PEDAGOGICAL AND MATHEMATICAL CAPITAL: DOES TEACHER EDUCATION MAKE A DIFFERENCE?

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This paper explores teachers' pedagogical content knowledge and mathematical content knowledge for both primary and secondary school teachers. Using established questions from other reviewed research, the research team posed a series of tasks for teachers to complete. The results surprised the researchers as they contradicted our expectations. Primary teachers scored lower on both Pedagogical Content Knowledge (PCK) and Mathematical Content Knowledge (MCK) while secondary teachers scored (significantly) higher on both scales. Preservice teachers scored similarly to primary teachers. To explore this further we subsequently added a further cohort – engineers who were expected to have strong mathematical knowledge but little to no pedagogical knowledge. The results reported pose serious questions in the current political and employment contexts. The data suggest to us the capital building that may be facilitated through teacher education programs needs to be questioned as do many of the assumptions that permeate the field.

TEACHER KNOWLEDGE

Creating an understanding of what makes for a good teacher in mathematics is a very vexed question. In a forum such as MES, there is a heavy emphasis on issues around access and equity – how do teachers cater for the diversity found in contemporary classrooms? Many researchers in this forum take issue with how teachers create learning contexts that enable students from socially diverse backgrounds to be able to access and succeed in mathematics. This paper digresses from this literature to focus critically on dimensions of teachers' knowledge that dominate the field - content knowledge and pedagogical knowledge. These two concepts have been appropriated by researchers in the field who have assigned various terms to the two broad constructs. There is a vast literature in this area and with competing findings. The research described in this paper raises some serious questions that we seek to pose based on the outcomes of part of a large research project conducted in Australia. Teaching mathematics is a complex process that goes beyond the simple collection of activities that keep students busy or engaged. Teachers need to have a complex contingency of sophisticated professional knowledge that unites the knowledge that must be taught/learned and effective ways in which that knowledge can be created for students. Knowledge of the discipline and effective and quality pedagogical practices are the touchstone to high quality teaching and learning.

Mathematics education, as a field (Bourdieu & Wacquant, 1992), has certain discourses and practices that are widely accepted as cultural truths. Some practices, over time, gain certain credibility and ultimately operate as powerful truths within the field, conveying power to those who accumulate and promulgate such truths. Within

the field of mathematics education there is a large literature around the knowledge, practices and beliefs that teachers hold in relation to the teaching of mathematics. For example, a google search for mathematics pedagogical content knowledge yields 763 000 hits, while mathematical content knowledge yields 16 700 000 hits. Much of this writing has been founded on the seminal work of Schulman (1986) where he made the clear distinction between what to teach (discipline knowledge) and how to teach that knowledge (pedagogical knowledge). His work has been taken up in mathematics education, with heated debate in contemporary times as to the importance or not of discipline knowledge. There has been an historical and on-going debate as to the primacy of either discipline knowledge or pedagogical knowledge. This is most evident in the maths wars in the U.S. (Schoenfeld, 2004) where mathematicians predominantly declare the primacy of discipline knowledge as essential to good teaching in mathematics. Other mathematics educators such as Boaler (2002) advocate strongly for the pedagogical knowledge of teachers. In their comprehensive study of the importance of mathematical knowledge, Hill and colleagues (Hill, Blunk, & Charalambous, 2008) argue for the importance of mathematical knowledge in fostering and supporting quality instruction in mathematics. At the same time, they recognise that this is mediated by other factors, one of which is pedagogy. There is considerable tension in the field as to the importance of the two constructs. Increasingly researchers are proposing other terms to reflect nuances within these broad constructs. It is beyond the scope of this paper to provide a comprehensive account of this literature so a broad albeit condensed overview, recognising the limitations of a conference paper, is undertaken.

The importance of teachers' mathematical content (or discipline) knowledge has been linked to student achievement (Hill, Rowan, & Ball, 2005) where it was found that there was a positive relationship between teachers' knowledge of mathematics and student achievement. Part of the reasoning for this relationship is that teachers with a deep knowledge of mathematics are better able to see relationships and networks in mathematics and build the mathematical understandings in their students. In contrast, in a large study of German teachers, researchers (Staub & Stern, 2002) found that not only was pedagogical content knowledge important but teachers' beliefs impacted significantly on the end performance of students. These authors found that teachers taking a particular pedagogical approach (cognitivist constructivist orientation) produced better outcomes than teachers with other approaches in the pedagogy.

Ball and Bass (2000) have suggested that pedagogical content knowledge often consists of routines and practices that are commonly used across mathematics – such as ways of teaching number, fractions and integers – which produces regularities in the teaching approaches commonly used in schools. However, they contend that much of teaching mathematics is also uncertain. For teachers to be able to cope with this uncertainty, they need to have strong discipline knowledge. It is one thing to know the regularities, the 'tricks', in teaching mathematics, but it is another thing to know how to deal with the uncertainties and application of mathematics.

In an international study of graduating preservice teachers (Blömeke, Suhl, & Kaiser, 2011) it was found that there were remarkable differences in how countries prepared their prospective teachers in relation to pedagogical knowledge and content knowledge in mathematics. They also reported that there were marked gendered differences in mathematical content knowledge (MCK) but not in mathematical pedagogical content knowledge MPCK). We (Lowrie & Jorgensen, 2015) have reported elsewhere the particular findings of our preservice cohort to include the intersection of pedagogical knowledge, discipline knowledge and teachers' beliefs as beliefs impact on teaching in profound ways.

Within this context, we draw on a number of Bourdieu's concepts to make sense of practices within the field of mathematics education. In the current context, at least in Australia, there is now a growing recognition that teachers must have strong discipline knowledge. There are now entry and exit requirements for preservice teachers who must demonstrate their competence in numeracy tests. A graduate who cannot pass the test (for literacy and numeracy) will not gain registration as a teacher. While this is somewhat contentious, it highlights a recognition that teachers should have fundamental knowledge in the disciplines – literacy and numeracy – that underpin the core work of educators. The field of teacher education is now recognising the importance of discipline knowledge. Teachers who can engage with the practices of the field vis a vis content and pedagogical knowledge and embody these practices into the repertoire of teaching skills into their teaching habitus are likely to be seen as better teachers or quality teachers. In so doing, these skills, as represented through their teaching habitus, can be converted to other benefits such as higher salary for being an advanced skills teacher, or assuming leadership roles in mathematics within the school and so on. Thus, these skills are embodied into the teacher habitus to become forms of capital that can be exchanged for other goods within the field – such as salary, certificates, status. One of the key features of teacher education, whether preservice or in-service, is to build the capital of the teachers so that they become valued members of the field. But fields are not static and change. At this point in time there are valued forms of knowledge that convey status (and capital) and these are centred around teacher knowledge which is seen to create quality practices.

This paper reports on the findings from part of a much larger study. Here we discuss the findings of a survey in which we explored teachers' knowledge of mathematics content and pedagogy. We anticipated that primary school teachers were more likely to have strong mathematics pedagogical content knowledge (MPCK) and would not be as strong in mathematics content knowledge (MCK), with the reverse being the case for secondary school teachers. The results surprised us. This paper is intentionally reflexive as we track through the data and the implications for our thinking about the relationship between MCK and MPCK.

METHOD

This paper is part of a much larger study funded through the Australian Research Council's Discovery Grant system (DP1200101495) and reports on the findings from the online survey. The survey was the first phase of the project where we initially sought to identify teachers' and preservice teachers' backgrounds in the teaching of mathematics. We sought to explore the backgrounds of teachers, where they taught, where they grew up, and their current levels of knowledge in terms of mathematics content and pedagogical content knowledge, as the larger project was concerned with the socio-geographic implications of mathematics education. In this paper we draw on the MCK and MPCK elements of the survey.

Participants

The participants are teachers from all over Australia. The teachers were invited to participate in the on-line instrument described below. Participants were solicited through various means that included approaching principals in schools to pass on the request for participation, advertising through social media including Facebook, advertising through various mathematics organisations, and through personal contacts through previous research and consultancy work. Teachers identified their teaching background via the profile section of the survey. Initially we only sought to include teachers and preservice teachers. As an addendum to the study, we included a cohort of engineers. The rationale for the inclusion of the engineers will be discussed later in the paper.

Participan t	Preservice teachers	Primary teachers	Secondary teachers	Engineers	Total
Number	162	100	139	31	432

 Table 1: Numbers of participants undertaking the survey.

The instrument

The 'Social and Geographic Dimensions of Mathematics Education' questionnaire comprised four main parts—the first is demographic data that broadly included location and qualifications; the second section contained 11 mathematical content knowledge questions (MCK); and the third section 9 pedagogical content knowledge questions (MPCK), while the final section focused on beliefs and dispositions to teaching mathematics. The second and third sections were timed sections, which supported an effectiveness and efficiency measure. This paper reports on data from section two (MCK) and section three (MPCK). The knowledge sections consisted of items sourced from national and international tests designed for middle high school attainment levels. All items were trialled by other researchers or assessment corporations (such as Australian Curriculum, Assessment and Reporting Authority, Australian Mathematics Trust, The Mathematical Association of America, and Educational Testing Service) so that we can assume reliability and validity of the test items, and thus can be used for comparative purposes. The test items have all been

published in peer-reviewed research papers within the mathematics education research community. Permissions were sought from authors and organisations to be able to use the test items in the survey. The selection of items was on the basis of ensuring a spread of content areas as well as a range of levels of mathematics. The mathematics content knowledge questions should be reasonably answered by junior secondary students, with some of the items being below this level. Questions were all multiple choice. For example, one question asked the respondents to select from four different representations of MABs of the number 32 which showed the underlying concepts of base 10 numeration (see appendix for test item).

FORMS OF CAPITAL

From here we discuss the MCK and MPCK as forms of capital that teachers have. We anticipated that primary teachers were likely to have more MCPK than secondary teachers as primary teachers are experts in pedagogy as they teach across many curriculum areas. Conversely, we anticipated that secondary teachers would have greater MCK than primary teachers as they (should) have been qualified in the discipline of mathematics whereas this is not the case for primary teachers. However, our results yielded a contradictory pattern which has resulted in us rethinking the results and building another dimension into the study, the inclusion of engineers. We write this next section as a reflexive piece as it was of considerable concern to the research team as to what we uncovered. To assist in the analysis of the data, we employed an external consultant to analyse the survey data.

As part of the survey, teachers were asked to rate themselves in teaching mathematics. On a scale of 1-10 where 10 was excellent and 1 was poor, 90.1% of the teachers rated themselves above 5, while 19.3% rated themselves on 9-10 on the scale. This suggests to us that the teachers, overall, had a very strong sense of themselves as being good teachers of mathematics.

In terms of their background in mathematics discipline, the level of mathematics studied at school varied between different teaching groups (p<.001). A majority of pre-service teachers had their highest level of maths at Year 12 General Maths (56.4%) while only 30% of primary practising teachers and 20% of secondary teachers reported highest level of maths studied at Year 12 General. Similarly, almost half (46.5%) of secondary teachers studied maths at Year 12 Specialised level, followed by primary teachers (34.7%) and last pre-service teachers at 8.0%.

MATHEMATICAL AND PEDAGOGICAL CAPITAL

The results for the survey are presented in the two areas – MCK and MPCK. The participants were assessed with 12 MCK and 11 MPCK questions. Each correct answer was given a score of 1, the responses were then summed up to give final score for each test domain. For each group of interest, we calculated the group's mean MCK and MPCK scores and the corresponding standard deviation. To identify if the mean between groups were statistically significant, we performed Turkey post-hoc tests. Initially we compare the MCK results for the primary teachers and secondary

teachers using a Turkey post-hoc test and found that there were significant differences between primary teachers (6.27 ± 3.12), and secondary teachers (7.57 ± 2.62) (p<0.015). This was not a surprise as we can see from the studies of mathematics reported in earlier research, primary school teachers were less likely to study high levels of mathematics whereas this is not the case for secondary teachers whose expertise should be in the area of mathematics. Our background data also supported this, so it was not a surprise to obtain these results.

When considering primary and secondary teachers' MPCK we were surprised to find a counter-intuitive finding. Here we found that secondary teachers scored significantly higher than primary school teachers where the Turkey ad hoc analysis showed that the difference between junior teachers (5.64 ± 2.27), to senior teachers (7.02 ± 2.12), showed a difference of 1.38, p<0.003. What was alarming for us was that this component had nine items. The mean score for the primary teachers was 5.64 which suggested that they were only, on average just reaching a pass equivalent – assuming that a pass is 50%. All of the items in the MPCK section related to concepts taught in the primary school curriculum. In contrast, secondary teachers, who do not teach these constructs, were able to obtain a mean score greater than the primary teachers despite not being responsible for teaching this content.

At this point, we questioned our results, wondering why primary school teachers were scoring significantly worse on both measures than secondary teachers, but more concerning is why the teachers were scoring counter intuitively on MPCK. We questioned how teacher education may have been implicated (or not) so then compared these data sets with our data from the preservice cohort. We anticipated that teacher education should be adding capital to participants and thus, teachers would score significantly better than preservice teachers who were only commencing learning the craft of teaching. We also anticipated that as practicing teachers both MCK and MPCK would be enhanced as teachers worked through their craft and gained more experience and confidence in teaching.

For MCK, we found that this increases with the different cohorts. Mean scores increase from pre-service teachers (4.94 ± 2.80) to primary teachers (6.27 ± 3.12) , to secondary teachers (7.57 ± 2.62) and the difference between each group is statistically significant using the Turkey post-hoc test (p<0.015). Similarly we found that there was an increase in MCPK across the cohorts pre-service teacher (4.95 \pm 2.18), to primary teachers (5.64 \pm 2.27), to secondary teachers (7.02 \pm 2.12). Interestingly (or alarmingly) the only significant differences were between the secondary teachers with the other groups (p<0.001). That is, there is no significant difference in MPCK between primary school teachers and preservice teachers.

So what does this suggest? In many team discussions we perplexed, contemplated, wrestled with these findings. Do they suggest that preservice teacher education is not adding capital – particularly in MPCK – to prospective teachers seeing that there are no differences in the MPCK between preservice and practising primary teachers? If

this is the case, what is happening in preservice teacher education? Or is it the case that there is no growth in MPCK as teachers move into their positions in schools. One would hope that as building the teaching capital of teachers is the primary purpose of teacher education that there would be a marked growth in the MPCK between teachers and preservice teachers. Or is it the case that teacher education is so successful that teachers exit their programs with most/all the knowledge they will demonstrate as a practicing teacher? Or is there a link between having strengths in MCK that flow to MPCK which we see in the secondary teachers but not in the primary teachers?

ENGINEERS: MATHEMATICAL AND PEDAGOGICAL CAPITAL

It was our assumption (to be read as hope) that preservice teacher education and inservice education does make a difference. To explore this further, we modified our research design to incorporate a cohort of engineers. These are professionals who must, by the nature of their work, have strong MCK but as they are practitioners in their fields, it is reasonable to expect that they have no teacher education. Including this cohort would help us better understand the role of MCK in supporting (or not) MPCK. To this end, we secured a cohort of 31 engineers who worked across many fields of engineering so it was a diverse group. As we were concerned with only the MCK and PCK of the engineers, they were only asked to complete the first three sections of the survey and any questions relating to beliefs about teaching and their experiences as a teacher were removed from the survey instrument. We did ask if they had had teaching experience but this was a negative response.

Unsurprisingly for the research team, the engineers scored the highest score in MCK (9.42 ± 1.67) and were statistically different from the other three cohorts using the Turkey post-hoc test (p<0.015). As part of the engineering qualification, engineers must study high levels of mathematics so this was not a surprise. However, what was of most interest to the research team was the score in MPCK. It is noted that none of the engineers had undertaken any studies in teacher education. All test items in the MPCK phase of the survey related to aspects of teaching mathematics. The mean scores for the engineers on MPCK was 4.67 ± 1.96 whereas pre-service teachers scored 4.95 ± 2.18 and primary teachers 5.64 ± 2.27 . There was no statistical difference between these three cohorts. The only statistical difference in MPCK was between secondary teachers and the other three cohorts. This says to us that despite never having undertaken any formal study of mathematics teaching and learning, engineers performed close to preservice teachers and primary school teachers in MPCK.

So what do these data tell us? Preservice, primary and engineers can be seen to have relatively similar MPCK, despite one cohort never having studied teaching. To try to understand what these data reveal, we will refer back to some of the research in MCK and MPCK. For this paper, we intend to conclude the paper with questions and

implications of these findings and ask what they mean for the field of mathematics education and mathematics teacher education.

How important is MCK?

Krause and colleagues (Krauss et al., 2008) have argued that high expertise in a domain provides much more opportunity for stronger integration of other domains of expertise and, consequently, strong MCK supports the development of MPCK. Wu (2011, p.381) makes a stronger assertion when stating 'what must not be left unsaid is the obvious fact that, without a solid mathematical knowledge base, it is futile to talk about pedagogical content knowledge.' These positions have been supported by the comprehensive work of Hill et al. (2008) where they argued that students' gains were most strongly correlated with teachers' MCK. What we see from the data in this study is that MCK may be critical for teaching.

From our analysis, we were compelled to ask ourselves whether or not the observed outcomes were a reflection of MCK knowledge. Is it the case that if a teacher (or engineer) has strong MCK that they have a better understanding of how to teach/learn mathematics. Such a proposition might help to explain our outcomes. Those teachers who have strong MCK were stronger in MPCK, and those (engineers) who never studied teacher education yet had strong MCK were similar to those teachers who had studied teacher education and/or had been involved in the teaching of mathematics. This could suggest that strong MCK ameliorates the effect that teachers with poor MCK spend learning how to teach. It does beg askance of how teacher education is adding pedagogical capital to teachers when outsiders (engineers) can perform comparatively similar to those who have studied educational practices. It begs the question to be asked as to whether or not MCK is sufficient for being a good teacher in mathematics?

How important is teacher education?

There are some researchers who question the possibilities of teacher education to actually make a difference to the professional work of teachers (Brouwer & Korthagen, 2005), in part, through the conservatism that is entrenched in schools (Brouwer & Korthagen, 2005), and mathematics departments (Guitterez, 1990). But our data suggest something potentially more sinister.

What was observed was that there was no significant difference in the MPCK of preservice teachers and primary school teachers. Initially we though that this might be a reflection of the high quality of initial teacher preparation and that graduates were exiting with pedagogical skills relatively commensurate with primary school teachers. However, the inclusion of the engineers suggested something else. The engineers' MPCK was not significantly different from the preservice teachers who would have 2-4 years studying pedagogy and primary teachers who varied from new graduates to long standing practitioners with ≥ 16 years practice. While their mean score was lower than teachers who had studied pedagogy and worked in classrooms, they were scoring relatively comparably to these cohorts despite never having formal qualifications in teacher education nor being a practitioner in schools. This begs askance of how much

capital is being added to teachers as a result of preservice, in-service and service in the classroom. Further it questions whether preservice teacher education may be better served through the inclusion of mathematical content courses rather than mathematics education courses.

How important is MCK for educators' MPCK?

It would seem to us that the inclusion of the engineers into this study raises very serious questions about teacher preparation and the importance of discipline knowledge in the quality of teachers. These findings suggest that MCK impacts significantly on MPCK. This finding poses serious challenges to teacher education programs. As researchers and teacher educators, these findings have challenged many of our assumptions about the content in teacher preparation courses. Our faith in mathematics education courses has been challenged and we have been caused to rethink the role of mathematics as a discipline and its impact on teaching. These findings suggest that MCK has significant influence on the capacity to teach mathematics well.

CAVEATS AND LIMITATIONS

In the study we have worked from the assumption that our test items are reliable given that they have a strong research basis to them. Our findings are reported against this assumption.

REFERENCES

- Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. *Multiple perspectives on the teaching and learning of mathematics*, 83-104.
- Blömeke, S., Suhl, U., & Kaiser, G. (2011). Teacher education effectiveness: Quality and equity of future primary teachers' mathematics and mathematics pedagogical content knowledge. *Journal of Teacher Education*, 62(2), 154-171. doi:10.1177/0022487110386798
- Boaler, J. (2002). *Expriencing school mathematics: Traditional and reform approaches to the teaching and their impact on student learning*. Mahwah, N.J.: Lawrence Erlbaum.
- Bourdieu, P. & wacquant, L.J.D. (1992). An invitation to a reflexive sociology. Chicago: Chicago University Press.
- Brouwer, N., & Korthagen, F. (2005). Can teacher education make a difference? *American Educational Research Journal, 42*(1), 153-224. doi:10.3102/00028312042001153
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372-400.
- Hill, H. C., Blunk, M. L., & Charalambous, C. Y. (2008). Mathematical Knowledge for Teaching and the Mathematical Quality of Instruction: An Exploratory Study. *Cognition and Instruction*, *26*(4), 430-511.

- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371-406. doi:10.3102/00028312042002371
- Krauss, S., Brunner, M., Kunter, M., Braumert, J., Blum, W., Neubrand, M., & Jordan, A. (2008). Pedagogical content knowledge and content knowledge of secondary mathematics teachers. *Journal of Educational Psychology*, 100, 716-725.
- Lowrie, T., & Jorgensen, R. (2015). Pre-service teachers' mathematics content knowledge: implications for how mathematics is taught in higher education. *Teaching Mathematics and its Applications., on line.* doi:doi: 10.1093/teamat/hrv008
- Schoenfeld, A. (2004). The math wars. Educational Policy, 18(1), 253-286.
- Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.
- Staub, F. C., & Stern, E. (2002). The nature of teachers' pedagogical content beliefs matters for students' achievement gains: Quasi-experimental evidence from elementary mathematics. *Journal of Educational Psychology*, 94(2), 344-355. doi:10.1037/0022-0663.94.2.344
- Wu, H. (2011). The mis-education of mathematics teachers. *Notices of the AMS*, 58(3), 372-384.

APPENDIX: EXAMPLE OF A TEST ITEM

1. The illustrations below show how four students – Alicia, Bobby, Carlos and Davilla – used base 10 blocks to represent the number 32.



Which of

the students used the blocks

to represent the number 32 in a way that does not indicate an understanding of the underlying concepts of the base 10 numeration system?

Tick the circle beside the correct answer

- (A) Alicia
- (B) Bobby 🔿

(C) Carlos

(D) Davilla

Source: Educational Testing Service: The Praxis Series (2011). Elementary Education: Curriculum Instruction, and Assessment (0011/5011)