RECONFIGURING MATHEMATICAL SETTINGS AND REPRESENTATIONS THROUGH WHOLE-BODY COLLABORATION

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We examine consequences of whole-body, multi-party activity for mathematics learning, in the contexts of number sense and ratio and proportion. Drawing on micro-ethnographic techniques, we compare two cases of whole-body, collaborative movement in mathematics activity. Informed by contemporary theories related to social space and embodied cognition, we illustrate how whole-body collaboration might transform how students experience, make sense of, and make use of spaces of learning. The analysis enriches our understanding of the changing spatial landscapes for learning and doing mathematics as well as how re-instating bodies in mathematics education can open up new forms of collective mathematical sense-making and agency.

INTRODUCTION

This study examines the consequences of whole-body, multi-party activity for mathematics learning, both in and out of the classroom. In particular, we report on and compare two cases of implementing whole-body, collaborative movement activities designed to engage learners in the mathematics of number sense and ratio and proportion respectively. By investigating how learners made sense of these mathematical concepts through physical action and interaction, we illustrate how whole-body collaboration can transform mathematics activity and learning, and shift aspects of disciplinary agency, embedded in representational infrastructure, to students' collective activity. The paper also contributes to research that seeks to expand sociocultural lenses on mathematics learning to take into account bodies and place.

To investigate these issues, we bring together (a) scholarship that attends to spaces of learning as both productive of and produced by human activity and (b) contemporary theories of embodied mathematical cognition that view mathematical thinking and learning as inseparable from body-based action, interaction, and experience. Attending to the dialectical dynamic between embodied interaction and place-making, we consider how whole-body, collaborative designs can disrupt representational infrastructure in a way that repositions learners in relation to mathematical content. We bring this theoretical focus on place, embodiment, and representational infrastructure into dialogue with two cases of whole-body, multi-party mathematical activity. In the first case, middle-school children in the context of special school programming participate in a series of Walking Scale Number Line activities taking place in the school's gymnasium. In the second case, elementary students during their regular mathematics class participate in a ratio-and-proportion activity called Whole and Half. In both cases there are deliberate designs to disrupt more typical uses of space and bodies in relation to school mathematics learning.

In what follows we introduce the theoretical perspectives we are bringing together with respect to spatial production, embodied cognition, and representational infrastructure as they relate to learning mathematics. We then summarize our research methods, followed with our analysis of each of this study's two cases. Finally, we discuss these findings together, focusing on how whole-body, multi-party activity can influence spatial production and the relationship between learners and relevant mathematics.

SPACE, EMBODIMENT, AND REPRESENTATIONAL INFRASTRUCTURE

In this paper we take up the argument that the physical spaces of learning should not be treated as static boxes waiting to be filled with human activity, but instead as complex, historically constituted, dynamically experienced, and socially produced settings (Leander, Phillips, & Taylor, 2010). In their study about arithmetic and grocery shopping, Lave and her colleagues (Lave, Murtaugh, and de la Rocha, 1984; Lave, 1988) noted that spaces do have durable material arrangements with design histories situated in some social, economic, and political context of use. They called this the *arena*. The arena, however, is experienced differently by individuals engaged in activity. They called this experienced space the *setting*, and argued that setting and activity are dialectically constituted, in the sense that "the setting both is generated out of grocery-shopping activity and at the same time generates that activity" (Lave et al., p. 73). Ma & Munter (2014) built on this relationship to consider how individuals, interacting together in activity, socially produce spaces, positing, in parallel with the dialectical relation between individual activity and setting, an analogous relation between collective activity and socially produced space.

In examining the dialectical relationship between collective activity and socially produced space, we deliberately focus on embodied aspects of that activity, including physical action, interaction, and experience. This choice is motivated by contemporary work in embodied cognition, communication, and experience, particularly as it applies to mathematics education. In particular, in this study we take an "interactionist" (Stevens, 2012) and "nondualist" (Nemirovsky, Kelton, & Rhodehamel, 2013) view of mathematical embodiment, understanding doing and learning mathematics as the body's (varyingly overt or covert) activity in its environment. We focus on how mathematical cognition is distributed across actors, material artifacts, and dynamically unfolding bodily activity (Hutchins, 2010).

From this perspective, the dialectical relationship between activity and socially produced space is necessarily infused with corporeal action, interaction, and experience. The multiplicity of space, of coexisting trajectories and stories-so-far (Massey, 2005), is a multiplicity of embodied selves under production, together in activity. Learners' bodies traverse these spaces individually but also in relation to each other, making up part of the dynamic material landscape while simultaneously producing the activity. A motivating principle for this study, then, is that possibilities for mathematical meaning-making are generated and constrained by this dynamic co-production of bodies and space, and our aim is to examine how deliberately novel deployments of whole bodies can create new opportunities for mathematics learning.

The whole-body designs in our two cases result in disruptions to representational infrastructure, the tools that allow representations to be produced, recognized, organized, manipulated, and interpreted. Examples of representational infrastructure in typical school mathematics include algebraic notation and the Cartesian Plane. In both cases in this study, number lines and intervals are key aspects of representational infrastructure that undergo destabilization and creative transformation in the context of whole-body, multi-party activity. A defining feature of infrastructure is that, when it works, it is invisible, or transparent to users (Star & Ruhleder, 1996). However, for learners, representational infrastructure is both a tool for learning and an object of learning (Hall & Greeno, 2008), or "simultaneously transparent and opaque" (p. 58, Kaput, Noss, & Hoyles, 2001). Kaput and colleagues (e.g., Kaput, Noss, & Hoyles, 2001) have argued that much of school mathematics is built on notation developed for the use of an "intellectual elite," advocating for developing more accessible representational systems. Our case comparison follows this line of reasoning by investigating two settings where representational infrastructure has been disrupted, then at least partially reconstituted to include whole bodies in interaction.

We take representational infrastructure to be socially constructed, historically sedimented, and flexibly used in local practice (Hall, Stevens, and Torralba, 2002). Breakdowns in representational infrastructure may provide analytical leverage for making infrastructure and local practices visible. At the same time, disruptions to representational infrastructure may open up possibilities for learning (Hall & Jurow, 2015). Ma (2016) built on this work, considering how a designed disruption to representational infrastructure might support conceptual agency by allowing geometry students to develop their own tools and routines using everyday objects and their own bodies for constructing large scale geometric objects (e.g., quadrilaterals) and relations (e.g., congruency). The case comparison presented here further investigates how learners might take up disruptions to representational infrastructure that involve whole bodies in interaction, as well as the consequences for mathematics learning.

METHODS

We draw on video recordings of learners engaged in the whole-body, multi-party activities in the contexts of Walking Scale Number Lines and Whole and Half. In both cases, we employed techniques from micro-ethnography (e.g., Streeck & Mehus, 2005) and multimodal interaction analysis (Jordan & Henderson, 1995) to understand how learners participated in and made sense of these activities through detailed sequences of talk, physical action, and socio-material interaction. We began with individual case analyses, focused on describing the emerging representational infrastructures and attendant mathematical activity with respect to the study's spatial, embodied lens. Then, taking a case-comparison approach (Hall & Horn, 2012), findings were put into conversation to bring into relief relationships among bodies, settings, and developing representational infrastructure. The study will be presented in this way below, to familiarize readers with individual case analyses before discussing their comparison.

WALKING SCALE NUMBER LINES

Our first case follows a group of students into their school gymnasium for special programming designed and provided by a dance educator, Malke, and a math educator, Max. The two, in conversation with other math educators and researchers (including the first author) planned activities that would place students as points on a giant number line represented by tape stretched across the gym floor, what we eventually began to call a Walking Scale Number Line (WSNL). Five groups of students between grades 2-8 experienced the activities over the course of two days, and Malke and Max revised their design after each group. Here we focus on a group of seventh and eighth graders.

The eleven students in this group gathered on a blue number line taped across the diagonal of the gym with evenly spaced yellow hashes. They were asked to choose a "home" position by choosing a yellow hash mark and taping their name tags in front of it. Students began by moving five units to their right, then two units to their left. Malke asked them where they were in relation to where they started, then Max pointed out that one of the students, Thad, was in the "exact middle" of the line (and, by design, the middle of the gym, as indicated by the basketball line markings and a picture of the school's mascot on the floor). This led to a sequence of dilation tasks where students were asked to double, triple, quadruple, and quintuple their distance from Thad. Finally, students were asked to identify their "opposite" if they had a student opposite, or just to name it if there was no student at that spot. The whole group was then tasked with finding a strategy of getting everyone to their opposites simultaneously in some efficient and safe (i.e., no crashing bodies) manner.

The WSNL setting placed material arrangements of the gym and students' past experiences in the gym into interaction with "familiar" mathematical tools (students were all familiar with number lines on paper). The open space of the gym typically used for play, competition, and performance was transformed by tape and the designed WSNL activities. The floor of the gym, painted with lines for basketball and four square, was temporarily augmented with number lines of brightly colored tape running parallel to the long wall and one long diagonal blue line, the "paper" for students' representations and problem solving. The gym arena along with students' moving bodies took on new meanings in the context of WSNL. Students' bodies became meaningful aspects of representational infrastructure for themselves and each other, beyond individual quantities moving and operating along the number line. Quantitative relationships were understood and talked about as spatial relations between students' home positions and bodies. They tracked their walking and described their locations as "two thingies over. From where I started," or "I'm where [Thad] was."

The familiar representational infrastructure of number lines was newly materialized as large-scale walkable physical phenomena embedded in the gym floor, tacitly agreed-upon attributes (of number lines and of the gym) no longer so readily available. Left and right, negative and positive were experienced variably, depending on individuals' embodied orientations in relation to each other; even "to Thad's left" became problematic as soon as Thad turned around to face the other way. Static aspects of the space (the wall, the stage) became stand-ins for direction (left, right), taking on mathematical meanings in the service of performing and describing operations.

Students' unique and distributed perspectives from their positions along the line also became resources for reasoning. Toward the end of the workshop the group discussed strategies for moving to their opposites all together without bumping into each other. Maggie, nine units to the right of Thad, thought they could all walk along the line, and when they encountered another person they would hold hands, lean back, and swing each other around (Figure 1, top). Thad suggested that if he held onto Morgan (two to his right) and Kian (two to his left) with either hand, he could just turn around and rotate them to their opposites (Figure 1, bottom). He then revised this to include the whole group: "Wait, we could all grab hands with each other, and then I spin around, and you spin around." Maggie and Thad solved the problem from their respective physical and mathematical perspectives in the material arrangements of the space—Maggie from nine units to the right of Thad, needing to get to nine units to the left, and Thad needing to stay put but have everyone on either side of him swap to the other.

Figure 1. Top row: Maggie (second from left) demonstrates holding Theresa's (far left) hands and swinging her around to switch places on the line. Bottom row: Thad (third from left) demonstrates holding Morgan and Kian's (on either side of him) hands and spinning around to place them in each other's home spots (their opposites).

In sum, in WSNL designers physically modified the arena (school gym) with tape to produce a large-scale version of the familiar number line. Together, students' bodies operated as quantities on the line, performing displacements, dilations, and an "opposite" routine. As students engaged in the tasks, and instructors responded to them, they developed new meanings for their bodies, the arena, and the spatial relations among them. Individual and distributed perspectives contributed to this meaning-making, and as the new representational infrastructure of the WSNL emerged, particular forms of mathematics became available. We next describe how representational infrastructure emerged in a different multi-party, whole body design.

WHOLE AND HALF

Our second case examines the incorporation of whole-body, multi-party activity into a 5th-grade mathematics classroom in 'North Lake' Intermediate school. The activities we examine here take place in the context of classroom preparation for an upcoming visit to a museum exhibition about ratio and proportion. On the day before the field trip, in anticipation of the exhibition's emphasis on physical movement and kinesthesis, 5th-grade mathematics teacher Ms. Collins assigned her students a suite of classroom tasks involving collaborative physical movement. We focus here on a pairs task called Whole and Half (W+H). To play W+H, one person creates an interval of space between two hands, or one hand and the floor. The second person must respond by placing a hand halfway between the ends of the interval. As Whole varies her hand placement, Half must keep up by moving her hand accordingly. Players can vary the game by alternating who plays Whole or Half or by experimenting with different proportions.

After introducing W+H, Ms. Collins launched the activity by directing students to "get out of your seats and start working," a directive that indexed how the activity of W+H entailed a marked reconfiguration of the routinely practiced space of the classroom. To meet the practical demands of whole-body collaboration, the students needed to de-center their mathematical activity away from its usual locus in the classroom's tightly packed rows of desks and toward atypical regions: spatial margins between the desks and the walls, a cluster of goldfish tanks used for science class, and the corridor in front of the desks, typically occupied by Ms. Collins, that houses the Smart Board.

Similar to WSNL, W+H recruited participants' bodies and body parts as meaningful components of a representational infrastructure in which (a) W's bimanual hand positioning embodied an interval-like whole, (b) H's single hand represented a half, and (c) the spatial relationship between W's and H's hands created a multi-party, body-based instantiation of a part-whole quantitative comparison. Expressing and holding constant a part-whole relationship became both a matter of intricate social and embodied coordination. Interactional breakdowns made particularly visible how participants were incorporating multiple bodies and the dynamic spatial relations among them into representational infrastructure. For example, just after Ms. Collins's directive to "get out of your seats and start working," Katie lingered at her desk making notes while her partner, Claire, skirted around her desk to the front of the room and, taking on the role of W, positioned her hands to materialize a diagonal whole in front of Katie's desk. But Katie, still writing in her notebook, left Claire hanging for about a quarter of a minute. Holding her hands still to keep the diagonal whole interval in place, Claire waited for Katie, growing increasingly impatient, re-iterating the activity's directive in physical terms ("stick your hand in between it"), and urging Katie to hurry up ("come on Katie"). This brief interactional breakdown was simultaneously a breakdown in representational infrastructure; without Katie's cooperation, Claire's whole lacked its comparative half and she could not complete the task.

Once Claire had elicited Katie's collaboration and the breakdown had been provisionally repaired, each student took a turn as W, producing sequences of bimanual intervals to which her partner, H, responded. Like many of the North Lake students, Claire and Katie discretized the activity, with W posing staccato progressions of intervals to H as a sequence of punctate tasks or challenges. Within this game-like appropriation of W+H, the students enacted progressions of representational innovations, leveraging bodily capacities and limitations as resources for authoring and revising an emergent representational infrastructure to heighten and diversify possible challenges. For instance, early in their engagement with the activity, after Claire had produced four different Wholes, Katie observed somewhat plaintively, "I'm not really having to move my hand very much." A few turns later, the students switched roles and Katie embodied a sequence of wholes that were progressively more challenging scenarios for Claire-as-Half. For example, taking advantage of Claire's finite reach, Katie positioned two interval wholes asymmetrically with respect to the median plane of her own body so as to be just out of reach for Claire (Figure 2a-b).

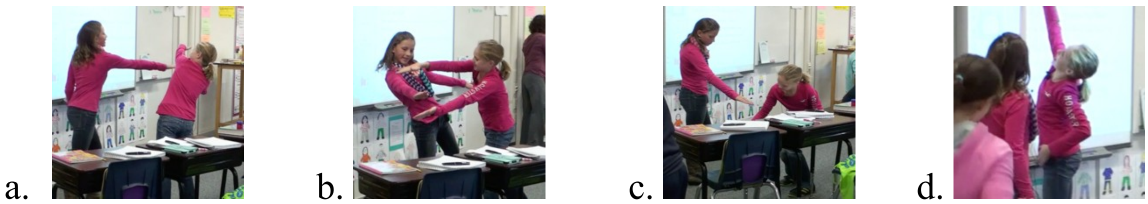

Figure 2. a-b: Katie-as-Whole (right) creates a W sequence that challenges Claire's (left) reach. **c**: Claire-as-Whole (left) uses her right hand and the floor to create W, treating the floor as the other end of the interval. **d**: Katie-as-Whole (right) indicates the top of the Smart Board as one end of W, while treating the floor as the other end.

To produce these hard-to-reach wholes, Katie not only made use of hers and Claire's physical possibilities and limitations, but did so in a way that opportunistically leveraged the newly "free" space around her body. Access to this space was facilitated by the spatial disruption to representational infrastructure. Displaced from the confines of desks, newly mobile intervals could occupy and incorporate alternate corridors and materials of the classroom arena, as students transformed mundane features, such as the floor (Figure 2c) or the upper edge of the Smart Board (Figure 2d), into meaningful components of a mathematical representation.

To summarize, W+H recruited students' bodies as components of dynamically shifting interval representations of part-whole quantitative relationships. Playing Whole and Half disrupted the routine spatial practices of the classroom, as participants relocated mathematical activity to new classroom regions and flexibly incorporated eclectic material features of the classroom into representations of part-whole relations. Participants creatively leveraged new possibilities for – and constraints on – physical movement in relation to the environment in order to make innovations and elaborations on the emergent representational infrastructure.

SELECT COMPARATIVE THEMES

We now bring these two cases together by highlighting select themes that emerged from comparative analysis: (a) the dynamics of friction and augmentation in spatial disruptions to learning environments, (b) ecologies of mobility and durability in disrupted representational infrastructures, and (c) the consequences of whole-body collaboration for learners' mathematical agency. First, both WSNL and W+H involved re-purposing the arena in which they took place. Yet, while WSNL deliberately capitalized on the histories of participation associated with the gymnasium (whole-body movement and performance), W+H was taken up in salient contrast to histories of classroom practice, attendant embodied and spatial routines, and the material arrangements of the classroom that both indexed and enabled those routines. Thus, the WSNL case predominantly made visible the ways in which whole-body, collaborative design can intentionally highlight, leverage, and augment a setting for doing and learning mathematics. W+H, on the other hand, made salient how this kind of design can lead to meaningful contrasts—or induce friction—with the built environment of the mathematics classroom and associated sedimented histories of embodied spatial practice. Together, these cases illuminate how both spatial friction and augmentation may be present in educational designs that disrupt the space-activity dialectic.

Second, both cases illustrate how whole-body, collaborative designs reconfigure the environments to which they are introduced, with important consequences for the representational infrastructure of learning and doing mathematics. Because the dialectical relationship between embodied activity and setting can influence possibilities for representing mathematical concepts and processes, disruptions to the activity-space dialectic are simultaneously disruptions to representational tools and practices. Thus, in both cases, representational infrastructures were disrupted and reconfigured to incorporate whole bodies, body parts, and new regions, materials, and features of the arena. Yet, the resulting reconfigured infrastructures were comprised of remarkably different material ecologies. While in both cases students' bodies were deliberately recruited for the mathematical content of the activity, our analyses unpack how bodies played significantly different roles in the emergent representational infrastructures. In WSNL much of the infrastructure was determined by the tape on the gym floor and, as a result, had a relatively immobile and durable quality. Bodies as points along the line became the dynamic part of the infrastructure and the possibilities and constraints for making sense of numeric operations hinged on the negotiated interplay between the static, durable frame of the tape-augmented gymnasium floor and collective physical movement. In contrast, in W + H, interval boundaries were not durably congealed but, rather, were partially constituted by moving hands such that the emergent representational infrastructure was less fixed to any one particular aspect of the classroom arena. In other words, in W+H, where the bodies went determined where the mathematics was. As intervals became unfixed and re-tethered to moving bodies, performers of Whole quickly and flexibly re-oriented (e.g. turn diagonally), re-scaled (e.g. stretch or shrink), and translated (e.g. move to the right) interval boundaries. Leveraging a newly mobilized representational infrastructure, students spontaneously and opportunistically incorporated material elements of the arena into dynamically changing intervals, producing a setting in which unexpected regions of the classroom might suddenly become salient and saturated with mathematical significance. The distinct representational ecologies we find in WSNL and W+H highlight two possibilities for how multiple bodies might play a part in representational infrastructure: as mobile parts framed by a materially stable, designed space, on the one hand, or as constituting the entire representational tool, on the other.

Finally, these analyses illustrate how whole-body, multi-party activity can create different kinds of opportunities for conceptual agency, the nature and extent to which learners are positioned as genuine authors or creators of mathematical ideas. In both cases, the incorporation of learners' bodies into representational infrastructure physically positioned learners as mathematical objects and learners' physical movements as mathematically significant operations or events. Because of this, students' repertoires of bodily movement—their possibilities for, constraints on, and histories of physical action—became resources for mathematical invention. Even the subtlest of bodily movements (such as changing the orientation of a palm in W+H) could be taken up as mathematically meaningful and incorporated into the local development of representational infrastructure. And, as the infrastructure shifted and evolved, participants differentially selected from these embodied repertoires (e.g. standing up on tip-toes or linking arms with a partner and spinning around) to author and negotiate new representational forms. Thus, we suggest that both activities collapse—or at least trouble—distinctions we might make among disciplinary, conceptual, and material agency (e.g. Pickering, 1995) in these contexts.

CONCLUSION: RE-CENTERING SPACES, BODIES, AND MATHEMATICS

This study represents an attempt to foreground and interrelate spatial and embodied perspectives on mathematical thinking and learning. In particular, we pieced together a framework that views mathematical representational tools and practices as emergent from a dialectic between embodied activity and interaction, on the one hand, and the social production of space, on the other hand. Using this framework, we drew on micro-ethnographic and case-comparative techniques to investigate how whole-body, collaborative activity can create new meanings for physical movement and interaction while simultaneously transforming how learners experience, make sense of, and make use of the spaces in which these activities unfold. Broadly, this study aimed to contribute to an understanding of the changing social spaces—both in- and out-of-school—for learning and doing mathematics as well as the detailed consequences of re-instating embodied physicality for mathematical thinking, learning, and agency.

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