their appearance, function and their interconnection within the local community (b) the mathematical rationale behind these practices, (c) the informal learning during the processes in local practices and (d) the integration of local and global knowledge that merge as a result of the anthropologist's interaction with the local community. (iii) We have to clarify the additional aims or side effects of the study. Ascher & Ascher (1997) emphasized that part of their research project was to revaluate the local practices of indigenous peoples. Another political commitment of field research can be to save or even to reinstate local practices in order to favor the sustainability of cultural diversity (whose goals are similar to biodiversity). (iv) The research can be introduced or pursued in order to respond to the peoples' wishes e.g. to help them practice academic mathematics; to include them in the local curriculum or to establish a local economy based on 'mass' production using their local practices.

These points of reference are related to contemporary theoretical debates on ethnomathematics. Within the framework of this article we will mention a philosophical debate on the concept of the 'Other' as introduced by Pais (2011). Studying local practices of indigenous peoples is politically sensitive (and subject to review by ethical commissions before a research proposal is accepted). The substantial criticism of Žižek (2009) on the desubstantialized Other is essential for understanding the evolution in the way ethnomathematics are practiced. It was the critical work of Ascher & Ascher (1997) which pointed out the hegemony of anthropological work. This was our starting point in examining new methodologies (e.g. critical ethnographical research) where the 'object' of the study becomes a stakeholder in the research's design as a collaborative and involved researcher. As researchers we need to be aware of the different perspectives when referring to 'the other'. What is interesting in the context of both empirical cases we will be presenting is the way local people interact with the researchers and how they perceive the researcher as the useful other in order (i) to set up a local curriculum based on their own practices or (ii) to develop an economical business model based on the commercialization of their local practices. From this perspective we can observe the eagerness of local people's involvement in globalization and how the 'Western' researcher becomes the exotic other. Taking into account that we always remain the 'other' when practicing ethnomathematical research, we hope to provide answers to our research questions, as detailed in the two empirical cases we will be presenting. The final research question –on a meta-level– is how two anthropological investigations in the field of ethnomathematics carried out in different places with different peoples can be compared. Our goal is not to contribute to a generalized universal ethnomathematical theory. Instead, we are looking for local practices as a way in which we can learn from each other in order to contribute to the ethnomathematics research field or speaking in terms of the more dynamic approach of U. D'Ambrosio (1997), the ethnomathematics programme.

CASE STUDIES: EMPIRICAL RESEARCH

String figure-making and sand drawing from Vanuatu

Eric Vandendriessche has undertaken ethnographic research that aims to compare, through an ethnomathematical approach, two activities –with a mathematical dimension– carried out in Northern Ambrymese society (Ambrym Island, Vanuatu, South Pacific). These activities can be described as string figure-making and sand drawing, and are locally termed using the same vernacular term *tu* (lit."to write"). This suggests that both activities are perceived by Ambrymese people as conceptually linked to one another. String figure-making consists in applying a succession of simple gestures –analyzable as "elementary operations"– to a string (knotted in a loop), using mostly fingers and sometimes feet, wrists or mouth. This succession of operations, which is generally performed by an individual and sometimes by two individuals working together, is intended to generate a "final figure" whose name refers to a particular being or thing. For over a hundred years this practice has been observed by anthropologists in many regions of the world, especially within "oral tradition" societies (Paterson, 1949; Maude, 1978). A few mathematicians (Ball, 1911; Storer, 1988) have also regarded string figure-making as a worthy topic within their discipline. The analysis Eric has carried out on the string figure-making processes that he observed in Vanuatu (and on those he collected in the Paraguayan Chaco and in the Trobriand Islands, Papua New Guinea) gives evidence of the expression of a mathematical rationality in this activity. String figures can be analyzed as the result of an intellectual process involving concepts such as "algorithm", "operation", "sub-procedure", "iteration" and "transformation" (Vandendriessche, 2015).

In Vanuatu, the practice of figure-making by drawing a continuous line with the finger, either in the sand or on dusty ground, –generally drawn through the framework of a grid made of perpendicular lines, without retracing any part of the drawing– also consists in an ordered succession of operations which can be seen as geometrical algorithms. Like string figure-making, the making of sand drawings has been observed in Vanuatu since the early twentieth century (Deacon & Wedgewood, 1934) and is still practiced in many places on the archipelago, in particular in (Northern) Ambrym. More recently, mathematician Marcia Ascher (1935-2013), drawing from these publications, has underscored the mathematical aspects of the practice of sand drawing (Ascher, 1988). Some conceptual tools have thus been created, that allow hypotheses to be built on the methods used by the partakers (in the past) for inventing such drawings and on the cognitive acts involved in this activity.


Figure 1: Left: Mata performing mel (a nut)– Right: sand drawing rem (yam), Ambrym, Vanuatu. © Vandendriessche

Eric's project (still in progress) is based on ethnographical investigations (carried out in 2006, 2012 and 2016, around the village of Fona, Northern Ambrym) aiming at collecting various types of data: 1) the procedures leading to the various figures 2) the vernacular (technical) terminology linked with the studied practices 3) the oral texts and/or discourses which are sometimes associated with the latter. This in-depth collection has been mainly collated through semi-structured interviews with the practitioners considered by the other members of the community as the more knowledgeable in string figure-making or sand drawing. At a second stage, the collected data have been analyzed –and put in perspective with other ethnographical sources– in order to comparatively analyze the mathematical dimension of the latter procedural activities in their relationships with other forms of knowledge in a given society.

Using and refining both the formal/modeling tools elaborated by Ascher for studying sand drawings and those Eric has developed for analyzing string figure procedures, enable him to comparatively study the cognitive acts involved in the creation of figures either made with a looped string or those drawn in the sand. In particular, the comparative analysis of both corpora (string figures/sand drawings) has brought to light numerous "sub-procedures" (i.e. ordered sequences of 'elementary/basic' operations/patterns used in a similar fashion in several procedures within the same corpus). This suggests comparable operational practices in the creation of sand drawings and string figures. Furthermore, the concept of 'iteration' (iteration of a pattern or a sub-procedure) and the concept of transformation (of the final figure 'geometry' i.e. combination of motifs) are ubiquitous in both practices. Finally, some Ambrymese string figures suggest that practitioners have elaborated some procedures –or paths– leading to identical or very similar final figures. In a related way, in the case of sand drawings, it can be shown that different paths have sometimes been created to obtain exactly the same final drawing, varying the order in which the various segments are drawn.

Some expressions are used by the practitioners to refer to the (basic) movements involved in both activities (operation applied on the string/basic patterns). Although these vernacular terms usually differ from one activity to another, they are action verbs in both cases. The existence of these expressions suggests a local perception of the notion of 'elementary operations' (string figure-making)/'basic patterns' (sand drawing) revealed by ethnomathematical analysis. Furthermore, vernacular expressions (uniform in both activities) explicitly express the property of symmetry (shared by a number of these figures) and the iteration of a pattern or a sub-procedure.

As the first observations on the two studied practices suggest they both are –or were– meant to record, memorize and/or express some "traditional" knowledge. In particular, string figures and sand drawings are both means of representing mythological entities, cosmological and environmental elements, and they are both subject to ritual prescriptions. In Northern Ambrym, string figure-making is said to be a feminine activity, whereas sand drawing is said to be a masculine one, and is more highly valued than the previous one. However, it is indeed a shared knowledge, almost everyone being able to perform a few of these figures. Moreover, the latter are preferably performed during the yam harvest (from February to July), while their usage is prohibited outside this period, the making of such figures being perceived as having a negative impact on the growing of the stem of the plant winding around the stake: it would favour the entanglement of the stem, slowing down the plant's growth.

About a decade ago, the Republic of Vanuatu (ex New Hebrides, a French & English Condominium until its independence in 1980) has begun an evaluation whose goal is to elaborate a National Curriculum taking into account the various local cultures and the different vernacular languages (about 120 in the archipelago). In that perspective, the *Vanuatu National Curriculum Statement* (2010) recognizes the value of traditional knowledge and practices (often in decline) –such as sand drawing and string figure-making in particular– and induces their integration in the national curriculum (still under revision). Therefore, the "Vanuatu Cultural Centre", the local institution working for the preservation and the promotion of different aspects of Vanuatu's culture, –drawing on Eric's ongoing research– has given its (mandatory) assent for this project, provided it leads to pedagogical applications. As the next/final stage of the latter project, pedagogical materials, related to the outcomes of the present research, will be produced in collaboration with local practitioners and mathematics educators, by means of a "critical dialogue" (or "mutual interrogation" *cf.* Adam, Alangui & Barton 2010), discussing the mathematical dimension of local practices as well as, in particular, their value in mathematics education. This should help local teachers in experimenting with the use of (culturally-related) mathematical string figure-making and sand drawing practices 'as such', in and of themselves, bringing into the mathematics classroom the cultural and cognitive complexity of these practices.

Handcrafted *cuias*, State of Pará, Brazil

This study, based on an ethnomathematical perspective, as well as sociological and anthropological theoretical contributions, is germane to post-doctoral research carried out by Mafra and supervised by Fantinato, in 2015 (Mafra & Fantinato, 2016). The main objective of this research was to study the techniques, processes and tools involved in the carving of orna mental patterns on the curved surfaces of vegetable gourds, called *cuias*.

Figure 2: Handcrafted cuias with tapajonic (foreground) and floral (background) patterns. © Fantinato & Mafra

This research was carried out in collaboration with a group of eight women artisans from the region of Aritapera, a rural area near the city of Santarém, state of Pará, Northern Brazil. In 2003, these craftswomen established an association –the Santarém Riverside Craftswomen Association (ASARISAN). The manufacturing of *cuias* is a traditional practice carried out by women of this region and is an indigenous tradition, performed primarily during the rainy season. The *cuias* are first collected from the *Cuieira crescentia cujete* tree. Next they are cut, scraped, and dyed, then decorated with various carved patterns. By using natural resources at each stage of production (fish scales for sanding, and an organic purple pigment for dyeing) this activity has no impact on the local ecological balance or the forest's regeneration.

The research was based on ethnographic fieldwork, carried out between 2014 and 2015. Data collection methods used included field notes, recordings, open interviews, photographs and video recordings. The study is a detailed description –of the making of *cuias*– approaching what has been defined as a "thick description" (Geertz 1973, Denzin 1989) i.e. a description of social events or behaviors, including context, detail, emotion, webs of social relationships, as well as the voices and feelings, of interacting individuals. As part of this research program, we also carried out an analysis of informal learning processes (Dasen, 2004) involved in the making of *cuias*. In particular, we attempted to use some conceptual tools drawn from informal educational studies with these craftswomen, focusing on informal learning processes and knowledge transmission, as related to the subjects' cultural practices.

We can define the Aritapera craftswomen group as a "community of practice" (Wenger, 1998), since they share the same knowledge, practices, and "histories of learning" (1998: 86). Indeed, they congregate to further common objectives, and their collective activity gives them a sense of belonging to a specific group. In the context of the craftswomen's collective, the process of knowledge transmission related to these *cuias* processes is enacted by "legitimated peripheral participation" (LPP) (Lave & Wenger, 1993). The apprentices joining the craftswomen group begin by gradually performing carving and ornamental tasks, guided by the craftswomen through a well-defined progressive teaching/transmission. The apprentices actually learn the kinds of patterns considered to be the easiest ones to handle.

There is a variation in the different devices and techniques applied to the carvings. Every craftswoman appears to master a wide range of creative possibilities (D'Ambrosio, 1997), regarding both the size and decoration of the *cuias*, depending on their various uses. We can find chiefly two kinds of ornamental patterns: the floral and the *tapajonic* (*cf.* Figure 2). The floral ornaments, based on indigenous techniques and European rococo, are very traditional, and were acquired through informal processes taught by grandmothers and mothers. Symmetrical patterns inspired by indigenous culture from the region of the Tapajós river, known as *tapajonic*, were first introduced in 2002 by a group of anthropologists, belonging to the "National Folklore and Popular Culture Center" (CNFCP/IPHAN). Slowly the *tapajonic* patterns became part of the craftswomen's repertoire of patterns. The different patterns drawn on the *cuias* demonstrate a rich modeling diversification, both individually and in craftswomen groups, thanks to the ongoing revival of their cultural heritage.

The material and immaterial order mechanisms applied in the Aritapera craftswomen's activities demonstrate instrumental actions both while gathering the raw material and in the final production phase. Their shaping and the manufacturing stages show conceptual and practical elements of knowledge culled from the visual perception of the person performing the action. The process involved in crafting the *cuias* seems to be deeply rooted in the physical/natural environment (for instance, stylized flowers, fruits and animals are part of the ornamental repertoire).

The procedures applied by the craftswomen are sometimes quite similar to those performed in academic calculations, when dealing with symmetry and proportionality. The first step carried out on the *cuias* with *tapajonic* patterns, for instance, is the carving of two parallel circumferences on the widest part of the bases which have a half scooped spherical shape. The determination of the midpoint in the drawing sequence is done by setting references using fingers and visual estimation.

Craftswomen usually perform calculations by estimation. Other cognitive activities that can be analyzed as mathematical are related to the making of bigger or smaller pieces, according to their storage capacity.

After the creation of ASARISAN, the craftswomen started using measuring instruments in order to classify the *cuias* according to four increasing sizes, when they are sold in sets used solely for ornamental purposes.

In the Aritapera community, the formation of ASARISAN, in 2003, introduced a new work concept amongst the craftswomen: a solely individual pursuit became a group endeavor carried out by the core of the community. *Cuia*s production increased, as well as the breadth of their repertoire (including graphic patterns in *tapajonic* style), as their ornamentation evolved and new use values for the product diversified.

The Craftswomen's skills relating to the making of *cuias* are dynamic. The traditional art of producing *cuias* using floral patterns was enriched by the adoption of new-found skills acquired by more formal means. In turn, these new proficiencies in *tapajonic* carvings, were assimilated by these women and generated new skills, as well as new indigenous patterns. Studying the range of artistic, technical, strategic and material possibilities, lead us to consider the use of *cuias* for future pedagogical usages, thus creating new pedagogical material particularly suited for teachers' training courses. Including the craft of producing *cuias* in the mathematics classroom would benefit the riverbank communities, around Santarém. This latter recommendation aims at combining it with many other mathematical instrumentation mechanisms, in such a way that knowledge gathered from the craftswomen's work could be used for pedagogical purposes. This should be done in an attempt to improve skills and school competences of students in mathematical practices related to measurement, counting and basic geometry. Working with this type of cultural artifact in a school curriculum might lead to an interdisciplinary integration among areas of knowledge. Over the medium to long term, the research's educational developments will be undertaken through collaborations between local mathematics educators and craftswomen, who both are keenly aware of the positive factor these pedagogical actions have, in strengthening and preserving their socio-cultural practices.

DISCUSSION

Our proposal consists in combining epistemological, ethnological and educational approaches in ethnomathematics, in order to enrich mathematics curricula with cultural context and local mathematical practices. This should be done by carrying out in depth studies of the cultural and cognitive aspects of local practices with a mathematical dimension –*cf.* string figure-making, sand drawing, handcrafted objects, etc., through a twofold perspective. Firstly, (participant-observer) ethnographic research should permit to determine how these (local) practices are embedded within specific cultural contexts, and how they are conceptualized and transmitted (to younger generations) by the local actors in particular. Secondly, the analysis of the latter (mathematical) practices, using modeling and epistemological tools, as well as the verbal interactions with the actors, should be carried out in order to gain a better understanding of the relationship between these practices and mathematics.

This dual approach seeks to promote the development of novel conceptual tools so as to apprehend, in a given society, the mathematical dimension of local practices in their relationships with culturally specific practices and logics, and in their affiliation with other particular expressions of symbolic and religious systems. Furthermore, carrying out –within the same society– a formal comparison of cultural and cognitive aspects of various practices with a mathematical dimension (*cf*. string figure-making and sand drawing), should contribute to the definition of a new epistemological framework for the study of such (ethno-)mathematical activities.

Based on the meta-analysis of empirical cases, we can learn from each other's findings pertaining to the value of local practices. Looking back at our points of reference we can detect many parallels on a methodological level. Ethnomathematical research is primarily interdisciplinary and is a qualitative research. Based on the principals of critical and long term ethnographical methods, we as academic researchers, try to integrate ourselves as much as possible in the local community in order to establish an interaction and make them part of the research's planning. Although we always will remain the Other, and perhaps for the local people an exotic Other, we realize that due to the interactive setup of the research's design, local people have the opportunity of formulating their own specifications. It is no longer the academic researcher who has to seek out incentives for the locals to get involved in research –which remains quite a paternalistic view. Instead, local people engaged in an international research design have the option of formulating their own goals. This became clear in both our examples. People from Vanuatu would like to devise a new curriculum, based on the findings of anthropological ethnomathematical work, in order to give young children from Vanuatu the opportunity of pursuing an academic education (i.e. University). The other example (from the riverbank communities, close to Santarém, Pará) is even more interesting in its demonstration of how 'local' people seize the opportunity to globalize their local praxis. They have begun setting up a truly business oriented association in order to produce increasingly diverse *cuias*. Moreover, they utilized the researcher, the anthropologist, to acquire more indigenous patterns (from other places) for the manufacturing of their *cuias* because of their awareness of the commercial value of the 'more geometrical' *tapajonic* figures. The production of the *cuias* once a single woman's activity has developed into a production unit organized by ASARISAN.

An interesting aspect of rethinking ethnomathematics when applying these deconstructed points of reference is that it places traditional thinking and accretioned interrelations on their head. This is a good vantage point to grasp the 'Other's' perspective. From this overturned position, we can consider *how local are local people* and if we, the academic researchers, aren't the *'exotic Other'*.

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BILDUNG, MATHEMATICAL LITERACY AND CIVIC EDUCATION: THE (STRANGE?) CASE OF CONTEMPORARY AUSTRIA AND GERMANY

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Civic education in terms of fostering societal participation and active citizenship is a goal that is prominently addressed in both conceptions of "mathematical literacy" discussed in the international context and (contemporary) conceptions of "Bildung/ Allgemeinbildung" specific to the German speaking countries. In this paper I will argue that in transitioning from such conceptions to contemporary reform efforts like educational standards, national regimes of testing and standardized exams the notion of mathematical literacy/Allgemeinbildung changes as well in a way that may be self-defeating as far as the goal of (critical) citizenship is concerned.

INTRODUCTION

To my understanding, the discussion about mathematical literacy in Germany and Austria has two distinct features: Firstly, it is kick-started by the results of Austrian and German students within large scale cross-national studies (TIMSS, PISA) widely construed to be unsatisfactory and it is focused almost exclusively on the specific understanding of mathematical literacy according to the PISA framework. Secondly, in implementing mathematical literacy into official curricula this conception is on the one hand mixed and (some have argued mis-)matched with the long standing German language tradition of "Bildung" and/or recent conceptions of "Allgemeinbildung" (see next section). On the other hand, the curricular implementation is accompanied by the establishment of regimes of assessment and the introduction of nationally centralized exams which for their part are heavily inspired by the testing regimes of TIMSS and PISA.

For example, the German National Educational Standards for Mathematics in Grades 5-10 (KMK, 2004) contain a section dedicated to "the contribution of the subject 'mathematics' to Allgemeinbildung" (ibid., p. 6) – which draws heavy inspiration from Winter (1995), even paraphrasing his widely referenced three *basic experiences* as well as five so called *leading ideas*– adopted more or less directly from the PISA-framework. Admittedly, the Austrian Nationally Standardized Written Final Examination ("Zentralmatura", cf. BIFIE, 2013) has no direct links to the frameworks of large scale cross-national studies but is also using a concept of "basic competencies" that focuses on such mathematical skills and abilities students should acquire "for their own good and for the good of society" (ibid., p. 3) and which are assessable by means of rather brief tasks aiming directly at the use of said skills and abilities preferably in real world contexts. Again, there are also direct references to a specific conception of Allgemeinbildung, in this case Fischer (2001).

By and large, both the standards in Germany and the written final exam in Austria are in fact politically legitimized with an understanding of "mathematical literacy for developing human capital" typically associated with the PISA framework (Jablonka, 2003, p.80). In this paper I will argue, that this is kind of a mismatch or even a contradiction with the more broad goals regarding civic education the aforementioned conceptions of Allgemeinbildung originally aim at. By means of a draft of an exemplary teaching unit I will then discuss why and how aspects especially crucial to civic education may be "lost in translation" when adoptions of such theoretical conceptions into national standards and frameworks for written final exams are concerned.

THEORETICAL BACKGROUND: CIVIC EDUCATION AS AN INTEGRAL PART OF "ALLGEMEINBILDUNG" IN MATHEMATICS

Before considering the aforementioned contemporary conceptions of Allgemeinbildung in mathematics, some explanatory remarks on the notoriously hard to translate German terms "Bildung" and "Allgemeinbildung" seem appropriate. The use of the term "Bildung" in the context of education and culture can be traced back to the first German translation of the 3rd Earl of Shaftesbury's (1710) "Soliloquy or Advice to an Author" from 1738. For this central writing on his philosophy of politeness "Bildung" (noun) and "bilden" (verb) were used as German translations for *formation*, *inner form* or *to form*, respectively (see Bothe-Scharf, 2010, p. 68). The classical German concept of "Bildung" as an *individual process of self-formation through confrontation with and acquisition of culture in an all-encompassing view* (arts, sciences, ethics, etc.; see Heymann, 2014, p. 248) was then developed throughout the 18th and 19th century and was closely related to the emerging self-image of the rising class of the bourgeoisie (see Horlacher, 2016, pp. 58–71). The opportunity for substantial Bildung as a civil right for *all members* of society while programmatically rooted in the 19th century discussion was still an unredeemed promise and thus a popular catchphrase of educational reform in the 1960s (see Dahrendorf, 1965).

The contemporary use of both the terms "Bildung" and "Allgemeinbildung" is a rather broad and fuzzy one. Heymann (2014, p.

248) discerns three principal meanings: Firstly, “Bildung” is used as an explicit referral to the aforementioned tradition and denotes a train of thought that is concerned with answering fundamental anthropological questions like “What constitutes the humanity of human beings?” and “How can external influences be used to let someone develop a personality which is strong enough to not be determined by external influences but follow his own insights?” Secondly, “Bildung” is also used in a more pragmatic way with respect to the aims of education in (public) schools. Here, the leading question is the fundamental pedagogical question what students should learn or how and what they should be taught in public schools. It is related to the first meaning of Bildung inasmuch as it asks what socially generalizable groundwork schools can (realistically) lay for individual processes of Bildung according to the first meaning. Many authors, including Heymann, prefer using “Allgemeinbildung” (literally translates to “general” Bildung) for this meaning of Bildung. Thirdly, in contrast to the normatively charged first and second meaning, “Bildung” is also used in a descriptive sense as a mere synonym for education, especially in compound words (for instance, the educational system is called “Bildungssystem”, the minister of education is the “Bildungsminister”).

As far as “Allgemeinbildung”/the second meaning of Bildung is concerned, considerations about Bildung usually include aspects of societal participation and active citizenship as important goals of formation public schools should aim to foster for all students. To that extent, Allgemeinbildung in mathematics is a functional equivalent to conceptions of “mathematical literacy”. This is particularly the case for the two contemporary approaches to Allgemeinbildung in mathematics discussed below.

Approach I: Heinrich Winter

Heinrich Winter’s three basic experiences paraphrased within the German educational standards are taken from an essay on “Mathematikunterricht und Allgemeinbildung” (transl. to: “Mathematics Instruction and General Education”; Winter, 1995). For the purpose of this paper, I will limit myself to the discussion of the first of said three basic experiences. Winter proclaims that for mathematics instruction contributing to the greater goal of Allgemeinbildung students need “to recognize and understand in a specific way such phenomena in the world around us, which everyone of us is or should be concerned with, be it within nature, society, or culture” (Winter 1995, p. 37). At first glance, this principle bears some semblance to the PISA definition of mathematical literacy. However, we would be wronging Winter considerably in stating that his conception of Allgemeinbildung is predominantly focused upon developing human capital. Winter is explicitly opposing a view of mathematics education that confines it to the domain of individual usefulness (see Winter, 1995, p. 38).

Mathematical modelling for example, which he presumes an important activity to foster the first basic experience, is no goal in itself for Winter, but a means of a specific kind of enlightenment mathematics may provide when applied to such phenomena in nature, society, or culture each and every one of us is or should be concerned with. To this respect, “Mathematikunterricht und Allgemeinbildung” is a direct continuation of Winter’s essay “Bürger und Mathematik” (transl. to: “Citizens and Mathematics”; Winter 1990). In this paper Winter discusses ways in which mathematics education may contribute to the traditional Kantian goal of enlightenment: the public use of reason. For Winter public mathematics education has always and still is standing in a dialectic tension between enlightenment and social conformity (see Winter, 1990, p. 135). Winter asserts that the division of labor, especially the growing tendency to base private and public decisions on expert opinions (which for their part have a growing tendency towards mathematization) may pose the greatest challenge for enlightenment and public use of reason in modern, democratic societies. “Bürger und Mathematik” contains a set of examples in which students are to be confronted with mathematical models from the domain of public affairs and welcomed to analyze critical features of these societal uses of mathematics. There can be little doubt that what Winter advocates here is closely related to a broader, critical understanding of mathematical literacy that is in line with Jablonka’s (2003) perspectives “mathematical literacy for evaluating mathematics” or even “mathematical literacy for social change”.

Approach II: Roland Fischer

Mathematical literacy for evaluating mathematics and as a prerequisite for social change is even more pronounced in Roland Fischer’s conception of Allgemeinbildung. Just as Winter, Fischer considers communication between experts and the lay public the greatest challenge general education faces in the modern democratic societies. Fischer argues that communication between experts and lay people is always asymmetrical: While expertise is precisely based upon the fact that the respective experts have a better understanding of the matter at hand than the lay people, it is mostly the lay people who have to make a decision. For example: A surgeon usually has a better understanding of the benefits and risks of a surgery than the patient. Nonetheless, it is the patient who has to decide and give written notice of his “informed consent”. Likewise, politicians (as elected representatives of the public) may consult experts, it is nonetheless their ‘job' to make and take responsibility for the actual decisions. Every democratic society is after all based upon the principle that in some way or form it is the (lay) public itself that decides upon its public matters. Nevertheless, that necessarily implies deciding upon proposals for problem resolution oneself would not be able to conduct and does not understand as well as the experts. To Fischer, educating students to become well-informed laypersons should therefore focus on prospectively enabling them to make decisions about the importance of (mathematical) activities and problem resolutions even and especially in such cases in which they are not able to judge (in detail) about their technical correctness or to undertake the respective activities by themselves. For establishing a line between the professional study of a subject and its study for the purpose of Allgemeinbildung, Fischer (2001) distinguishes between three domains of knowledge in a subject:

> "Firstly, *basic knowledge* (notions, concepts, means of representation) *and skills*. Secondly, more or less creative ways of *operating* with knowledge and skills within applications (problem solving) or for the generation of new knowledge (research). Thirdly, *reflection* (What is the meaning/wherein lies the significance of these concepts and methods? What can be achieved with them, what are their limitations?)." (Fischer, 2001, p. 154)

Fischer then concludes that experts have to be well versed in all three domains, while the education of laypersons should focus on the first and third domain. One criticism towards Fischer's conception has been to dispute whether one can reflect meaningfully upon mathematics without actually "doing" said mathematics. It is nonetheless *the* typical mode of confrontation with professional knowledge in a society that is based upon the division of labor. Furthermore, Fischer sees mathematical modelling and problem solving as important activities *in the mathematics classroom*. But, Fischer contests that being able to do (elaborate) mathematics can be *a goal in itself* for the purpose of general education. So, mathematical modelling or operating is seen by Fischer as a *means* to the end of acquiring basic skills as well as developing reflective knowledge, which is of particular interest for future citizens as well-informed lay public.

ILLUSTRATIVE EXAMPLE: EXPLORING AND REFLECTING MEASURES OF POVERTY IN THE MATHEMATICS CLASSROOM

If we accept the goal of fostering critical citizenship by supporting students' present and future public use of reason with respect to argumentations relying on mathematizations as crucial to Allgemeinbildung or mathematical literacy, it seems fitting to actually engage students in examples from the realm of public and political matters. An example I have worked on myself is the at-risk-of-poverty rate and its underlying socio-economic and mathematical models (cf. Vohns, 2013).

To the extent permitted by the brevity of this paper, I will try to outline a teaching unit focusing on this topic in the mathematics classroom. The unit is aimed at students at the 9th/10th grade (age 15 to 16 years) and is designed in a way that prototypically adheres to the notion of "mathematical literacy for democratic citizenship" according to the two approaches discussed above. While the core mathematical content of this teaching unit (measures of central tendency, discrete distributions) is covered in the Austrian syllabus ("curricular validity" for the abovementioned grades can be assumed in principle), I will also use this example to address the question of crucial components of this unit which *are not* (and are not likely to ever be) covered by educational standards and central examinations in mathematics.

The teaching unit may start with conflicting accounts of "poverty" as a phenomenon that is on the one hand usually seen as multidimensional in social sciences, combining economic, social, cultural and psychological aspects and on the other hand occasionally discussed in the media on grounds of a single number reported (precise to the first decimal place) by the Federal Statistical Office: the at-risk-of-poverty rate. Any discussion of mathematical models of poverty should take its time to discuss the underlying socio-economical models (the so-called "real model"-stage, cf. Leiss et al. 2010). Students should become aware that applying any kind of mathematical model to a socioeconomic phenomenon is dependent upon processes of structuring and simplification. For instance, the "real model" used to describe poverty in the European Union aims at identifying "persons, families and groups of persons whose resources (material, cultural and social) are so limited as to exclude them from the minimum acceptable way of life in the Member State in which they live" (EEC, 1985). For this purpose, the Statistical Office of the European Union (EUROSTAT) distinguishes three different dimensions of the so-defined poverty: relative income poverty, severe material deprivation and low work intensity (cf. EUROSTAT, 2015). At-risk-of-poverty rates are one measure for the first of these three dimensions. Finally, the "mathematical model" for income poverty called "at-risk-of-poverty rate" is "the share of people with an equivalised disposable income (after social transfer) below the at-risk-of-poverty threshold, which is set at 60 % of the national median equivalised disposable income after social transfers" (ibid.).

Structuring and simplification processes such as the one above may on the one hand rest upon more or less sound economical and/or sociological theories justifying the inclusion and exclusion of specific aspects of the broader phenomenon of poverty. On the other hand, the succession of "real models" and "mathematical models" is not as clear-cut as didactical modelling cycles tend to imply. In reality, a "real model" of poverty is already influenced by questions of how easily data is includable in a "mathematical model" regarding objectivity, suitability for cross-national comparisons, and collectability under restrictions of budget and time. According to Yasukawa (1998) we should further ask, who is concerned with poverty models and why the concerned parties have an interest in mathematizing this phenomenon. Measuring poverty is primarily a "problem" of social statisticians usually working at government-run statistical offices. Governments then use the results in public social reporting and social scientists may use them for further research. Eventually, these measures may also be a part of public policy formation regarding e.g. the amount of social benefits and transfers. A possible starting point for investigating the mathematical properties of at-risk-of-poverty rates in the classroom is examining the heavy criticism their publication draws with uncanny regularity from conservative and market-liberal economists. Let us consider the following prototypical arguments:

A) "The crux of relative poverty models: If poverty is defined in relation to the mean (or median) [of disposable income – A.V.], one has to accept that – save for a uniform distribution [of disposable income – A.V.] – there will always be poverty regardless of how rich a society becomes" (Beck & Prinz, 2004, p. 51).
B) "Someone living in a tax haven with an average yearly income of 1 Million Euro is already 'at-risk-of-poverty' by definition if he earns 590 000 Euro or less per year" (Sinn, 2006).
C) "If ten-thousands of rich people would immigrate to Germany, the local population would become 'poorer' – at least according to the definition of poverty used by the statistical office and the ministry of social affairs" (Knauß, 2012).

Substantially scrutinizing such claims in the mathematics classroom means asking (and answering) questions such as:

1. How do at-risk-of-poverty rates relate to measures of average income?
2. Why and for which countries are at-risk-of-poverty rates at all calculated depending on measures of average income? Does it make any difference, whether these rates are calculated depending on either mean or median income?
3. How do at-risk-of-poverty rates depend on other properties of the income distribution function? Specifically: Is equal distribution of income in fact a prerequisite for statistically overcoming poverty?
4. How do peculiarities of the actual data collection processes for income measures affect both (the robustness of) measures for average income and consequently at-risk-of-poverty rates?
5. What alternative and complementary models and measures of poverty are used in public social reporting and the social sciences? What are the assets and drawbacks of the various models and measures?

Our exemplary teaching unit would have to introduce or revive different measures of central tendency (arithmetic mean, median). Afterwards, it should concern students with the question of how the skewness of a distribution and outliers (e.g. persons with extremely high income) affect these measures. For this purpose, one may begin with simplified income distributions (cf. Eggen, 2006). A spreadsheet may be useful for investigating different scenarios in relation to the aforementioned three counter-arguments against relative poverty rates. Turning back to the real world measurement of poverty, the teaching unit should also consider that the income data used for the calculation of poverty rates stems from sample surveys which have their specific drawbacks, in general as well as for the matter at hand (cf. Vohns, 2013).

Coming back to the two approaches to Allgemeinbildung discussed above, it should already have become apparent that our exemplary teaching unit has the potential to sustain a better understanding of “how mathematical modelling works” and of “what kind of enlightenment it provides” (Winter, 1995, p. 38) regarding the socioeconomic phenomenon of poverty. In scrutinizing counter-arguments found in the media, it is also directly linked to the realm of public matters and a prime example of the “mathematical literacy for evaluating mathematics” Winter advocates. Turning to Fischer’s approach, we can identify his three domains of knowledge: Firstly, knowledge about central measures and their robustness against outliers and skewed distributions as well as effects of random sampling are in fact elements of (reflective) basic knowledge that is covered by the Austrian Educational Standards and/or the catalog of competencies for the final central examination. Secondly, the validity of counter-arguments has to be investigated by mathematical operating (Fischer’s second domain of knowledge). Here, many calculations can be handed over to technology (e.g. spreadsheets). But, thirdly and lastly, the linchpin of the previously outlined teaching unit for contributing to civic education is the question whether it supports the development of reflective knowledge both regarding poverty models in particular and the use of mathematical models in public reasoning in general. It should be quite recognizable that the teaching unit aims to provide opportunities for mathematical-, model-, and context-oriented reflections (see Skovsmose, 1998). Whether students actually engage themselves in context-oriented reflections, I consider the litmus test for a possible civic educational effect of this teaching unit.

Abstracting from the case of poverty models, students could and should become aware that criticism towards a specific mathematical model used in public reasoning can either be mathematically invalid (e.g.: counter-argument A) or mathematically valid. Yet, even that does not automatically imply its relevance (e.g.: counter-argument B, C). If someone wants to deny the relevance of a specific measure, it is a frequently used argumentation tactic to focus on mathematically correct drawbacks of said measures. However, such argumentations may use exaggerated scenarios of little practical relevance. Likewise, people who want to use such measures for their argumentations are likely to gloss over actual drawbacks and try to obfuscate the finer details of their mathematical construction. In the first case, the "mathematical model" acts as a proxy or scapegoat for a kind of criticism that is in fact questioning the relevance of the "real model" or the existence of the social phenomenon in itself. In the second case, the conviction that the social phenomenon is relevant acts as a shield against any scrutiny the "real model" or the "mathematical model" may very well deserve. Every one of us is prone to such a biased evaluation of mathematical models in a context we feel deeply involved in. Allgemeinbildung would therefore ultimately manifest itself in an awareness for this fact and appropriate countermeasures.

But educational standards and central examinations are much more likely to focus on basic knowledge and –if anything– mathematical- and model-oriented reflections. They almost have to omit context-oriented reflections, as such reflections either need a very careful consideration of the specific context mathematics is applied to (which is not realistically possible under the constraints of a written exam) or are likely to be on a similar meta-level as the above statement about our very selective skepticism towards different models. However, such a meta-level is usually too far removed from peoples' preconceptions of "doing mathematics" and/or mathematical competence as to be included in a mathematics exam. In consequence, such restrictions run the risk of both basic *and* reflective knowledge becoming rote and inconsequential "textbook" knowledge and in turn raise doubts about the seriousness of conceptions of Allgemeinbildung or mathematical literacy as such.

CLOSING REMARKS

I want to close this paper by summarizing and emphasizing the key issues in fostering civic education through mathematics instruction which I presume the most pressing at least for the contemporary discussion in Austria and Germany:

1. The recent tendency of focusing mathematics education on the achievement of basic skills and knowledge for every student is an ambivalent endeavour: Basic skills and knowledge are considered a prerequisite for becoming a well-informed layperson able to act as an active, reflective citizen. However, there is no precise line between basic skills and more elaborate skills akin to Fischer's domain of operating. Mathematics instruction may therefore be both in danger to draw too much and too little attention to the achievement of mathematical knowledge and skills as a basis for meaningful reflections and evaluation of mathematics.

2. The focus on assessment and central examinations furthers this problem in respect to reflective knowledge, as components especially relevant from a civic education standpoint are the least likely to be realistically implemented under the constraints of central examinations and standardized (cross-)national testing.
3. The two aforementioned issues have and still do contribute to already prevalent qualms about the seriousness of mathematical literacy and civic education efforts and have enforced serious doubts about both the style and amount of application-oriented teaching currently recommended in the mainstream of mathematics education in the German speaking countries. Both educational scientists of humanistic background and mathematicians have raised concerns regarding a supposed negligence of pure mathematics –each with their own agenda.
4. Setting aside the first three issues, the development of "good practice"-examples for fostering model- and context-oriented reflections directly relevant to civic education is still in its infancy. This is especially true in regards to empirical research investigating the actual engagement of students in questions aiming at context-oriented reflections and transfer of reflective knowledge to similar problem contexts.
5. As Jablonka & Gellert (2010) have pointed out, there would still be a long way from a reasonable stock of good examples to a consistent curriculum construction. Even mathematics educators who are convinced in mathematical literacy for evaluating mathematics and as part of civic education rarely dare to challenge established mathematical curricula fundamentally. This may contribute to a growingly problematic mismatch of mathematical content and educational goals in turn feeding back to the above issues.

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EXAMINING RELATIONS BETWEEN STUDENTS' PERCEPTION OF "BEING KNOWN" AND THEIR MATHEMATICS AND RACIAL IDENTITIES

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The authors investigated the relation between students' perception of "being known" and both their racial identity and their developing sense of identity as one who is interested and competent in mathematics. During the 2014-2015 and 2015-2016 academic years, middle and high school students (N = 373, 59% self-identified as a student of color) completed surveys about their experiences interacting with their current mathematics teacher. The analyses revealed that students' perceptions of how well their teacher knew them was related to self-reported value of mathematics and the perception they could be successful in advanced mathematics –and that the former relation was stronger for students of color than for white students.

This study examines a central component of adolescents' sense of belonging in classrooms –the perception of being known. The construct of being known is focused on transformative instructional interactions between a teacher and student. The perception is derived from interactions with an individual teacher who, through their teaching practice, creates a context in which the multiple, and sometimes contradictory, identities of youth are welcomed, valued, and explored. Informed by theories of inequality as well as theoretical perspectives across disciplinary boundaries, the concept of being known connects the history of racial categorization and segregation in the U.S. to psychological experiences of trust and belonging.

DIFFERENTIAL RACIALIZED EXPERIENCES INFORMING THE BEING KNOWN CONSTRUCT

Within social organizations, including schools or classrooms, trust exists when group members act according to and are secure in the expected futures that are constituted by interactions with others and/or by the symbolic representations of those expected futures (Lewis & Weigert, 1985). Enduring ideas about racial groups shape perceptions of who is valued and who is not, who is capable and who is not, and who is "safe" and who is "dangerous" (Carter, Skiba, Arredondo, & Pollock, 2014). Specifically, the construct of race and the continued prevalence of racism influences how students determine who is a member of their community, who is an ally, and who might be acting in their best interest.

For adolescents ascribed a nondominant racial status, working under conditions of potential threat to their social identities –wherein personal narratives of competence and belonging are at risk– compromises students' efforts to perform well on both cognitive and social tasks (Schmader, Johns, & Forbes, 2008) and makes student resistance an understandable reaction against the struggle to feel respected and valued (Toshalis, 2015).

In the context of these differential racialized experiences in schools, common purpose (e.g., teachers' expertise in facilitating learning) and interests (e.g., evidence of teachers' willingness to serve as an ally) may affect perceptions of being known. Perceptions of being known create a different (psychological) set of conditions under which students can better learn under conditions of threat. Explaining why adolescents, particularly students of color, extend or withdraw trust in classrooms or perceive they do or do not belong –and understanding the consequences of those choices and perceptions– is central to advancing equitable teaching practice in mathematics education. In this analysis, we provide an initial examination of the relation between students' perception of being known and aspects of their mathematics identities and dispositions.

STUDENTS' IDENTITY AND DISPOSITION IN MATHEMATICS EDUCATION

For at least the last 15 years, there has been increasing interest in student identity in mathematics education research. This has especially been the case in equity-focused work, which has attempted to understand the intersecting relations among individuals' multiple identities (e.g., Varelas, Martin, & Kane, 2013) and forms of instructional practice that attend productively to the co-development of these identities (e.g., Aguirre, Mayfield-Ingram, & Martin, 2013). Of course, in mathematics education, students' *mathematics* identities are of particular concern. Aguirre et al. (2013) defined mathematics identities as

> the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives (p. 14).

These "dispositions" are often considered a kind of knowing. In the U.S., authors of the prominent 2001 National Research Council report, "Adding It Up," included *productive disposition* as one of 5 "strands of mathematical proficiency." The authors suggested that students likely to be successful in mathematics

> have a set of attitudes and beliefs that support their learning. They see mathematics as a meaningful, interesting, and worthwhile activity; believe that they are capable of learning it; and are motivated to put in the effort required to learn (Kilpatrick, Swafford, & Findell, 2001, p. 171).

One consequence of not developing such a disposition, they argued, is that students might shy away from taking more advanced mathematics courses, particularly in the high school years, which, in turn, might limit future opportunities in mathematics-related fields. Thus, a crucial goal for schools and teachers is ensuring that students –and particularly students of color and members of other typically marginalized groups– experience school mathematics in ways that support them in seeing the value of learning and pursuing mathematics, and in feeling confident that they can continue to be successful, particularly in more advanced courses.

As we know, not all students are supported in developing positive and productive identities and dispositions in mathematics. For example, Solomon (2007) has studied the relations between these aspects of students' identities and their sense of belonging –in terms of both local communities (e.g., classroom) and connections to broader communities of disciplinary practice. Her analysis revealed gender differences in undergraduate mathematics students' experiences, including that women were more likely to describe experiences of exclusion.

Taking a more large-scale approach, English-Clarke, Slaughter-Defoe, and Martin (2012) surveyed African American secondary students to examine relations between aspects of their racial and mathematical identities, finding that students who perceived greater usefulness of mathematics were more likely to provide positive accounts of the success of African American students in mathematics generally.

With the present analysis, we intend to build on previous findings such as these by examining relations between aspects of students' mathematics identities and dispositions (the extent to which they value and feel they can be successful in mathematics) and an indicator of belonging (their perception of being known by their teacher), and consider whether such relations differ between white students and students of color.

METHOD

Study Context

The data reported on here come from a larger study conducted in a large urban district in the Northeastern region of the United States focused on racial equity in secondary mathematics and closing the opportunity gap for marginalized mathematics students in grades 6-12. Students of color comprised approximately two-thirds of the school district's population. Of those, the majority were African-American, with Latino/a students and Asian/Pacific Islander students each comprising less than 5% of the total population. In that larger study, two cohorts of secondary mathematics teachers participated in consecutive 2-year professional development (PD) efforts.

Participants

During the first cohort's PD years (2014-15 and 2015-16), survey responses were collected from middle and high school students in the classrooms of participating teachers (in both cohorts). Each year, teachers identified a focal class for the project, each student of which was invited to participate in the survey. Students who completed the survey included those who returned signed permissions forms (mean 8.56 per class). The final sample included 383 students of 36 teachers (all but 4 of whom were white). In total the 383 students responded with 57 distinct combinations of responses regarding their racial or ethnic identities, with 65% of the students identifying as students of color (73% of whom as Black and/or African-American).

Measures

Measures of adolescents' perceptions and experiences were obtained from pencil and paper surveys administered during the 2014-15 or 2015-16 academic years. Adolescents' perceptions of being known were measured using the Adolescent Perceptions of Being Known scale (APBK; Chhuon & Wallace, 2014; Wallace, Ye, McHugh, & Chhuon, 2012). The scale is based on long-term development work, beginning with focus groups of and in-depth interviews with urban youth about experiences of belonging and school engagement and continuing to pilot testing and standard statistical analysis for survey development. (For an extended description of the scale's development and theoretical underpinnings, see Wallace et al., 2012.) The response structure for this scale was a 5-point Likert-type scale with the response options "Strongly Disagree," "Disagree," "Not Sure," "Agree," and "Strongly Agree." APBK scores were averaged across the ten items, M = 3.81, SD =.685. Cronbach's alpha (an estimate of internal consistency) was .87 for the present study.

With the same survey, we assessed two constructs related to students' mathematics identity: the extent to which students value mathematics (or view their mathematics class as being worthwhile) and the extent to which students think they could be successful in advanced mathematics courses. The first, value, was measured using a motivation subscale created by Kosovich, Hulleman, Barron, & Getty (2015) that consists of 3 items (e.g., "I think my math class is useful"). The response structure for this scale was a 6-point Likert-type scale with the options "Strongly Disagree," "Disagree," "Slightly Disagree," "Slightly Agree," "Agree," and "Strongly Agree." Scores were averaged across the 3 items (M = 5.26, SD =.76). Cronbach's alpha was .80 for the present study.

Students' perceptions of whether they could be successful in advanced mathematics were measured using one item adapted from

English-Clarke et al. (2012), with response options the same as those for the value scale. Scores represent the endorsement of a single item ("I think I could do well in advanced math," *M* = 4.64, *SD* =1.40).

Data Analysis

We constructed two regression models to examine relations between variables and, by investigating statistical interactions, whether those relations were different, on average, for students of color compared to their white counterparts. We fit hierarchical models in order to account for the nested nature of our data source (i.e., students clustered in classrooms with the same teacher). Results of initial correlational analysis suggested that average value and advanced course taking responses might have differed between years. We therefore included a dichotomous year variable in each of our primary models.

With Model 1, we examined the relation between APBK and students' reported value of mathematics, controlling for year and including an interaction between APBK and whether respondents self-identified as a student of color (SOC). In Model 2, we examined the relation between APBK and students' perception of whether they could be successful in advanced mathematics, again including an APBK-SOC interaction, and a control for year of data collection. For this model, however, we employed logistic regression because of the nature of the dependent variable, which we created by dichotomizing students' perceptions of whether they could be successful in advanced mathematics. We did so to avoid treating a single Likert-type item as a continuous variable, and because of the non-normal distribution of responses (*M* = 4.64; *Mdn* = 5; *Skewness* = -1.00) and the item's six choice response structure, which forced some level of agreement or disagreement with the single statement assessing this perception.

Table 1. Results of hierarchal multiple regression analyses predicting valuing mathematics and success in advanced course-taking. Note: Linear regression coefficients listed for Model 1; logistic regressions coefficients listed for Model 2 (*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$; $^{+}$ $p < 0.10$).

Predictor		Valuing mathematics		Success in advanced course-taking
APBK		0.33*** (0.08)		0.40 (0.40)
SOC		–1.01** (0.39)		–3.17^{+} (1.82)
APBK x SOC		0.23* (0.10)		0.81^{+} (0.49)
Year		–0.10 (0.08)		–0.72^{+} (0.37)
Constant		4.21*** (0.35)		1.63 (1.67)

RESULTS

Table 1 lists the model results. The results of the first model suggest that, on average, with other variables held constant, a one-point increase in students' APBK responses is associated with a 0.33 increase in their reported valuing of mathematics. The statistically significant interaction, however, suggests that for students of color, the increase in value is 0.55.

The results of Model 2 similarly suggest that the relation between APBK and students' perceptions that they could be successful in advanced mathematics courses is stronger for students of color. Table 1 also lists the coefficients produced from Model 2, which, in this case of logistic regression, are log odds, which can be converted to odds by exponentiating them. The results suggest that, on average, with other variables held constant, for white students, a one-unit increase in APBK responses is associated with a 49% change in the odds (or a 0.40 change in log odds) of perceiving they could be successful in advanced mathematics, but the relation is not statistically significant. For students of color, a one-unit increase in APBK responses is associated with a 335% change in the odds (or a 1.20 change in log odds) of perceiving they could be successful in advanced mathematics. The relation is trending toward significance.

To help with interpretation, we converted the estimated odds of Model 2 to probabilities at particular values of the independent variable. Figure 1 presents the estimated probabilities that students would positively endorse likely success in advanced mathematics for each APBK response level, for white students and students of color separately. These results suggest that, on average, even at the lowest response level of APBK, there is an 88% probability that white students perceive they could be successful in advanced mathematics, with slight increases at each subsequent APBK response level. For students of color, however, at an APBK value of 1, the average probability of perceived likely success in advanced mathematics is only 42%. The difference quickly diminishes, though, as APBK values increase, with equal probability (96%) for both groups of students at an APBK response level of 4. We note that these results should be treated cautiously, given the marginal statistical significance of the interaction variable.

DISCUSSION

In this study, we found adolescents' perceptions of being known to be related to aspects of their mathematical identities or disposition, a widely-shared goal for student learning and school mathematics experiences in the field of mathematics education, and one that was officially endorsed as a strand of mathematical proficiency by the U.S. National Research Council (Kilpatrick et al., 2001). It is commonly accepted in mathematics education that relationships with teachers influence the development of such productive dispositions in students. Yet the sentiment that "relationships matter" is often left ambiguous in terms of particular perceptions youth develop in response to interactions with their teachers. Such a broad and undifferentiated construct inhibits the generation of the evidence needed to understand how experiences in classrooms do or not lead to such dispositions.

This study begins to provide some insight into how relationships with teachers matter for mathematics learning. Our findings suggest that students' perceptions derived from experiencing a particular teacher's practice may influence their feelings about the subject itself and their likelihood of being successful in it. While the findings may not be surprising –especially since they likely align with many readers' own experience as mathematics students– they do represent an uncommon investigation. This is particularly the case with respect to investigating racial differences –an affordance of the perception of being known scale used in this study, which was directly informed by empirical studies of differential racialized experiences.

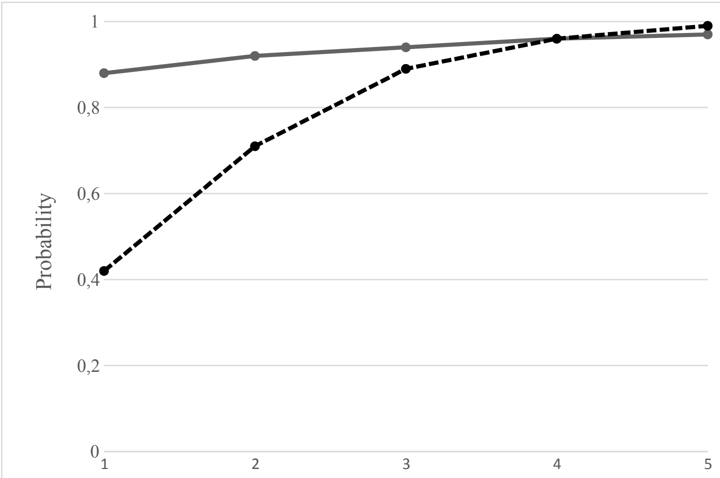

Figure 1: Estimated Probabilities that Students Perceive Likely Success in Advanced Math for Each APBK Response Level

In recent years, mathematics education research has expanded to more inclusive conceptions of instructional practice. We know more about how teachers' scaffolding provides adolescents with tools to successfully complete tasks and prepare for future learning (e.g., Horn, 2012; Staples, 2007). We know more about promoting and facilitating multiple ways of explaining things and drawing on previous shared experiences of joint problem-solving in order to shift the focus from diagnosing and remediating –or trying to "fix" individual students– to an emphasis on supporting adolescent learning so that students can stretch their current levels of competence (e.g., Aguirre et al., 2013; Hand, Penuel, & Gutiérrez, 2013). What is less clear is whether and how, through mathematics-specific aspects of practice, teachers build the kind of relationships through which students feel they are "known." Considering what lies "inside" or "outside" the typical mathematics education terrain could help to inform both future research and teacher education.

LIMITATIONS

Before concluding, we wish to acknowledge a number of limitations of our study. First, our data source represents cross-sectional survey results collected from two cohorts of students. Because of our study design, we are not able to make claims about the direction of the relation between variables. In other words, it may be possible that teachers get to know the students who already value mathematics and feel they can be successful in it. The significant interaction between for students of color renders this interpretation less plausible for the value outcome. Still, future studies could address this limitation by collecting and investigating multiple measures of both outcome and predictor variables. These multiple measures would facilitate more complex path modeling techniques. We should note that for this study, we did have a subset of students (n = 103), for whom we had prior year achievement data and found no correlation between APBK and prior achievement.

A second limitation of this study relates to sampling. In many cases, only a few students in each focal classroom completed the survey. So, while the diversity of students in our overall study sample was strong in terms of factors central to our hypotheses, student achievement levels biased toward higher achieving students. For example, 75% of the 103 students in the subsample for which we have previous year's achievement scores were categorized as "proficient" or "advanced" based on mandatory statewide assessments, which is higher than average for the district from which students were sampled. Teachers were asked to pick a focal class that was not an advanced class, but even with this exclusion criterion, it is possible that students with higher state test scores were more likely to complete the survey. It is uncertain how this bias might influence our findings.

Third, although analyses of survey responses of large numbers of students can help to uncover trends that might have otherwise gone unnoticed, an inherent limitation of such approaches is that we are unable to provide insight into how the students in our sample experienced the very relationships that we asked them to report on, which more ethnographic studies could provide. Additionally, one of our outcomes of interest was represented by a single survey item. Although were careful not to treat it as a continuous variable, our analyses could have been strengthened by using a reliable, multi-item survey scale.

CONCLUSION

These results suggest that teacher education –which has recently been focused on training pre-service teachers to enact "high leverage" practices– might attend more practically to recognizing teaching practices that enhance students' psychological experiences. Questions arise from our findings about the ways that teachers might need to develop relationships with their students rooted in knowing the students –not just in terms of their mathematical understandings and performance, but as developing people with the fundamental wisdom to invest in relationships that are generative and self-enhancing and withdraw from ones that offer less desirable experiences than readily available alternatives.

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STATISTICS EDUCATION: AN ALTERNATIVE FOR CHILDREN IN LITERACY CYCLE TO DEVELOP THE NUMBER SENSE

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We present theoretical framework, methodology and the investigation cycle (teeth brushing and water consumption) of our doctorate research developed in the Mathematics Education Post-Graduate Program at UNESP - Rio Claro - Brazil. We elected to guide the search the following question: Is it possible, through Statistics teaching context of tasks, children's literacy cycle develops number sense? The Statistics and Mathematics investigation, the discovery, reflection and validation stand out because they are basic elements in the process of knowledge construction. In our analysis, we infer that the tasks, the methodology chosen contribute to the development the number sense.

INTRODUCTION

In this document, we present the theoretical reference, development, investigation and reflection about the application of the investigation cycle "Teeth Brushing and Water Consumption", developed in our doctorate research.

This investigation cycle addresses the importance of oral hygiene and conscious consumption of water, inserts the children into the Statistics Investigation universe, performing data collection, organizing them into tables and displaying them in charts and presenting them to the interviwed children. Important skills that allows them to understand the world that surrounds them, contributing for the formation of autonomous, critical people and intervening in today's society.

By presenting this Cycle of investigation, we exemplify the conception of Statistical Education which we assume is directed to social praxis, to the study of real problems and situations in a perspective of contextualized, reflexive and critical investigation.

To guide the research, we aim to identify and understand the contributions that the Statistics teaching brings to the promotion of the development of number sense to children studying in the 1st year of literacy cycle. Thus, the focus of research is the development of number sense, the means by which this construction will be seen is the Statistical Education and the research cycle instrument.

THEORY FUNDAMENTALS

Number sense:

The number sense is an expression that appears in the literature of mathematical education about 20/25 years ago, usually associated to the knowledge observed in educational contexts or linked to the active life of any citizen (CASTRO E RODRIGUES, 2008, p. 117)

The National Council of Teachers of Mathematics (NCTM, 1989/1991). brings for the first time the expression of Number Sense that indicates that it is an essential goal of the first years of schooling. When it comes to topics that should be given more attention, the document states that the mathematics curriculum should include concepts and skills related to integers so that children develop number sense (NCTM, 1991, p. 48). The document stresses that development in the number of direction is on an intuition of numbers formed from the various meanings of the numbers (NCTM, 1991, p. 50).

In 2000 and 2007 the National Council of Teachers of Mathematics - NCTM reaffirms the importance of number sense, considering it essential to its development from kindergarten until the end of high school. The document indicates that the development of number sense should be the core of mathematics education and that students should acquire a broad knowledge of numbers (NCTM, 2007, p. 34). In our research, we rely on the theoretical framework of McIntosh et al. (1992). The number sense from the perspective of the authors include:

> The general understanding of the individual on the numbers and operations, along with the ability and inclination to use this understanding in a flexible manner to make mathematical judgments and to develop useful strategies to cope with the numbers and operations. It reflects an ability and tendency to use the numbers and quantitative methods as means of communication, information processing and handling. (MCINTOSH et al 1992, p. 3).

To articulate a framework to explain, organize, and inter-relate some of the basic components of general number of direction, identify key components and organize them according to common themes, Mcintosh et al. (1992) organize a framework, whose main indicators are: knowledge and facility with numbers, knowledge and ease with operations and application of knowledge and facility with numbers and operations in calculation situations (MCINTOSH et al. 1992, p. 5).

The knowledge and facility with numbers: - encompasses the sense of regularity of numbers, the multiple representations of numbers, the sense of relative and absolute magnitude of the numbers and finally the use of reference systems for assessing a response or around a number to facilitate the calculation. Knowledge and facility of the operations: - includes the understanding of the effect of operations, properties and relations between them. The application of knowledge and facility with numbers and operations in calculation situations: - includes the understanding to relate contexts and calculations, awareness of the existence of multiple strategies, the power to use effective representations and sensitivity to review data and results.

Number sense and social interactions: Understanding that numbers can have different meanings and can be used in diverse contexts, the development of number sense happens throughout life and not only at school, it enables to understand the numbers and their relationships, and to develop useful and effective strategies for each use in your day-to-day, in your professional life or as an active and critic citizen where they live.

The children in the literacy cycle are developing number sense as they are learning and understanding the different meanings and uses of numbers and how they are interconnected. The number sense is therefore broader than the knowledge of the number (CASTRO E RODRIGUES, 2008, p.11).

Statistics Education: Mathematical results and statistical data are a constant reference for debates in society. Thus mathematics and statistics become part of the language with which political, technological and administrative suggestions are presented (BORBA and SKOVSMOSE 2001 p. 127). The conscious exercise of citizenship goes well, the reflective and critical understanding of the data.

The National Council of Teachers of Mathematics (1991) recommends that, since the early years of Basic Education, Statistics and Probability content is worked out. In Brazil, only since 1997, with the publication of the National Curriculum Parameters (PCN), the concern with the teaching of Statistics, was presented in the early years, thus becoming a major breakthrough for Basic Education.

The *Guidelines for Assessment and Instruction in Statistics Education*-GAISE (2007, p.6) Informs that the goal of Statistical Education is to help students develop statistical thinking. Regarding statistical thinking, the document emphasizes that, for the most part, it deals with the presence of variability; Solving statistical problems and understanding, explaining and quantifying data variability.

On the literacy cycle, Statistics Education should already be taught. For this to occur, "the child literacy should have contact with the reading and interpreting table and charts and to the collection, organization and representation of this data" (BRAZIL, 2012, p. 83). For example, if a child asks: "What is my colleagues' favorite ice cream?" The teacher can direct this question to an investigation and also cause new questions to research from the collected data. The Statistics Education plays the role of investigating this question.

Statistics education and social interactions: When students have the opportunity to confront the tasks and situations of the classroom where they are encouraged to investigate statistically, subjects of their interest, allowing a look at issues, political, environmental, social and economic, a great wealth of resolution strategies. Lopes (2004, p. 85) highlights that the research process favors the development of important skills on the students who participate in it, such as: Statistical Literacy, Statistical Reasoning and Statistical Thinking. These three important competencies together will encompass the global understanding of Statistics. To them we add the ability to deal with variability, which enables people to make a decision centered on the understanding of events, because they can quantify the variability of the data, and this makes them able to explain and argue their decisions. This is a fundamental skill for a critical citizen. (GAISE (2007, p.6)

Mathematics Investigation: In teaching and learning contexts, as Ponte et al. (2006, p. 9), means, so alone, we formulate questions that interest us, for which we do not have already an answer. It means working with issues that arise in early obscurity but we seek to study in an organized manner.

For Ponte et al (2006, p. 105) students must work with real problems, participating in all phases of the process that got its start by the choice of methods and data collection involving the organization, representation, and interpretation of data and culminates with the talking of final conclusions. The author characterizes this process of "Investigation Cycle".

In our research, we adopted this term to represent the entire investigation process carried out for each set of tasks and lessons necessary for the development of each theme. The "investigation cycle" is the means/locus in which this construction was carried out and/or developed.

Mathematical research and social interactions: The student-to-student interaction tends to be much stronger in a research class. Faced with challenging tasks, students spontaneously interact with one another, which strongly encourages learning (PONTE et al, 1998, p. 120). Another important social interaction that is highlighted by the authors is the group work, which has important potentialities to facilitate the interaction between the students and to promote the development of the capacity for cooperation. With greater participation of students, the class becomes more productive. The authors emphasize that the classroom becomes a place where not only one learns pure mathematics but also learns to discuss, to argue and to live a more democratic interpersonal relationship. The authors tell us that this mode of interaction also changes the role of the teacher in the classroom, rather than "solitary actor" appears as mediator of class activities.

METHODOLOGY: RESEARCH PROCEDURES AND ANALYSIS

Our research, qualitative and interpretative and was conducted during 2014 and 2015 Weekly meetings of 2/3 periods.

Subjects: 30 children (14 boys and 16 girls aged between 6 and 8 years old) in a class of literacy cycle, who didn't study the kindergarten, however, their literacy process started in 2014. Most of them are children of civil construction employees, who are part of the low class society.

Research location: the Escola Municipal Dr. Gladsen Guerra de Rezende, located in Uberlandia, Brazil. The school is administrated by the City Hall through Secretaria Municipal de Educação, ruled by the municipal, state and federal laws. 1.580 people study at this school on the following educational levels: Elementary School, Part-time Elementary School for adults and elderly.

The research was divided into five stages: bibliographic study; Development of Investigation Cycle and its tasks; Application of the tasks (data collection); Transcript of classes and analysis of data.

The classes were conducted to encourage students to ask questions, establish relationships (Statistical / mathematical concepts / number sense), build justifications and develop the spirit of inquiry.

During the planning of investigation cycles and implementation of their tasks in the classroom, we used different features of the infant universe that allowed children to recognize different information, in different conditions and settings and daily contexts that involved the ideas of number and Statistics.

In a math activity of investigative nature, students start from a challenging question, consider alternative strategies, discuss with their peer, test their ideas and then communicate, discuss and reflect with the whole class (JESUS e SERRAZINA, 2005, p. 7). During the achievements of the tasks, we respect all stages of this process.

As a summary, we show that for preparation of tasks, we rely on theoretical frameworks Number Sense and Statistics Education. For the development of tasks, we seek the theoretical foundations of mathematics investigation and, finally, for analysis, we interconnected these three references with our categories of analysis, which is the number sense indicators proposed by Mcintosh et al. (1992).

RESEARCH IN ACTION: APPLICATION DEVELOPMENT AND CONSIDERATIONS ON THE INVESTIGATION CYCLE

Brazil has 13.7% of the planet's fresh water. The largest water resource in our country is the Amazon basin, which contains 80% of all Brazilian water. Unfortunately, it is far from the great urban and industrial concentrations (Uberlândia is located in this region, specifically in the Southeast), and this implies that fresh water is a commodity of extreme value for the distant regions of that wealth. When brushing your teeth for five minutes with the faucet not too open, 12 liters of water is wasted. If you soak the brush, turn off the faucet and rinse the mouth with a glass of 250 ml water, save 11.5 liters of water.

Therefore, the aims of this investigation cycle were to alert children to a conscious consumption of water in the face of a day-to-day activity and to work on statistical investigation in such way as to enable children to develop the sense of number. The investigation cycle took place in three weeks, on November 18 and 25 and December 2, totaling 7 periods.

Task 1 - establishing the survey theme:

According to the theoretical option done in an investigative nature task, students must initially start recognizing the situation and a challenging issue. To emerge the question - "how do you brush your teeth?" We created a problem-solving environment from a conversation wheel whose theme was water consumption, the importance of oral hygiene and the way that children brush their teeth and we finally look forward to understanding the questionnaires.

Once established what to investigate is therefore necessary to discuss with the group, the population to be studied. In this sense, we dialogue with the children about what population we would investigate and what the representation of this population in the survey (sample or census) is. The children chose to survey the three rooms of the 1st year and interview all students who were there at the date of data collection.

Task 2 - Data collection:

As a tool for data collection, we used a questionnaire form (Figure 1) consisting of three categories represented by three children performing teeth brushing activity in three different ways. For children to feel safe on how to collect the data, we conducted the first survey with them. For data collection in other rooms of the 1st year the children were divided into 3 teams. The tasks were distributed as follows: 1) to introduce themselves to the classroom; 2) explain the importance of the research; 3) distribute the questionnaire; 4) explain the questionnaire; 5) help children who were in doubt; 6) collect the questionnaire.

Figure 1: questionnaire - how do you brush your teeth?

		
Open faucet	Open the faucet only to flush your mouth out.	Use a glass of water

Task 3 - Data organization:

Each team received the room records from where they had collected the data. On a round of conversation, we talked about the actions to be taken (separate forms, count by making the corresponding records). We give each child the table that we prepared for the activity.

While performing the task from the dialogue established with the teams, we try to make children aware of the importance of organizing the data establishing with them an investigative attitude towards the results. Ponte et al (2006, p.27) points out that this type of action should be encouraged by the teacher (in this study by the researcher), especially with children.

When performing tab, the children used and improved their knowledge related to matching ideas, count, number and data organization table (Image 1). During this task, many times children reworked the initial strategy for organizing and counting. This tendency to solve a problem by exploring it in different ways in Mcintosh et al, (1992, p.8), allows comparison of different methods before making a final assessment or seeking another point of view. These crisscross knowledges provide children to develop the number sense.

Task 4: Making a bar chart.

We resumed again a time of research presenting to the children another challenging question, which is: is there another way to present the data in addition to the table? To test and verify hypothesis proposed that the children meet with their team to make a bar charts (Image 2). The bar chart was built using their own answer sheets.

To end the activity, Jesus & Serrazina (2005) and Ponte et al. (2006), claim that this is the time to communicate, discuss and reflect with the whole class. In this sense, we ask the teams for the analysis, interpretation and presentation of results found in their search for the groups that were interviewed.

Image 1: table

Reference: researcher file

Image 2: bar charts

Reference: researcher file

CONSIDERATIONS ON THE INVESTIGATION CYCLE

This investigation cycle allowed an interconnection between concepts (mathematical and statistical) and procedures (mathematical and statistical investigation) that contributed to the development of the number sense of the children participating in the research. We interrelated these concepts and procedures with the indicators of a good number development presented by Mcintosh et al (1992, p.5). The results are presented below:

Investigation cycle and number sense

Knowledge and ease with numbers: The understanding of the numerical system helps the student to organize, compare and mentally order the numbers. In this sense contributed the following concepts and procedures - *Correspondence*: (association of the answer sheet to the pictorial record in the table); *Counting*: (number of records / answers in each category, analysis of data variability, treatment and organization of data); *Writing of number*: (record of quantities in table and graph); *Comparison*: (interpreting the variability of the data, comparing the frequency of each category, through situation problems).

Multiple representations for the number: It includes the recognition that numbers can be thought and manipulated in various ways. In this item we record - *Data presentation*: two different ways of representing the research result (table and bar graph); *Counting*: construction of the table (pictorial representation and numerical record).

Awareness of the existance of multiple strategies: The student must bring with him three skills of this component, create and / or invent strategies, apply different strategies and know how to select a strategy. This search for different strategies allows the comparison of different methods before making a final evaluation or seeking another point of view. For this indicator we count on the contribution of the following procedures - *Sample / Census / Population*: when defining the population to be investigated and what its representativeness in the research we did a round of conversation where different strategies were thought; *Variable / Category* (definition of the theme and construction of the questionnaire), *Data collection* (planning execution, division of the team).

Inclination to review data and reasonableness of outcome: When the solution is found, a person with Number Sense will examine their response to the original problem, taking into account the numbers that appear in the problem as well as the question asked. This indicator was widely used in the *analysis of the results* of the research: interpreting the variability of the data and verifying the reasonableness of the results.

The research cycle and social interactions:

Task 1 - Participating in the round of conversation allowed the children to talk a little about themselves and to know a little about their colleagues. They have learned that the round is a democratic space where everyone can speak but that organization is needed in such a way that each one speaks in turn.

Task 2 - Data collection allowed the children to share what we had talked about in the round of conversation (presentation of the survey) and the knowledge about the questionnaire (individually explaining the children who were having difficulty answering it). We observed that children who participated in the research presented more autonomy than the others, both in their speech and in their actions.

Task 3 – During the tabulation of the questionnaires and filling in the table, the children counted and recounted. With each different result a dialogue was established between them to determine who had been right. At that moment they used different strategies of counting the individual hour and collective time to reach a common result, when the child who had been successful celebrated with shrieking "I got it." The accomplishment of this task allowed the children to live harmonically with the error, since this was understood from the dialogue with the group's colleagues.

Task 4 - The accomplishment of this task allowed the children of the other two rooms that participated in the research to understand by means of explanations of other children as one does the reading of a chart. In addition, the child can share what he has learned with his peers allows him to acquire confidence and autonomy, a fact that is perceptible if we compare the first presentation at the time of data collection with the presentation of the results.

FINAL CONSIDERATIONS

We consider that the set of actions developed in our research are of great importance to allow the emergence of behavioral indicators of a good number sense development, which will reflect throughout life for mathematical and statistical learning and for living in society.

The social interactions promoted by the research are of fundamental importance for the construction of a reflexive and critical citizen, they are: to perform a Statistical Investigation (collect, organize, analyze and present data); Knowing and recognizing social use of numbers; to express your ideas on the topics discussed on the rounds of conversation and team activities; to work collaboratively; to live with colleagues respecting each individual's differences.

With the partial results of this research, announced through this article, we reinforce that it is in the field of the study of real problems and situations, in a perspective of contextualized, reflexive and critical investigation that Statistical Education is called to give its contribution that allows children to develop interpretive skills to argue, reflect, and critique.

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TEACHING MATHEMATICS FOR SOCIAL JUSTICE: TRANSFORMING CLASSROOM PRACTICE

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This discussion paper draws together a theoretical analysis of the potential of school mathematics to be either exploitative or empowering and the findings from a recent research project. It reports on how the participatory action research project impacted upon the thinking and classroom practice of a group of five secondary mathematics teachers. It considers the implications for transforming classroom practice in relation to teaching mathematics for social justice on a wider scale.

INTRODUCTION

The world is facing a crisis involving financial instability and political turmoil. Increasing income inequality is leading to greater social unrest, disquiet and a lack of trust in political leaders who have failed to deliver on their promises to promote social mobility and democratic reforms. So what has this got to do with mathematics education? I suggest two links. Firstly, school mathematics contributes towards perpetuating inequities existing within society, and secondly, mathematics education has the potential to develop the thinking and skills required for future generations to address global challenges we face. In this paper, I explore these two claims further, before discussing implications for the transformation of mathematics teachers' classroom practice with reference to the findings from a recent research project.

SCHOOL MATHEMATICS AND SOCIAL INEQUITY

Bourdieu argues that the primary function of schooling is to ensure that social divisions and unequal power relations are reproduced from one generation to the next (Bourdieu & Passeron, 1990). He claims that children from wealthier backgrounds acquire greater 'cultural capital' through their upbringing, placing them in a better position to make the most of the opportunities offered, and behave in ways that are valued, by schools (Jorgensen, Gates, & Roper, 2014). This process is disguised by presenting schooling as a meritocracy in which success is attributed to the natural talent of some students, rather than to structural advantages they may be afforded within the education system. An example of how mathematics plays a leading role in this process is the prevalence in England of 'setting', in which students of similar prior attainment are grouped together, despite little or no empirical justification for doing so. Setting relies on the notion that mathematical ability is innate and fixed, which is used to legitimise placing some children in lower sets, where they often experience an impoverished curriculum that reinforces their belief that they are weak mathematically (Black, Mendick, & Solomon, 2009).

Nardi and Steward (2003) highlight disturbingly high levels of alienation from mathematics amongst secondary (age 11 to 16) students. Noyes (2012) attributes this to the predominance of a transmission-led orientation towards mathematics teaching. Skovsmose (2011) warns of the ascendancy of the *exercise paradigm*, in which the teacher explains a mathematical procedure before students practice a series of almost identical closed questions and are then tested on their understanding. Not surprisingly, school mathematics is frequently perceived by students as being boring and irrelevant (Boaler, 2009; D'Ambrosio, 2006). However, children from wealthier families, often with higher levels of *cultural capital*, are more predisposed towards learning mathematics, even if its purpose is unclear, since they are more likely to appreciate its status as a *critical filter* that regulates access to higher education and future employment (Black, Mendick, & Solomon, 2009). A disengaging mathematics curriculum can therefore exacerbate gaps in achievement between children from different social groups, explaining the enduring correlation between mathematics attainment, participation and family income (Boaler, Altendorf, & Kent, 2011).

The international mathematics education community has called consistently, for over thirty years, for a more engaging mathematics curriculum, with a greater focus on *progressive* teaching approaches (Cockcroft, 1982; NCTM, 1989). These involve encouraging collaboration, discussion, investigation, communication, justification and reflection amongst students. Proponents of such approaches argue that they result in deeper levels of conceptual understanding, an appreciation of why mathematical procedures work and how to apply them to solving problems in unfamiliar contexts (Boaler, 2009; Swan, 2006). These skills prepare students better for problems they encounter in real life, and are increasingly demanded by universities and employers (ACME, 2011). So why have calls for more progressive teaching approaches been consistently ignored by educational policy makers? Part of the reason is that politicians, who tend to be preoccupied with the contribution mathematics makes to promoting economic growth, have begun to intervene to a much greater extent in curriculum change (Wright, 2012). Skovsmose (2011, p. 9) argues that the exercise paradigm cultivates a *prescription readiness*, preparing students for "*participating in work processes where a careful following of step by step instructions without any question is essential*". Gutstein (2006, p. 10) claims that the current disempowering mathematics curriculum merely reflects a capitalist economy's need for *"low-skilled, compliant, docile, pleasant, obedient service workers"*. It is hardly surprising, given the current economic and political crisis, that governments are not overly keen on establishing curricula that promote widespread critical thinking amongst their populations. However, for those with a genuine concern for issues of equity and social justice, there is a clear need for a more engaging and empowering mathematics curriculum that will provide learners with the type of mathematical skills and understanding they require to lead an active and fulfilling life, to avoid being exploited, and that will enable society to solve the problems it faces on a global scale.

MATHEMATICS EDUCATION FOR EMPOWERMENT

D'Ambrosio (2006) contends that, through colonisation, a form of *academic* mathematics was imposed by Europe on the rest of the world, subordinating indigenous cultures and displacing other more meaningful forms of mathematics. He argues, therefore, that mathematics educators have a responsibility for helping to address the growing crises facing humanity. Given schooling's tendency to reproduce inequities within society, and the apparent reluctance of governments to challenge this situation, what can the mathematics education community do to disrupt this cycle? How can researchers and teachers work together to develop an engaging and empowering mathematics curriculum, based on a *humanist vision* of education, that advances equity, social justice and sustainable development (UNESCO, 2015)?

Gutstein (2006) claims that progressive teaching approaches are a necessary, but not sufficient, pre-condition for such a curriculum. Bernstein (2000), however, highlights how these pedagogies can be problematic as the *rules of the game* in the mathematics classroom are less clear to students when more open-ended approaches are adopted. This can further disadvantage children from working-class backgrounds, who generally find it more difficult to identify relevant meaning from classroom tasks (i.e. follow *recognition rules*) and respond in an appropriate manner (i.e. follow *realisation rules*). However, the potential of progressive pedagogies for promoting engagement with mathematics suggests that, rather than avoiding them altogether, strategies should be explored for making the *rules of the game* more explicit. This reflects Skovsmose's (2011) argument that *critical mathematics education* should be preoccupied with students reflecting *on* mathematics, i.e. considering its nature and status in society, as well as *with* and *through* mathematics, i.e. using it as a means to explore their own situation and participating in mathematical inquiries that involve making their own decisions. Boaler (2009) describes school mathematics as an impoverished re-contextualisation of mathematics in which students have limited opportunities to experience the work of real mathematicians. Black, Mendick and Solomon (2009) argue that progressive teaching approaches encourage more students to develop positive relationships with mathematics and to study it beyond compulsory level. An empowering curriculum, however, necessitates going beyond inducting students into the somewhat artificial world of school mathematics by enabling them to exploit mathematics as a way of making better sense of their world. Drawing on Freire's ideas of *conscientisation*, Gutstein (2006) advocates *reading and writing the world with mathematics* in which genuine mathematical understanding develops alongside students exploring social issues and taking part in social action.

Critical mathematics education demands that greater consideration is given towards power relations that exist between researchers and teachers, as well as between teachers and students. Much educational research is conducted *on*, rather than *with*, practitioners and fails to take into account the constraints they face in the classroom (Bishop, 1998), often being conducted in *prototypical* classrooms in which social justice issues are less obvious (Skovsmose, 2011). Whilst there is growing interest in researching social justice issues in mathematics education, many such studies tend to be theoretical or philosophical in nature (Wright, 2015). Proponents of action research argue that working collaboratively with practitioners to develop an *understanding of theory-in-practice* generates knowledge that is more likely to be relevant to other practitioners and lead to positive social change (Torrance, 2004). Action research has the added benefit for participants of deepening understanding of their own situation and developing a critical understanding of research processes, making them better "*equipped to engage with and be discerning consumers of research*" (BERA, 2014, p. 5). Poststructuralist researchers, however, offer a critique of action research by maintaining that *universal truths* (such as *empowerment*) are non-existent and that *situated truths* exist only within a discourse (MacLure, 2003). A response from a critical perspective would be to reject the notion that knowledge generation can be objective and accept the partiality of action research as a practice that is "*explicitly political, socially engaged, and democratic*" (Brydon-Miller, Greenwood, & Maguire, 2003, p. 13). From this perspective then, poststructuralist research, at best, merely seeks to explain the status quo whilst, at worst, offers an excuse for ignoring existing inequality, injustice and exploitation.

Skovsmose and Borba (2004) offer a *critical research* model of participatory action research, which shares a "*research-resonance within critical mathematics education*" (p.209). It recognises mathematics education and research as fundamentally political practices. It rests on the assumption that the *current situation* should not be taken as given and that a more desirable alternative, i.e. an *imagined situation*, should be sought. It incorporates processes that help teachers to develop a critical understanding of the *current situation* and to investigate an *arranged situation*. This involves trying out aspects of the *imagined situation*, whilst taking into account the constraints of the *current situation*, in order to examine the feasibility of the *imagined situation*.

THE RESEARCH PROJECT

I report below on a research project, based on the critical research model, which aimed to explore how a concern for social justice amongst mathematics teachers could be translated into classroom practice. Invitations were sent out to those who had recently completed the initial teacher education course on which I was a tutor. It was made clear that participants should be committed to the following framework for *teaching mathematics for social justice*, reflecting the theoretical discussions above:

1) *Employ collaborative, discursive, problem-solving and problem-posing pedagogies which promote the engagement of learners with mathematics;*
2) *Recognise and draw upon learners' real-life experiences in order to emphasise the cultural relevance of mathematics;*
3) *Promote mathematical inquiries that enable learners to develop greater understanding of their social, cultural, political and economic situations;*
4) *Facilitate mathematical investigations that develop learners' agency, enabling them to take part in social action and realise their foregrounds;*
5) *Develop a critical understanding of the nature of mathematics and its position and status within education and society. (Wright, 2015, p. 27)*

A research group was established in June 2013 comprising five teacher researchers, Anna, Brian, George, Rebecca and Sarah (all pseudonyms), who were nearing the end of their first year as newly-qualified secondary mathematics teachers, and myself. All five taught in ethnically diverse comprehensive schools in inner-city London, with above average numbers of students who spoke English as an additional language, had statements of special educational need, and were eligible for free school meals. The first meeting of the research group focused on teacher researchers engaging with the theoretical ideas and research findings underpinning the project and relating these to their own practice. The remaining six meetings involved planning and evaluating a series of classroom activities, as part of three participatory action research cycles, and reflecting further on thinking and practice in light of these experiences. My role was mainly that of facilitator, for example by introducing relevant research findings and inviting teacher researchers to present selected readings to the rest of the group for discussion. The research group drew on previously existing resources to develop their own ideas to try out in the classroom (see Wright, 2016 for a collation of these). Teacher researchers made use of notes kept in research journals, and feedback collected from student surveys, when presenting their evaluations of classroom trials to the rest of the group for discussion.

I conducted a series of interviews with each teacher researcher, at the start, mid-point and end of the project. These were *empathetic* in nature, i.e. based on building relationships of trust to allow for the emergence of more meaningful representations of teacher researchers' views (Fontana & Frey, 2008). Data was collected through audio-recording and transcribing the research group meetings and interviews. The collaborative nature of the project meant it was not appropriate to collect data from observing lessons, as I felt this would have adversely affected the power dynamics between myself and teacher researchers. Instead, I sought to co-construct the stories of teacher researchers' participation in the project through interaction and dialogue (Kvale & Brinkmann, 2009). A thematic analysis was carried out on the transcripts, which involved breaking them down into units of meaning, summarising each of these and then assigning it a category. Inductive coding was used for this purpose, i.e. an initial reading of the data was used to derive the categories, examples of which included *students' engagement* and *constraints on teaching*. The categories were then used to compare units of meaning by looking for commonalities, differences and relationships between them, allowing themes to emerge (Gibson & Brown, 2009). The thematic analysis was iterative in nature as emerging themes were related back to the underlying theories to generate new analytical questions that influenced my choice of future interview questions. Initial findings were presented back to teacher researchers during meetings and interviews for their comment and to prompt further discussion. Whilst there is insufficient space here to describe the research project methodology in full, more detailed accounts can be found elsewhere (Wright, 2015). I summarise below the findings that are most relevant to the discussions in this paper.

SUMMARY OF RELEVANT FINDINGS

All five teacher researchers reported significant improvements in the engagement of students with mathematics resulting from the activities and teaching approaches tried out during the research project. This was most noticeable amongst lower-attaining students and those who previously lacked confidence or behaved poorly in mathematics lessons. A particularly striking example was the response of one student in Anna's Year 8 bottom set, who was in her last week before being moved to a special school because of her poor behaviour. The class had been asked to discuss how a total hourly wage bill of £100 should be shared out between five workers in contrasting jobs, before relating this to wealth distribution in real life:

> But in terms of her enjoyment of the project … she was asking so many questions, she was putting forward so many views, she was working in a team. She was just like a dream child for the whole project. (Anna, Meeting 3)

The positive response of students reinforced the teacher researchers' commitment to the progressive pedagogies employed during the project. However, one aspect of the framework that they acknowledged having not previously considered was the promotion of students' agency. Rebecca described the transformation in her thinking arising from the *Making a Change Project* she devised, in which groups of students were asked to choose a social justice issue of interest to them, explore it in detail, identify a change they would like to see made and then use mathematics to back up their argument. The first time she tried this activity, she was frustrated when students made unrealistic demands, such as amending the school rules on body-piercing, and when logistical difficulties meant some students did not complete the task. However, when she fed back her experiences at the next research group meeting, the idea was embraced enthusiastically by other teacher researchers and became the focus for the next action research cycle. The group designed a more structured activity that retained the element of student choice whilst providing more guidance on how to generate a powerful mathematical argument. For example, students were asked to contrast two statements, such as *"one in five people go to bed hungry each night"* and *"there are lots of people in the world who are hungry"* (Wright, 2015, p. 69) to help them appreciate the difference between mathematical and non-mathematical statements. Rebecca reported how, when she tried the activity again, students came up with more realistic demands for change and made more effective use of mathematics to support their arguments. Feedback from students suggested the activities helped them recognise the relevance of mathematics to their everyday lives and they welcomed the opportunity to exert greater control over their own learning.

All five teacher researchers articulated how they initially focused their concern for social justice issues on raising the mathematics attainment of disadvantaged children (the initial teacher education course they chose to study had a policy of placing them in schools in the most deprived areas of London):

> I've chosen to teach in a school where it's classed as a challenging school, because the kids stereotypically wouldn't be expected to achieve very much.

> ... So I think, in the sense of bringing about social justice through education, I'm involved in that just through being at this school. (Anna, Interview 1)

However, through engaging with relevant research literature, the teacher researchers began to appreciate the complexity of the relationship between social justice and mathematics education, and how structural issues could act as barriers to learning for their students. They began to ask themselves questions, often for the first time, about the nature of mathematics and the processes of schooling. Whilst George was alone in expressing concerns about setting (which was used in all their schools) at the start of the project, all five became increasingly critical of this practice. Rebecca began to question the rationale and fairness of grouping together students with generally weaker communication skills and poorer dispositions towards learning, whilst Anna proposed doing away with setting altogether were she to become head of department.

The teacher researchers also began to recognise their own tendency to teach lower-attaining students in a more structured way, often due to challenging behaviour they exhibited, which Rebecca described as the biggest constraint on trying out different approaches. Brian highlighted the importance of establishing a balance between encouraging all students to develop critical understanding and independence, and the need for lower-attaining students to acquire the social norms and dispositions towards learning required to become successful learners. He highlighted the value of building relationships of trust so that students would have faith in teachers when they were asked to question commonly accepted beliefs about learning mathematics:

> I'm able to almost take a step back ... and say 'Why are we doing this?' ... And why will there be some lessons that seem irrelevant, and some lessons that seem completely unrelated to your lives? What's the broader aim there?' ... And working with them in a way that allows them to both access what we're trying to learn in terms of skills sets and mathematical content, but also develop the broader skills that they need for life, and that allow them to make the most of opportunities that are there for them. (Brian, Interview 3)

The teacher researchers began to identify the conditions that deterred them and others from adopting more engaging and empowering approaches to learning mathematics. They highlighted the high levels of monitoring of their performance by managers, with a focus on short-term and easy-to-measure progress of students, and the pressure to get through the scheme of work in order to prepare students for regular tests. However, they reported how the collaborative nature of the research group provided the incentive and mutual support necessary to begin to overcome these constraints:

> It's given me the confidence to step off the scheme of work treadmill, of getting through different topics or chapters … and it's also provided that additional incentive to do it, and to take the risk … you know you're going to be allowed to talk about it in a way that says that messing up doesn't matter (Brian, Interview 3)

The teacher researchers concurred that the research project radically changed both their thinking and classroom practice. They described how the sustained nature of the research project, and its focus on relating theory to practice, had a greater impact on their teaching than other professional development they had experienced. They particularly welcomed the opportunity to meet with colleagues from other schools, which exposed them to a range of different perspectives. They acknowledged the key role I played in introducing them to research theory, challenging their assumptions and providing a safe environment in which to develop ideas. All five, acting on their own initiative, became involved in disseminating ideas from the research project more widely within their schools, reflecting growing confidence and satisfaction with the direction in which their practice was developing. They experienced increasing levels of interest in the research project from other teachers as news spread about its positive impact on students' learning. George described this as the *multiplier effect*.

CONCLUSION

The research project provides evidence to support the claim made by Boaler (2009) and others that progressive teaching approaches can increase students' engagement with mathematics, particularly for those alienated from the subject or lacking confidence in their own ability. It highlights the need for teachers to establish relationships of trust so that they can challenge students' assumptions about learning and help them appreciate the *rules of the game* in the mathematics classroom (Bernstein, 2000). Enhancing the extrinsic motivation of all students to achieve in mathematics, whilst cultivating the behavioural and learning dispositions they need for success, can compensate for generally lower levels of *cultural capital* and intrinsic motivation possessed by those from disadvantaged backgrounds (Bourdieu & Passeron, 1990). The project therefore expounds on strategies that can potentially reduce gaps in attainment between students from different social groups. It points to how students can develop the mathematical skills and agency required to present a powerful argument for change, without necessarily taking part in the kind of direct social action advocated by Gutstein (2006). It therefore offers a vision of a teaching approach that fosters the type of critical mathematical understanding required for young people to better understand their situation and environment, and that equips them to take action in future to help resolve the crises currently threatening society.

The research project also highlights the potential of the *critical research model* (Skovsmose & Borba, 2004) for carrying out systematic and collaborative research with practitioners that generates relevant knowledge transferable to a wider range of contexts. It does so by focusing on typical constraints that teachers face in the classroom and showing how these can be overcome with the aid of mutual support provided by the research group. Through engaging with underlying theories, and relating these closely to classroom practice, it demonstrates how teacher researchers are able to acquire agency and self-efficacy as they gain a deeper understanding of their current situation and greater control over the extent and direction of their developing practice. It also shows how teachers benefit from being involved *in* research, rather than merely engaging *with* research findings, for example by developing a critical understanding of research processes. The reluctance of governments to advance pedagogies that promote critical thinking, particularly in times of crisis, magnifies the importance of *bottom-up* approaches to transforming classroom practice such as that proposed by the critical research model.

It should be noted that the research project was unfunded and relatively small in scale. Whilst there was significant interest shown by other teachers, the project was restricted to those who had already expressed a commitment towards teaching mathematics for social justice. Despite its limited scale, however, it provides a useful model and starting point for future larger-scale funded research projects. These could generate opportunities for even greater levels of collaboration, for example by providing additional time for discussion, reflection and peer observations amongst teacher researchers. Whilst the period of one year, over which the research project was sustained, witnessed significant impacts on thinking and classroom practice, an extended time frame would enable the longer-term impact on students' achievement and on teachers' professional development to be evaluated. Future projects might also explore how to transform the thinking and practice of teachers beyond those already exhibiting the same levels of commitment to teaching mathematics for social justice as those in this project, for example, by working with all mathematics teachers within a school, or with entire mathematics departments across a number of schools. The following question is posed for further discussion at the MES9 conference with a view to formulating ideas for developing future research projects of this nature:

> What would a research project look like that would transform classroom practice, in relation to teaching mathematics for social justice, on a wider scale?

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HISTORICIZING “MATH FOR ALL”

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This paper is a historical and comparative study of the “reason” of “math for all” through juxtaposing two documents advocating mathematics for all children with a core curriculum for school mathematics separated by seventy years: Post-War Plans (NCTM, 1944; 1945) and Principles to Actions (NCTM, 2014). The purpose is to reveal how the discursive assemblage of school mathematics reforms produces particular kind of mathematically able bodies and their others, how their differences are made an object of intervention then and now.

Today, it is difficult to imagine a school curriculum without mathematics. However, almost one hundred years ago, mathematics educators in the United States were worried about the extinction of school mathematics as a required subject in the curricula (Hedrick, 1936). In our age, accelerated after the Second World War (WWII), it has become awkward to think a person without certain knowledge of mathematics. Contemporary reforms are offered as an innovative resource for “systemic excellence” that will bring development and progress for children and society (NCTM, 2014). New methods of teaching and curricular ideas are presented to ensure the high quality of “mathematics for all”. Fair learning opportunities, a degree of autonomy, collaboration, group work and responsibility have become the desired practices in mathematics classrooms rather than administration or individualization. New roles have emerged for children such as problem solver, decision maker, collaborator and so on. These roles have been less to do with mathematics but more about the production of the categories of distinctions and differentiations among the “kinds of people”, in Hacking’s (2007) term. What I shall argue throughout this paper, the discursive assemblage of school mathematics does not only entail learning mathematics but also produces mathematically able bodies by ordering particular modes of life.

In policy documents, reforms and research practices, efforts to include are made under the discourse of “math for all” for “progress” and “development” of the child and the society. Although the language of “all” seems to be inclusive, there is an assumed space for children who are less than or who are not there (yet). Embodied in this assumption is a set of historical distinctions and cultural rules of knowledge that differentiate and divide people on a hierarchy. While the recent reforms and research practices in mathematics education continue to offer “solutions” or “actionable practices” to “maximize” their capacity, this paper considers how the discursive assemblage of school mathematics does configure mathematically able bodies embodying particular modes of living and acting in the world. My purpose, specifically, is to demonstrate that how mathematically able bodies and their others are made up within the entangled relationships of various discursive mechanisms of school mathematics.

“Math for all” discourse in mathematics education has historical parallels. The habit of using mathematics in everyday life, for example, in order to be and act as an “efficient” and “intelligent” citizen can also be traced back to the 1940s, right after mathematics was introduced to the curricula as a required subject, for the preparation of life following the WWII. Modernization of daily life, at that time, required “possession of knowledge” that make citizens act rationally and mastering mathematics was seen essential to understand machines, housing, business, transportation, public and personal health, investment, insurance or taxation (NCTM, 1944, p. 227). As it does today, the idea of math knowledge for all permeated mathematics education practices in planning the postwar society to build modern secular nation with the particular kind of mathematically able bodies.

In what follows, after describing my analytical strategy, I study the particular ways of these two school reforms on mathematics, *Post-war Plans* (NCTM, 1944; 1945) and *Principles to Actions* (NCTM, 2014), how they operationalize their interventions to make visible the practices that divide, order and hierarchize kinds of people. Then, I examine how the processes of abjection are actualized through their interventions, technologies and tactics to normalize the particular kind of mathematically able body and to include the Others, who are different than the norm(s) or one another, locating them unlivable cultural spaces. I conclude with the limits of “math for all” for progress and development by making visible the parallels and discontinuities.

ANALYTICAL STRATEGY

The historical analysis employed in this paper is done with a sensibility of the “epistemic habits” employed in these discursive practices that produces the hierarchical ontologies for children and society (Stoler, 2009). How we come know what we know, that is “systems of reason” (Popkewitz, 2008), are going to be analyzed to understand how fictitious and hierarchical categories are formed between different kinds of bodies. This requires a particular notion of history and archival documents that legitimize its truths, which is not an accumulated record of people’s actions or intentions. Historically comparing these two school mathematics reforms treats them as an event, which can be analyzed in the within the assemblage of mathematics education discursive practices through its complex entanglement with the multiplicity of historical processes (Foucault, 1991, p. 75).

The method is called a "history of the present" that is not about the past, but how "the past is intricately woven in constituting the present" (Popkewitz, 2013, p. 440) in a grid of discursive practices. This grid enables to see how various discursive practices come together while the assemblage has its own intelligibility, which makes the conduct of people possible and "reasonable". The assemblage is somewhat similar to "rhizome", as explained by Deleuze and Guattari (1987), consisting of multiplicity of lines, segments that are connecting heterogeneous ways with non-hierarchical entry and exit points. The rhizome is not a generative or reproductive model for tracing structures, but it is a map, which is open to constant modification or any kind of social transformation. Therefore, the discursive practices in this study are not treated as skewed or biased sources but as the sites of epistemological and political anxiety (Stoler, 2009) that includes hopes and fears for the subjects of schools (Popkewitz, 2008). This comparative style of reasoning, as Popkewitz puts, targets the Other through processes of abjection that encloses different possibilities of conduct.

In this study, body is considered as a spatiotemporal configuration and has historically become possible and real in the material-discursive practices (Barad, 2003). Instead of assuming the subject pre-existing before the "school mathematics", the mathematically able bodies might suggest a subject constituted while doing school mathematics as an ongoing discursive practice. As Butler (1990) elaborates, the demarcation between bodies is the "result of a diffuse and active structuring of the social field [in which] this signifying practice effects a social space for and of the body within certain regulatory grids of intelligibility" (p.178). So, the differentiations between different kinds of bodies (i.e. mathematically able) are the effects of the regulatory mechanisms and cultural rules of the knowledge that authorize the discursive formation of people.

Turning the analytical lens on the styles of reasoning embedded in these reforms enables to think about how the practices of school mathematics make the subjects as the objects of the study, the categories of difference as a site of intervention and what kind of pre-emptive realities are produced. These discursive practices are not purely language but ought to be understood "as places of what is being said and done, rules imposed and reasons given, the planned and the taken for granted meet and interconnect" (Foucault, 1984, p. 75). I will be looking at the keywords emerged through the analysis of hopes and fears in the discursive statements as indices of relations of power (Stoler, 2009, p. 33). What becomes the commonsense of school mathematics, teaching and learning assembled with the sociopolitical conditions of the society and the pedagogical and curricular technologies that make the mathematically able bodies possible is historicized to exhibit how the legitimizing practices for and of the bodies are the effects of contingent historical forces, political conditions and scientific knowledge.

MATHEMATICS AND LIFE

School reforms play an important role in cultivating the inner qualities of child and order (im)proper modes of life for the self and society (Popkewitz, 2008). Not surprisingly, the "reason" of "math for all" is one of the cornerstones in the process of ordering, dividing and normalizing of particular ways of living. Post-war Plans for school mathematics, for example, was aiming at the development "mathematical competence for the ordinary affairs of life" where using mathematics was prerequisite for "intelligent" and "efficient" citizenship. While the hope is to improve mathematical instruction for all, what these statements actually entail ordering of the proper modes of life to build the modern nation through mastery of mathematics in schools and then use of this knowledge in everyday life. In other words, disuse of mathematics in daily life would generate unintelligent and inefficient citizens who become different than the desired mathematically able body.

Although it is usually taken as a commonsense that mathematics is part of life, what makes it possible is an effect of historical and social transformations. After the 18th century, as Porter (1995) points out, quantification of every aspect of life emerged as new forms of governing the self and society. The collapse of sovereign empires required new technologies and new forms of power to govern people rather than brute force. Use of numbers in daily life begins through the modernity in order to maintain the social cohesion with a degree of reason and rationality. Mathematics and particularly numbers become intelligible technologies for constructing citizens of modern nations who can act in efficient and productive ways since then. This way of "knowing" entailed a logic based on numbers for the planning of the society and the people (Poovey, 1998) and produced mathematical ways of being and acting in multiple spheres of daily life.

Given the fragile position of mathematics in the school curricula at the beginning of the 20th century, the "social aim" of mathematics was emphasized in the reform efforts to make the it worthwhile for everyone not particular group of elites. To illustrate, it was the "responsibility of mathematics to provide training that will make the pupil intelligent and efficient in dealing with the problems that he may meet in the home and in his everyday reading and conversation" (NCTM, 1945, p. 204). Embodiment of mathematical modes of living became the motor of development and civilization, which maintains the historical concern of governing people in modern nations and planning the society. The "civilized" standards of the rules of conduct did not contradict with the American narrative such as being reasonable and enlightened for national progress (Popkewitz, 2008). Besides these hopes, the design of the child as future citizen incorporates fears of darkness and backwardness. The double gesture embedded in this assemblage generates an exclusionary matrix and enables the abjection of those who might act outside of these regulated spaces. It, then, fabricates mathematically able body that can function in modern societies through the embodiment of particular modes of life and produces mathematical ways of being and acting.

School mathematics was referred as a "mirror for civilization" (NCTM, 1945, p. 205) and a practical tool for solving problems of everyday life "in a world in which the questions 'how much?', 'where?', and 'how many?', have to be answered again and again accurately and with great precision and [are] as important as the ability to communicate" (p. 227). Mathematical precision, as Porter (1995) argues, has historically been regarded as a sign of competence and the moral character of the daily life. Conditions for governing responsible and ethical citizens were generated through these moralities for a peaceful and stable society in the planning for postwar society.

The "civilizing" feature of mathematics also underlies the reason of the contemporary mathematics education reforms. The aim, now, is to form future citizens as "productive full participants of the society" who can make instant access to information and understand the problems in the world such as meeting the entry-level workplace expectations and the daily responsibilities of household management and citizenship (NCTM, 2014, p. 3). Nonetheless, different than past, which desired a rational being, there is a shift to rational decisions by "flexible" human kind with a "growth mindset". While mastering the mathematics in school and then using them in a practical situation was prominent before, the daily life practices are brought into the schools. That is, children need to mathematically deal with the hypothetical or real situations in the schools. In the contemporary math classrooms, for example, children are expected to "identify important quantities in a practical situation and map their relationships [in order] to draw conclusions" (CCSSM, 2010, p. 7).

Informed and rational conclusions derived from mathematical models have become one of the new standard for maintaining societal cohesion and producing self-directed individuals. Not only possession of mathematical knowledge, but seeing mathematical relationships in an efficient system is necessary for drawing informed conclusions. That is, the question of what mathematical knowledge is possessed by an individual has transmuted into what individual can do with mathematics in everyday settings. How to live and act incorporates a cultural logic of rational choice, embedded in a style of systematic thinking emerged in the social sciences (Heyck, 2015). That is, individuals do not necessarily have to adjust themselves into the system or into the society as rational beings, but they have to change themselves in a way to make sure they are making rational choices and acting accordingly to make the system work efficiently. Therefore, according to contemporary reforms, there needs to be a "flexible" individual with a "growth mindset" who can easily adapt to the diverse circumstances with rational decisions and actions based on the mathematical relationships and associations to avoid system distortion.

Using mathematical tools for a practical situation is not merely to acquire mathematical knowledge, but to use that knowledge to make informed "conclusions" for the uncertain future. While there are important historical parallels in terms of ordering and regulating the life in both reforms, the modeling activities in contemporary reforms can be considered as a new one. How can we explain these changes? One approach is to consider the emergence of contemporary risk-based security calculations (Amoore, 2011). That is, a specific form of abstraction derived through data or quantities of a situation and reveal associations to calculate uncertainty and to unfold future (p. 2). For example, in one of the modeling activities, children are expected to identify the fatalities per year, per driver and per vehicle-mile traveled to find a good measure of highway safety (CCSSM, 2010, p. 58). What being produced here is the pre-emptive realities for the unknown future and actionable practices for the present, decided by mathematical models and quantitative relationships through the notions of safety and risk. Obviously, these projects do not only intent to develop an understanding of mathematical content but also the standards of participating in daily life.

Concerns for safety and risk resonate with the redemptive narratives of American exceptionalism where the pursuit of happiness is the goal. We should also note that these concerns are performative. Normalization of particular modes of life via appropriate risk analysis simultaneously targets those "unlivable", "insecure" and "unhappy" cultural spaces (since mathematical models do not foresee those or they are too risky). The deployment of data over unknown future, as Amoore argues, generates the capacity to act on and through people, populations and objects in the face of uncertainty. Besides the emergence of math modeling, contemporary mathematics education reforms, such as "math success for all", also have been gone through similar calculations to identify children who are "at risk" in order to "maximize" their capacities through actionable variables such as "achievement gap".

Modeling processes become an important calculative practice for producing flexible people who can "engage with the world in a mathematical way" or who can make "informed decisions" through quantifying the various circumstances and appropriate risk analysis for ordering of the life. While the aim is to mathematize the world for "progress", "development" or "safety", in fact, these technologies are mathematizing the bodies. The modes of life are ordered, differentiated and normalized through seeing and acting within the world in mathematical ways. The movement between the real and mathematical world becomes blurred and, at the same time, generates an exclusionary matrix between kinds of people. They produce mathematically able bodies and their distinctions from their others, which has to do with not only understanding the mathematical content but also ethical and moral qualities of "mathematically modeled life".

ACTUALIZING ABJECTION: MATHEMATICALLY ABLE BODIES AND THEIR OTHERS

In the post-war period of the United States, mathematical illiteracy was impossible to ignore not because of the importance of learning mathematics but forming a modern nation with rational, efficient and civilized kinds. The "social" aim of mathematics was becoming an important part of the curricula. Concurrently, evaluation practices were not just for measuring mathematical learning through paper and pencil tests but include other procedures such as interviews, observations and examination of work products in order to capture "the best kind of evidence that children are realizing the social aim of mathematics" (NCTM, 1945, p. 203). If the child was not eager or motivated to learn mathematics in school and then efficiently apply in practical situations in daily life, he/she was in needed to be tracked into "social mathematics" courses to make it attractive to these kinds whose "needs" were not met in regular tracks and who did not appreciated the importance of social aims. Within this pursuit of national belonging, difference was a societal fear as it might interrupt the cohesion:

> Society dare not neglect the problem, because its institutions are forever in danger when the uneducated masses become restive by the annoying gap between the things that they can have and the things they want. Mechanizing a nation that is not mathematically literate is a dangerous business (NCTM, 1944, p. 230)

The promise of a better future, living in a peaceful environment, is embedded in these reforms for the postwar society. Nonetheless, this practical utility of school mathematics was used as a mean to motivate the child through the political anxieties in the making of modern secular nation. The Social Question, as Popkewitz (2008) puts, is concerned with the loss of moral order in urban settings where redemptive themes in the educational reforms are to rescue these populations. Changes in the urban schools such as European immigrants and people coming from rural areas and settling in the cities make the possession of mathematical knowledge as a way of living and acting rationally and efficiently at that time. School reforms embodied universal standards and norms of conduct not only for the child, but also for the family and the community through ordering the everyday life. The cultural theses revealed from these reforms embodied a "sublime of American exceptionalism" positing who the child is and should be as future cosmopolitan citizen: a rational and enlightened being (Popkewitz, 2008, p. 45). The narratives of this sublime include a national exception, being a "light" to the rest of the world. Nonetheless, the unity incorporates double gesture that is hope for progress and civilization coupled with the fear of the "uneducated masses" for the civil life after the wartime. As school mathematics becomes commonsensical knowledge for proper way of acting in the lives of mathematically able bodies, it equivalently functions as a process of abjection and generates cultural space(s) for the Other. Mathematics as "mirror for civilization" embodied a psychological distinction between kinds of children. That is, tracking was not based on mathematical knowledge that children had; on the contrary, tracks were designed on the basis of children's "motivation" or "interest" to use math as a practical tool in everyday life.

In the contemporary reforms, different than past, we can see a shift to self-assessment strategies such as keeping track of own learning or development of metacognitive awareness that enables children to control their own actions. In this sense, making visible the goals of instruction becomes important so that "students become more focused and [are] better able to perform self-assessment and monitor their own learning" (NCTM, 2014, p. 12). Children develop their own tools to ensure whether they attain the goal of the lesson. This attainment might be either in or out of the school, merging different spaces that make the exercise of power diffusive. In present reforms, therefore, new forms of assessments are institutionalized and continuous control mechanisms are built despite the expressions of freedom and flexibility (Deleuze, 1992). The novel technologies of governing in societies of control, to borrow from Deleuze, have facilitated the making of unfinished cosmopolitan through self-directed mathematically able bodies. The ongoing risk analysis with mathematical models hierarchizes a particular mode of life. While the tactics for dividing people have changed from external assessment to self-assessment strategies, the normalization and abjection of kinds of people are still there. In the contemporary case, children need to monitor their own progress and differentiate oneself in relation to the pre-established hierarchy of learning progression. If the child cannot attain the goal of the lesson, he would be in need for further studying or remedial instruction by herself, in the harshest case.

The written forms of assessment have not disappeared; however, they have different function than the past. The tests, student products, observations, interviews are used for the planning of teaching and making instructional decisions because “effective mathematics teaching” requires to be drawn on the “evidence of students’ current mathematical understanding” (NCTM, 2014). Hence, mathematics teachers need to be “adaptive” for diverse situations, have a “growth mindset” to flexibly change or modify the instructional decisions and be reflective on their own teaching practices to improve. While in the previous reforms, examinations were for tracking children into a particular curriculum, the contemporary reforms use assessment for the continuous control of the self, both the child and the teacher. Therefore, contemporary discursive assemblage of school mathematics is performative in a sense that it produces new subjectivities of teaching.

Mathematically able bodies and their Others are not separated into different tracks nowadays. Nevertheless, math teachers need to differentiate their instruction for different kinds. Put it differently, classroom space is differentiated for the different bodily needs. While these pedagogical and social inscriptions produce its kinds, they simultaneously invent its Others. The shared space is differentiated according to sociomathematical norms that are considered as mathematically appropriate ways of participating in the learning communities (Yackel & Cobb, 1996). These “acceptable” mathematical explanations and justifications do not only characterize its kinds but also prefigure what normal/abnormal is through the “mathematical” standards of participation. They create normal spaces in mathematics classrooms for the mathematically able bodies to be fit in.

Psychological distinctions produced in the contemporary reforms reveal important historical parallels and but require different calculations to “see” the mathematical ability. Mathematically able bodies are differentiated from their others through the processes that lead to rational choices and decisions such as showing “perseverance” on the tasks or “interest” in modeling activities. Historically, pointing out the psychological constructs has always been a way to talk about differences (Danziger, 1997). Then the goal of “elimination persistent racial, ethnic, and income achievement gaps” (NCTM, 2014, p. 2) is an illusion as the differences are actualized through these corporeal regulations and the categories of psychological distinctions that authorize who the child is and should be.

LIMITS OF "MATH FOR ALL" FOR PROGRESS AND DEVELOPMENT

Comparing the assemblages of these two documents, separated by seventy years, is not to show the accumulation of scientific ideas or "advancement" of the discursive assemblage of school mathematics; on the contrary, the purpose is to reveal the important historical parallels thereby making visible how the discourse "math for all" is productive in the making kinds of people. In addition to this, my purpose of comparing these two realities of schooling is not to take "math for all" as an object or a thing to be replaced by something else but to locate some of the historical and cultural trajectories that enabled the circulation of "math for all" in mathematics education and how this discourse can become constitutive of what is "dangerous" or "good" at a given time-space in the name of progress and development. School mathematics entangled with the societal hopes and fears produces cultural theses for the modes of living and fabricates mathematically able bodies that can function in modern societies in "civilized", "secure" and "happy" ways.

The double gesture embedded in the reason of schooling, simultaneously, enables the abjection of those who think or act outside of these regulated spaces. The phrase "all children" functions as a pivoting point to distinguish two human kinds in the standards and research: those who have all the capabilities to learn math and their disadvantaged others (Popkewitz, 2004, p. 23). Those who are "different" than the desired mathematically able bodies are psychologized and they have become both an object of governing and a site for intervention not because they lack the ability to learn mathematical content but because they lie outside the "moral" qualities of a reasonable and mathematical life. The mathematics curriculum in the past and the mathematics teaching in the present are differentiated for "disadvantaged" others who have different "needs" and "interest" under the phrase of "math for all" to include them in a space that is normalized within the discursive assemblage of school mathematics. The discursive analysis reveals that there is something continuous in the reason of "math for all" that order particular corporeal regularities for governing and divide people on a hierarchy. The continuing trend in the discursive assemblage of school mathematics remains to restrain these "pathologic" cases, identified through different rationalities and calculations, which might disrupt the harmony.

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