PRIORITIZING VISUAL SPATIAL MATHEMATICAL APPROACHES IN FIRST NATION EARLY YEARS CLASSROOMS

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ABSTRACT

This paper describes the Math for Young Children project (M4YC), a professional development research project that works with teams of educators in an adapted lesson study approach to promote, design and assess visual, spatial approaches to mathematics learning in early years classrooms. Since 2011, our work has focussed on supporting communities historically underserved by the educational system. We present methods and results of our ongoing work in First Nations communities in the Treaty 3 territory of Northwestern Ontario.

INTRODUCTION

The research and professional development work we have been carrying out in Ontario, Canada in our Math for Young Children project (M4YC) investigates the potential of visual/spatial approaches to support early years mathematics learners, particularly those historically underserved by the education system. We situate our work in the growing literature on the importance of spatial reasoning and spatial kinaesthetic approaches to mathematics learning in the early years (e.g., Davis, 2015: Newcombe 2010; Sinclair & Bruce, 2015). Further, our work resonates with a proposal of Gates presented in a 2015 MES paper in which he speculates on the potential for reducing SES-related differences in early math learning by shifting the focus in mathematics teaching from its current emphasis on symbol, language and text, to a focus on the visual aspects of mathematics. Teaching approaches to school mathematics, he asserts, depend on, "language and textual communication to the near exclusion of other modes of communication – most notably the visual" (p. 517). Gates argues that a more equitable approach to mathematics instruction may be one that places greater emphasis on the visual aspects of teaching and learning mathematics.

In this paper, we discuss these issues in the context of our work in the Math for Young Children project. We begin with an overview of M4YC in its first two years; its rationale, goals and professional development approach as well as details of the visual/spatial lessons and activities that were co-designed as part of the process. The main focus of this paper, however, concerns our M4YC work in First Nation communities, and we present the methods and results of research we have been conducting to assess student learning and to learn more about the potential of this visual spatial approach. Finally, given the overall success of this project, we speculate on the implications of this work and consider how a rigorous spatial geometry curriculum can provide a culturally responsive context to support early years mathematical learning in First Nation communities.

THE MATH FOR YOUNG CHILDREN PROJECT

Since 2011, we have been collaborating with teams of educators, including early years teachers and their students, to design, implement and assess new spatial approaches to early mathematics learning, particularly in the area of geometry.

In the first few years of the M4YC project, we worked in low-resource urban school districts with schools ranking among the lowest in provincial math scores. During that time, we collaborated with more than 12 professional learning teams, each comprised of university researchers, school board mathematics coaches, Ministry personnel, and early years educators (released for seven days during the school year), and their more than 2,000 Kindergarten to Second Grade students. Our approach to this professional development involved an adapted form of Japanese Lesson Study (Lewis, Perry, Murata, 2006) with four key adaptations: a) teachers engaging with mathematics, b) teachers conducting clinical interviews with their students, c) teachers co-designing exploratory lessons, and d) teachers creating resources, lessons and activities for wider use in early years classrooms (Moss, et al., 2015).

Our over-arching goal for the M4YC project has been to re-think/re-envision what might be the most equitable, culturally responsive and effective starting point for early mathematics learners. Specifically, an essential part of our goal has been to work in collaboration with classroom teachers to shift the focus of early mathematics instruction away from the typical emphasis on number to one that is more focussed on children's intuitive mathematics, their emerging spatial knowledge, and their embodied and aesthetic experiences. In previous papers (e.g., Moss, Bruce & Bobis, 2015), we have written about our rationale and motivations for the M4YC project. Below we offer a brief review.

The starting point for our work has been the well-known literature on SES-related disparities in early math readiness. It is now well recognized that young children have strong intuitions about, and many informal understandings of, "everyday mathematics" (Ginsburg, Lee & Boyd, 2008) that are foundational to formal school mathematics. It has been reported by many (e.g. Case & Okamoto, 2005), that despite similar starting points, by the time children enter formal schooling, there are already striking SES-related disparities in readiness to engage in mathematical activity and this lack of readiness can have cascading effects on children's future mathematics learning. Recent findings on the predictive nature of early math competence for later *overall* academic success (e.g., Duncan et. al., 2007) speak even more strongly to the urgency of working towards improving the situation. Indeed, given this inequity, the U.S. National Research Council (NRC) has urged educators and policymakers to provide young children with, "extensive, high-quality early mathematics instruction that can serve as a sound foundation for later learning in mathematics and contribute to addressing long-term systemic inequities in educational outcomes" (Cross, Woods, & Schweingruber, 2009 p. 2). Despite such calls, current practices in many early years school settings (at least, in North America) are usually limited to a focus on number.

Why geometry and spatial reasoning

Our choice of geometry and spatial reasoning was based on many factors. Initially, we were looking for a curriculum-based math focus that would allow students to engage in visual/spatial ways of doing math. We were aware that geometry is an underserved area of study in early years classrooms (e.g., Sinclair & Bruce, 2015; Van den Heuvel-Panh uizen and Buijs, 2005) and, while geometry is inherently spatial, typically geometry instruction in early years is limited to sorting and labelling shapes (Clements and Sarama, 2011).

Research from cognitive sciences also contributed to the rationale for our spatial focus. Mix and Cheng (2012), among many others, have confirmed the close link between spatial reasoning and mathematics performance and have, for example, shown that the ability to visualize, to engage in perspective taking, and to rotate figures mentally, not only predicts overall mathematics abilities, but also success in other school subjects as well. Furthermore, while it has been believed that spatial reasoning is a fixed talent, there is now conclusive evidence that spatial reasoning is malleable and can be improved with practice and training in people of all ages (Uttal et al., 2013). And finally, our reason for focussing on geometry and spatial thinking relates to student motivation and interest, and is linked to what Sinclair (2001) refers to as, "the aesthetic dimension of student interest" (p. 25).

The collaborative creation of lessons, activities and teaching materials

Because there was a scarcity of teaching materials that focussed on spatial reasoning and geometry for young students, our work with our M4YC teams increasingly focussed on lesson design. Following the model of Japanese Lesson Study, all of the lessons and activities were co-created by the participants and put through a careful iterative process involving the implementation of these new lessons in many classrooms and settings, followed by critiquing and reflecting, revising, testing and re-testing. Although the content of the lessons and activities were often above typical grade level expectations, the design of the contexts, inviting guided playful pedagogy and thoughtfully designed scaffolds, made these lessons accessible and were enthusiastically taken up with our diverse K-2 students.

For example, in one lesson, very popular with students from all grade levels, entitled *The Magic Keys*, the students were charged with finding the 12 unique shapes, that can be composed with five squares and that comprise the full set of pentominoes. This is a considerable challenge that involves children grappling with concept of congruence, while also visualizing and conceptualizing the transformations of rotation and translation. In the second part of this lesson, once the 12 pentomino shapes have been discovered, the students were asked to visualize and then justify which of the 12 pentominoes, when folded, could make an open box – thus adding more visualizing, in this case from two- to three-dimensions.

Indeed, all of the lessons that were developed in the M4YC project involved significant challenges and sophisticated mathematics: *The 3D Cube Lesson*, for example, challenged students to find and construct the full set of unique shapes comprised of five cubes, requiring mental and physical transformations in three-dimensions; The *Symmetry Game* involved the students in recognizing and creating reflectional symmetry around vertical and horizontal axes; *The Tile Lesson* provided students with spatial approaches to grid structure for area measurement; and, The *Garden Patio Lesson* focuses children's attention on composing, decomposing, and transforming area. Each of these lessons, it should be emphasized, were designed to be taught in playful, inquiry-based ways combining play and specific curriculum goals (Fisher, et al, 2013).

In addition to the lessons, the teachers and researchers also designed a series of quick image activities that focussed on supporting and enhancing children's spatial thinking as well as reinforcing geometry concepts. Designed as brief (10-15 minutes) and easy-to-implement challenges, the quick challenge called for short bouts of intense visual-spatial attention from the students. These activities included drawing, building, copying, and visualization exercises. The main aim here was to develop the children's ability to engage in various features of spatial visualization, including the ability to generate, recall, maintain, and manipulate or transform visual-spatial information in mind and with the aid of manipulatives. For a full description of these lessons and quick image activities (Moss et al., 2016).

MATH FOR YOUNG CHILDREN NORTH WEST (2013 – PRESENT)

In our third year of the M4YC project, we were invited by a rural school district in Northwestern Ontario to work with teachers and students in schools serving a high percentage of Ojibwe students from neighbouring First Nation communities (Nigigoonsiminikaaning First Nation, Seine River First Nation and Naicatchewenin First Nation). The majority of students travelled by bus to attend these public schools. The school board was interested in being part of a research study that could provide a better understanding of the potential of this spatial geometry approach to early mathematics learning for their students. The school district had in place First Nation instructional leaders involved with curriculum development, as well as Educational Counsellors from each First Nation community whose role it was to act as a liaison between community and school. We welcomed these new partnerships and the opportunities to learn from First Nation communities.

Community Involvement

There were a number of ways that the research team and the various communities came together. Elders welcomed the research team to the communities with a traditional fish fry. In the words of an educational counsellor from one of the First Nation communities, "We wanted to make sure that the researchers knew the communities the children came from even before the math work was to begin." Other invitations were the "Fall Harvest" an annual gathering for local schools in which

Elders and community leaders share important traditions and cultural practices enabling us to develop an awareness and understanding of Indigenous histories and perspectives. The Family Math Nights, which included a fish fry, local drummers and an Elder-led ceremonial opening, were co-planned with First Nation communities and included both culturally relevant and school-based math activities. As our time with the project continued, new connections were forged with the communities.

The PD Process and the Educators' Involvement

Our PD process in the Northwestern Ontario school district, while retaining much of our initial PD design, outlined above, involved several significant differences that reflected the particular context. However, an important difference was the composition of this new professional learning community we have come to refer to as the NW Team. As in our previous work in M4YC, we included teachers, principals and math coaches, along with the research team, but as mentioned above, also included were the First Nation early years consultant, the First Nation curriculum coordinators, the school board's Anishinaabe Language teacher and the education counsellors from the four First Nation communities in the district. This was the first time that this group of educators had been invited to a mathematics professional development process in the school district and this turned out to be a significant factor in building new connections between the school and its First Nations communities. Important features of the earlier PD model remained intact. These features included a focus on teacher-led clinical interviews (Ginsburg, 1977) and the design of exploratory lessons, both of which, as we had learned (Moss et al, 2015) encouraged teachers to listen more closely to their students, and to be better able to interpret and analyse their students mathematical reasoning – generally, to become better able to relate to their students. The PD process involved seven full day meetings with three visits by the M4YC team and two interim meetings on Skype.

Table 1: A brief overview of a record of events of the PD sessions.

Sessions	Record of Events
Visit 1: Day 1	Team engage in geometry and spatial reasoning challenges. Teachers are introduced to research on spatial reasoning and geometry, and conduct clinical interviews.
Visit 1: Day 2	Teachers and researchers co-teach selected lessons in classrooms from our full team observations. Teachers choose quick image activities to try with small groups.
Skype Day 3	Each teacher shares and presents examples of lessons field-tested in their classrooms. Team plans new lesson collaboratively, which is implemented in real-time via Skype. Team reflects and begins to design further lesson.
Visit 2: Day 4	Team works on more geometry and spatial reasoning challenges, teachers present further examples of lessons they have adapted and designed.
Visit 2: Day 5	Team introduces new lessons and new quick image activities, again co-teaching in a variety of classrooms.
Skype Day 6	Each teacher shares and presents examples of lessons they field-tested in their classrooms. Team plans new lesson collaboratively which is implemented in real-time via Skype. Team reflects on lesson and plans next steps.

Visit 3: Day 7	Team works collaboratively to document and communicate the professional learning process and to create a bank of new resources for other educators.

RESULTS: HOW DID THE M4YC PROJECT WORK IN THE NORTHWEST?

In this section, we first present qualitative results pointing to the effectiveness of the project and then present a brief overview of quantitative results of quasi-experimental studies we carried out to assess change in the children's mathematics learning over the course of the project. We then describe the way the DGC spread in the communities and the invitations the M4YC team has received to expand the work.

As researchers, we spent a substantial amount of time in each of the Kindergarten to Grade Two classrooms, either co-teaching lessons, or observing children working in small group activities and engaging in quick image activities. What became evident was that this spatial approach, the DGC, suited the children who were highly engaged and appeared to rise to the challenges the activities presented. The children proved capable of engaging in transformational geometry, in visualizing and conceptualizing congruence in three dimensions, mathematics not usually addressed in early years classrooms. Many of the teachers noted that those children whom they had viewed as "lower achieving" appeared to flourish with this new approach.

While it was widely recognized that the children enjoyed and seemed to thrive academically and emotionally with the DGC approach, there was, as well, a collective interest by the team, the school district and the communities to learn more about the effectiveness of the DGC on students' developing mathematical understandings. Each year, we tested the Kindergarten to Grade Two children on a range of measures at the beginning and the end of the school year, and we compared their progress to comparison groups of children (who in turn would become part of the experimental group in the following year and whose teachers would participate in the M4YC PD).

Our pre- and post-measures included tests of geometry and spatial thinking, numeration and arithmetic as well as KeyMath, a standardized curriculum-based math achievement test. The details of this research are beyond the scope of this paper, (see Hawes, et al., submitted), but briefly, each year the results were extremely impressive and followed a similar pattern of achievement. While we expected that the students who participated in the DGC would make good gains in both geometry and in measures of spatial reasoning, what was unexpected were the significant gains that these students made, in comparison with the control group, in areas of mathematics not emphasized in the project, including basic numeration, arithmetic, and problem solving. Furthermore, our testing also showed that the students' scores were well above the expected Canadian norms, a very strong achievement given that the schools placed at the lower end of the provincial standardized math scores (Hawes et al., under review).

Maybe more than any other indicator of success of the project was how the M4YC approach was incorporated and adapted in new ways by the participating educators in First Nation communities. First, the day cares in each of the First Nation communities incorporated M4YC activities into their programs, creating a first ever math focus in their early years day care programs. Second, in some communities, the DGC became the focus of after-school programs for school-age children. Third, Family Math Nights have served as a model and annual event for other schools and communities in the district. (Please see Caswell et al 2013 for video link to Family Math nights.) Finally, the M4YC NW project, which began in 2013 in two schools, is now a collaborative endeavour in all eight elementary schools in the district. Maybe, most significant, however, in terms of reach, was the invitation in 2015 to become partners with a federal First Nation education authority, Seven Generations Education Institute's First Nation Student Success Program to work collaboratively with K-3 teachers and indigenous educational leaders in four First Nation federal schools.

Implications/Speculations

Given the success outlined above, we speculate on a number of contributing factors. In our initial urban work, we analysed how the M4YC process of PD supported teachers to gain a deeper content knowledge and broader conceptualization of geometry and spatial reasoning and, importantly, how the project empowered them as designers, even as researchers (Moss et at 2015). This has also been a feature of our work with the NW Team. As Kincheloe & Steinberg (2008) remind us about collaborative professional development and research in partnership with Indigenous communities:

> PD should produce new levels of insight amongst the participants; in particular, that PD should demand that educators at all academic levels become researchers.

We have also analysed why the DGC was appropriate for the young mathematics learners, particularly those who might otherwise have struggled with the subject. This list includes how a spatial approach: provides multiple entry points for the learners; proves very motivating; involves embodied experiences; and for many children their spatial reasoning and awareness develops in advance of their number sense.

Finally, the fundamental factor when we look across all of the work is that of establishing relationships. First, our PD model and "processes of partnering" (Bang, 2016) were structured to be collaborative and non-hierarchical and allowed for genuine relationship building between researchers, teachers, and community members. The PD model with the inclusion of clinical interviews and lesson design afforded teachers new ways of relating to and understanding their students.

Our work differs from that typically considered "culturally responsive." Our goal was not to mathematize cultural practices but rather to form a collective in which every member of the team could bring expertise to the table. A significant contribution from Jason Jones, the Anishinaabemowin (Ojibwe language) coordinator for the district, was his creation of a new word for math that encapsulated the PD process: *Gaa-maamawi-asigagindaasoyang*, meaning, “Gathering to learn and do mathematics together.” The inclusion of the Anishinaabemowin teachers in the math PD was a first for the district and offered a chance for non-Indigenous members of the team to become aware of the verb-based features of the language (Lunney Borden, 2009) and, more importantly, to show solidarity in the struggle to revitalize and reclaim Anishaabemowin that was taken away from First Nation peoples during the era of cultural genocide.

Elder Mike Kabatay of Seine River First Nation, on reviewing our PD process and viewing the research results, remarked, “You’ve reawakened something that is already in our children.” This statement reflects his understanding of a way of thinking that was repressed as part of the residential school era and that is now in the process of being “reawakened.” Elder Mike Kabatay’s statement also reflects how the focus on visual spatial mathematics created a space for children to engage with math in a way that they may not have previously experienced. Former National Chief of the Assembly of First Nations, Chief Shawn A-in-chut Atleo referred to our project in terms of reparation between community-school relationships, “This *is* reconciliation.”

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