BILDUNG, MATHEMATICAL LITERACY AND CIVIC EDUCATION: THE (STRANGE?) CASE OF CONTEMPORARY AUSTRIA AND GERMANY

Andreas Vohns

Alpen-Adria Universität Klagenfurt

Civic education in terms of fostering societal participation and active citizenship is a goal that is prominently addressed in both conceptions of "mathematical literacy" discussed in the international context and (contemporary) conceptions of "Bildung/ Allgemeinbildung" specific to the German speaking countries. In this paper I will argue that in transitioning from such conceptions to contemporary reform efforts like educational standards, national regimes of testing and standardized exams the notion of mathematical literacy/Allgemeinbildung changes as well in a way that may be self-defeating as far as the goal of (critical) citizenship is concerned.

INTRODUCTION

To my understanding, the discussion about mathematical literacy in Germany and Austria has two distinct features: Firstly, it is kick-started by the results of Austrian and German students within large scale cross-national studies (TIMSS, PISA) widely construed to be unsatisfactory and it is focused almost exclusively on the specific understanding of mathematical literacy according to the PISA framework. Secondly, in implementing mathematical literacy into official curricula this conception is on the one hand mixed and (some have argued mis-)matched with the long standing German language tradition of "Bildung" and/or recent conceptions of "Allgemeinbildung" (see next section). On the other hand, the curricular implementation is accompanied by the establishment of regimes of assessment and the introduction of nationally centralized exams which for their part are heavily inspired by the testing regimes of TIMSS and PISA.

For example, the German National Educational Standards for Mathematics in Grades 5-10 (KMK, 2004) contain a section dedicated to "the contribution of the subject 'mathematics' to Allgemeinbildung" (ibid., p. 6) – which draws heavy inspiration from Winter (1995), even paraphrasing his widely referenced three *basic experiences* as well as five so called *leading ideas* – adopted more or less directly from the PISA-framework. Admittedly, the Austrian Nationally Standardized Written Final Examination ("Zentralmatura", cf. BIFIE, 2013) has no direct links to the frameworks of large scale cross-national studies but is also using a concept of "basic competencies" that focuses on such mathematical skills and abilities students should acquire "for their own good and for the good of society" (ibid., p. 3) and which are assessable by means of rather brief tasks aiming directly at the use of said skills and abilities preferably in real world contexts. Again, there are also direct references to a specific conception of Allgemeinbildung, in this case Fischer (2001).

By and large, both the standards in Germany and the written final exam in Austria are in fact politically legitimized with an understanding of "mathematical literacy for developing human capital" typically associated with the PISA framework (Jablonka, 2003, p.80). In this paper I will argue, that this is kind of a mismatch or even a contradiction with the more broad goals regarding civic education the aforementioned conceptions of Allgemeinbildung originally aim at. By means of a draft of an exemplary teaching unit I will then discuss why and how aspects especially crucial to civic education may be "lost in translation" when adoptions of such theoretical conceptions into national standards and frameworks for written final exams are concerned.

THEORETICAL BACKGROUND: CIVIC EDUCATION AS AN INTEGRAL PART OF "ALLGEMEINBILDUNG" IN MATHEMATICS

Before considering the aforementioned contemporary conceptions of Allgemeinbildung in mathematics, some explanatory remarks on the notoriously hard to translate German terms "Bildung" and "Allgemeinbildung" seem appropriate. The use of the term "Bildung" in the context of education and culture can be traced back to the first German translation of the 3rd Earl of Shaftesbury's (1710) "Soliloquy or Advice to an Author" from 1738. For this central writing on his philosophy of politeness "Bildung" (noun) and "bilden" (verb) were used as German translations for *formation*, *inner form* or *to form*, respectively (see Bothe-Scharf, 2010, p. 68). The classical German concept of "Bildung" as an *individual process of self-formation through confrontation with and acquisition of culture in an all-encompassing view* (arts, sciences, ethics, etc.; see Heymann, 2014, p. 248) was then developed throughout the 18th and 19th century and was closely related to the emerging self-image of the rising class of the bourgeoisie (see Horlacher, 2016, pp. 58–71). The opportunity for substantial Bildung as a civil right for *all members* of society while programmatically rooted in the 19th century discussion was still an unredeemed promise and thus a popular catchphrase of educational reform in the 1960s (see Dahrendorf, 1965).

The contemporary use of both the terms "Bildung" and "Allgemeinbildung" is a rather broad and fuzzy one. Heymann (2014, p. 248) discerns three principal meanings: Firstly, "Bildung" is used as an explicit referral to the aforementioned tradition and denotes a train of thought that is concerned with answering fundamental anthropological questions like "What constitutes the humanity of human beings?" and "How can external influences be used to let someone develop a personality which is strong enough to not be determined by external influences but follow his own insights?" Secondly, "Bildung" is also used in a more pragmatic way with respect to the aims of education in (public) schools. Here, the leading question is the fundamental pedagogical question what students should learn or how and what they should be taught in public schools. It is related to the first meaning of Bildung inasmuch as it asks what socially generalizable groundwork schools can (realistically) lay for individual processes of Bildung according to the first meaning. Many authors, including Heymann, prefer using "Allgemeinbildung" (literally translates to "general" Bildung) for this meaning of Bildung. Thirdly, in contrast to the normatively charged first and second meaning, "Bildung" is also used in a descriptive sense as a mere synonym for education, especially in compound words (for instance, the educational system is called "Bildungssystem", the minister of education is the "Bildungsminister").

As far as "Allgemeinbildung"/the second meaning of Bildung is concerned, considerations about Bildung usually include aspects of societal participation and active citizenship as important goals of formation public schools should aim to foster for all students. To that extent, Allgemeinbildung in mathematics is a functional equivalent to conceptions of "mathematical literacy". This is particularly the case for the two contemporary approaches to Allgemeinbildung in mathematics discussed below.

Approach I: Heinrich Winter

Heinrich Winter's three basic experiences paraphrased within the German educational standards are taken from an essay on "Mathematikunterricht und Allgemeinbildung" (transl. to: "Mathematics Instruction and General Education"; Winter, 1995). For the purpose of this paper, I will limit myself to the discussion of the first of said three basic experiences. Winter proclaims that for mathematics instruction contributing to the greater goal of Allgemeinbildung students need "to recognize and understand in a specific way such phenomena in the world around us, which everyone of us is or should be concerned with, be it within nature, society, or culture" (Winter 1995, p. 37). At first glance, this principle bears some semblance to the PISA definition of mathematical literacy. However, we would be wronging Winter considerably in stating that his conception of Allgemeinbildung is predominantly focused upon developing human capital. Winter is explicitly opposing a view of mathematics education that confines it to the domain of individual usefulness (see Winter, 1995, p. 38). Mathematical modelling for example, which he presumes an important activity to foster the first basic experience, is no goal in itself for Winter, but a means of a specific kind of enlightenment mathematics may provide when applied to such phenomena in nature, society, or culture each and every one of us is or should be concerned with. To this respect, "Mathematikunterricht und Allgemeinbildung" is a direct continuation of Winter's essay "Bürger und Mathematik" (transl. to: "Citizens and Mathematics"; Winter 1990). In this paper Winter discusses ways in which mathematics education may contribute to the traditional Kantian goal of enlightenment: the public use of reason. For Winter public mathematics education has always and still is standing in a dialectic tension between enlightenment and social conformity (see Winter, 1990, p. 135). Winter asserts that the division of labor, especially the growing tendency to base private and public decisions on expert opinions (which for their part have a growing tendency towards mathematization) may pose the greatest challenge for enlightenment and public use of reason in modern, democratic societies. “Bürger und Mathematik” contains a set of examples in which students are to be confronted with mathematical models from the domain of public affairs and welcomed to analyze critical features of these societal uses of mathematics. There can be little doubt that what Winter advocates here is closely related to a broader, critical understanding of mathematical literacy that is in line with Jablonka’s (2003) perspectives “mathematical literacy for evaluating mathematics” or even “mathematical literacy for social change”.

Approach II: Roland Fischer

Mathematical literacy for evaluating mathematics and as a prerequisite for social change is even more pronounced in Roland Fischer’s conception of Allgemeinbildung. Just as Winter, Fischer considers communication between experts and the lay public the greatest challenge general education faces in the modern democratic societies. Fischer argues that communication between experts and lay people is always asymmetrical: While expertise is precisely based upon the fact that the respective experts have a better understanding of the matter at hand than the lay people, it is mostly the lay people who have to make a decision. For example: A surgeon usually has a better understanding of the benefits and risks of a surgery than the patient. Nonetheless, it is the patient who has to decide and give written notice of his “informed consent”. Likewise, politicians (as elected representatives of the public) may consult experts, it is nonetheless their ‘job' to make and take responsibility for the actual decisions. Every democratic society is after all based upon the principle that in some way or form it is the (lay) public itself that decides upon its public matters. Nevertheless, that necessarily implies deciding upon proposals for problem resolution oneself would not be able to conduct and does not understand as well as the experts. To Fischer, educating students to become well-informed laypersons should therefore focus on prospectively enabling them to make decisions about the importance of (mathematical) activities and problem resolutions even and especially in such cases in which they are not able to judge (in detail) about their technical correctness or to undertake the respective activities by themselves. For establishing a line between the professional study of a subject and its study for the purpose of Allgemeinbildung, Fischer (2001) distinguishes between three domains of knowledge in a subject:

> “Firstly, *basic knowledge* (notions, concepts, means of representation) *and skills.* Secondly, more or less creative ways of *operating* with knowledge and skills within applications (problem solving) or for the generation of new knowledge (research). Thirdly, *reflection* (What is the meaning/wherein lies the significance of these concepts and methods? What can be achieved with them, what are their limitations?).” (Fischer, 2001, p. 154)

Fischer then concludes that experts have to be well versed in all three domains, while the education of laypersons should focus on the first and third domain. One criticism towards Fischer’s conception has been to dispute whether one can reflect meaningfully upon mathematics without actually "doing" said mathematics. It is nonetheless *the* typical mode of confrontation with professional knowledge in a society that is based upon the division of labor. Furthermore, Fischer sees mathematical modelling and problem solving as important activities *in the mathematics classroom*. But, Fischer contests that being able to do (elaborate) mathematics can be *a goal in itself* for the purpose of general education. So, mathematical modelling or operating is seen by Fischer as a *means* to the end of acquiring basic skills as well as developing reflective knowledge, which is of particular interest for future citizens as well-informed lay public.

ILLUSTRATIVE EXAMPLE: EXPLORING AND REFLECTING MEASURES OF POVERTY IN THE MATHEMATICS CLASSROOM

If we accept the goal of fostering critical citizenship by supporting students' present and future public use of reason with respect to argumentations relying on mathematizations as crucial to Allgemeinbildung or mathematical literacy, it seems fitting to actually engage students in examples from the realm of public and political matters. An example I have worked on myself is the at-risk-of-poverty rate and its underlying socio-economic and mathematical models (cf. Vohns, 2013).

To the extent permitted by the brevity of this paper, I will try to outline a teaching unit focusing on this topic in the mathematics classroom. The unit is aimed at students at the 9th/10th grade (age 15 to 16 years) and is designed in a way that prototypically adheres to the notion of "mathematical literacy for democratic citizenship" according to the two approaches discussed above. While the core mathematical content of this teaching unit (measures of central tendency, discrete distributions) is covered in the Austrian syllabus ("curricular validity" for the abovementioned grades can be assumed in principle), I will also use this example to address the question of crucial components of this unit which *are not* (and are not likely to ever be) covered by educational standards and central examinations in mathematics.

The teaching unit may start with conflicting accounts of "poverty" as a phenomenon that is on the one hand usually seen as multidimensional in social sciences, combining economic, social, cultural and psychological aspects and on the other hand occasionally discussed in the media on grounds of a single number reported (precise to the first decimal place) by the Federal Statistical Office: the at-risk-of-poverty rate. Any discussion of mathematical models of poverty should take its time to discuss the underlying socio-economical models (the so-called "real model"-stage, cf. Leiss et al. 2010). Students should become aware that applying any kind of mathematical model to a socioeconomic phenomenon is dependent upon processes of structuring and simplification. For instance, the "real model" used to describe poverty in the European Union aims at identifying "persons, families and groups of persons whose resources (material, cultural and social) are so limited as to exclude them from the minimum acceptable way of life in the Member State in which they live" (EEC, 1985). For this purpose, the Statistical Office of the European Union (EUROSTAT) distinguishes three different dimensions of the so-defined poverty: relative income poverty, severe material deprivation and low work intensity (cf. EUROSTAT, 2015). At-risk-of-poverty rates are one measure for the first of these three dimensions. Finally, the "mathematical model" for income poverty called "at-risk-of-poverty rate" is "the share of people with an equivalised disposable income (after social transfer) below the at-risk-of-poverty threshold, which is set at 60 % of the national median equivalised disposable income after social transfers" (ibid.). Structuring and simplification processes such as the one above may on the one hand rest upon more or less sound economical and/or sociological theories justifying the inclusion and exclusion of specific aspects of the broader phenomenon of poverty. On the other hand, the succession of "real models" and "mathematical models" is not as clear-cut as didactical modelling cycles tend to imply. In reality, a "real model" of poverty is already influenced by questions of how easily data is includable in a "mathematical model" regarding objectivity, suitability for cross-national comparisons, and collectability under restrictions of budget and time. According to Yasukawa (1998) we should further ask, who is concerned with poverty models and why the concerned parties have an interest in mathematizing this phenomenon. Measuring poverty is primarily a "problem" of social statisticians usually working at government-run statistical offices. Governments then use the results in public social reporting and social scientists may use them for further research. Eventually, these measures may also be a part of public policy formation regarding e.g. the amount of social benefits and transfers. A possible starting point for investigating the mathematical properties of at-risk-of-poverty rates in the classroom is examining the heavy criticism their publication draws with uncanny regularity from conservative and market-liberal economists. Let us consider the following prototypical arguments:

A) "The crux of relative poverty models: If poverty is defined in relation to the mean (or median) [of disposable income – A.V.], one has to accept that – save for a uniform distribution [of disposable income – A.V.] – there will always be poverty regardless of how rich a society becomes" (Beck & Prinz, 2004, p. 51).
B) "Someone living in a tax haven with an average yearly income of 1 Million Euro is already 'at-risk-of-poverty' by definition if he earns 590 000 Euro or less per year" (Sinn, 2006).
C) "If ten-thousands of rich people would immigrate to Germany, the local population would become 'poorer' – at least according to the definition of poverty used by the statistical office and the ministry of social affairs" (Knauß, 2012).

Substantially scrutinizing such claims in the mathematics classroom means asking (and answering) questions such as:

1. How do at-risk-of-poverty rates relate to measures of average income?
2. Why and for which countries are at-risk-of-poverty rates at all calculated depending on measures of average income? Does it make any difference, whether these rates are calculated depending on either mean or median income?

3. How do at-risk-of-poverty rates depend on other properties of the income distribution function? Specifically: Is equal distribution of income in fact a prerequisite for statistically overcoming poverty?
4. How do peculiarities of the actual data collection processes for income measures affect both (the robustness of) measures for average income and consequently at-risk-of-poverty rates?
5. What alternative and complementary models and measures of poverty are used in public social reporting and the social sciences? What are the assets and drawbacks of the various models and measures?

Our exemplary teaching unit would have to introduce or revive different measures of central tendency (arithmetic mean, median). Afterwards, it should concern students with the question of how the skewness of a distribution and outliers (e.g. persons with extremely high income) affect these measures. For this purpose, one may begin with simplified income distributions (cf. Eggen, 2006). A spreadsheet may be useful for investigating different scenarios in relation to the aforementioned three counter-arguments against relative poverty rates. Turning back to the real world measurement of poverty, the teaching unit should also consider that the income data used for the calculation of poverty rates stems from sample surveys which have their specific drawbacks, in general as well as for the matter at hand (cf. Vohns, 2013).

Coming back to the two approaches to Allgemeinbildung discussed above, it should already have become apparent that our exemplary teaching unit has the potential to sustain a better understanding of “how mathematical modelling works” and of “what kind of enlightenment it provides” (Winter, 1995, p. 38) regarding the socioeconomic phenomenon of poverty. In scrutinizing counter-arguments found in the media, it is also directly linked to the realm of public matters and a prime example of the “mathematical literacy for evaluating mathematics” Winter advocates. Turning to Fischer’s approach, we can identify his three domains of knowledge: Firstly, knowledge about central measures and their robustness against outliers and skewed distributions as well as effects of random sampling are in fact elements of (reflective) basic knowledge that is covered by the Austrian Educational Standards and/or the catalog of competencies for the final central examination. Secondly, the validity of counter-arguments has to be investigated by mathematical operating (Fischer’s second domain of knowledge). Here, many calculations can be handed over to technology (e.g. spreadsheets). But, thirdly and lastly, the linchpin of the previously outlined teaching unit for contributing to civic education is the question whether it supports the development of reflective knowledge both regarding poverty models in particular and the use of mathematical models in public reasoning in general. It should be quite recognizable that the teaching unit aims to provide opportunities for mathematical-, model-, and context-oriented reflections (see Skovsmose, 1998). Whether students actually engage themselves in context-oriented reflections, I consider the litmus test for a possible civic educational effect of this teaching unit.

Abstracting from the case of poverty models, students could and should become aware that criticism towards a specific mathematical model used in public reasoning can either be mathematically invalid (e.g.: counter-argument A) or mathematically valid. Yet, even that does not automatically imply its relevance (e.g.: counter-argument B, C). If someone wants to deny the relevance of a specific measure, it is a frequently used argumentation tactic to focus on mathematically correct drawbacks of said measures. However, such argumentations may use exaggerated scenarios of little practical relevance. Likewise, people who want to use such measures for their argumentations are likely to gloss over actual drawbacks and try to obfuscate the finer details of their mathematical construction. In the first case, the "mathematical model" acts as a proxy or scapegoat for a kind of criticism that is in fact questioning the relevance of the "real model" or the existence of the social phenomenon in itself. In the second case, the conviction that the social phenomenon is relevant acts as a shield against any scrutiny the "real model" or the "mathematical model" may very well deserve. Every one of us is prone to such a biased evaluation of mathematical models in a context we feel deeply involved in. Allgemeinbildung would therefore ultimately manifest itself in an awareness for this fact and appropriate countermeasures.

But educational standards and central examinations are much more likely to focus on basic knowledge and – if anything – mathematical- and model-oriented reflections. They almost have to omit context-oriented reflections, as such reflections either need a very careful consideration of the specific context mathematics is applied to (which is not realistically possible under the constraints of a written exam) or are likely to be on a similar meta-level as the above statement about our very selective skepticism towards different models. However, such a meta-level is usually too far removed from peoples' preconceptions of "doing mathematics" and/or mathematical competence as to be included in a mathematics exam. In consequence, such restrictions run the risk of both basic *and* reflective knowledge becoming rote and inconsequential "textbook" knowledge and in turn raise doubts about the seriousness of conceptions of Allgemeinbildung or mathematical literacy as such.

CLOSING REMARKS

I want to close this paper by summarizing and emphasizing the key issues in fostering civic education through mathematics instruction which I presume the most pressing at least for the contemporary discussion in Austria and Germany:

1. The recent tendency of focusing mathematics education on the achievement of basic skills and knowledge for every student is an ambivalent endeavour: Basic skills and knowledge are considered a prerequisite for becoming a well-informed layperson able to act as an active, reflective citizen. However, there is no precise line between basic skills and more elaborate skills akin to Fischer's domain of operating. Mathematics instruction may therefore be both in danger to draw too much and too little attention to the achievement of mathematical

knowledge and skills as a basis for meaningful reflections and evaluation of mathematics.

2. The focus on assessment and central examinations furthers this problem in respect to reflective knowledge, as components especially relevant from a civic education standpoint are the least likely to be realistically implemented under the constraints of central examinations and standardized (cross-)national testing.
3. The two aforementioned issues have and still do contribute to already prevalent qualms about the seriousness of mathematical literacy and civic education efforts and have enforced serious doubts about both the style and amount of application-oriented teaching currently recommended in the mainstream of mathematics education in the German speaking countries. Both educational scientists of humanistic background and mathematicians have raised concerns regarding a supposed negligence of pure mathematics – each with their own agenda.
4. Setting aside the first three issues, the development of “good practice”-examples for fostering model- and context-oriented reflections directly relevant to civic education is still in its infancy. This is especially true in regards to empirical research investigating the actual engagement of students in questions aiming at context-oriented reflections and transfer of reflective knowledge to similar problem contexts.
5. As Jablonka & Gellert (2010) have pointed out, there would still be a long way from a reasonable stock of good examples to a consistent curriculum construction. Even mathematics educators who are convinced in mathematical literacy for evaluating mathematics and as part of civic education rarely dare to challenge established mathematical curricula fundamentally. This may contribute to a growingly problematic mismatch of mathematical content and educational goals in turn feeding back to the above issues.

REFERENCES

Beck, H., & Prinz, A. (2004). *Die Soziallüge*. Frankfurt a. M.: FAZ Buch.

BIFIE (2013). *Die standardisierte schriftliche Reifeprüfung in Mathematik*. Retrieved from https://www.bifie.at/node/1442

Bothe-Scharf, M. (2010). Bildung – the formation of a genteel character? In D. Gaus, & E. Drieschner (Eds.), *‚Bildung' jenseits pädagogischer Theoriebildung* (pp. 67 – 78). Wiesbaden: VS Verlag für Sozialwissenschaften.

Dahrendorf, R. (1965). *Bildung ist Bürgerrecht*. Hamburg: Nannen-Verlag.

EEC (1985). *On Specific Community Action to Combat Poverty* (Council Decision of 19 December 1984) 85/8/EEC, Official Journal of the EEC, 2/24.

Eggen, B. (2006). Das Kreuz mit der Armut in einer reichen Gesellschaft. *WSI-Mitteilungen* (3) 2006, 168 – 170

EUROSTAT (2015). *People at risk of poverty or social exclusion*. Retrieved from http://ec.europa.eu/eurostat/statistics-explained/index.php/People_at_risk_of_poverty_or_social_exclusion

Fischer, R. (2001). Höhere Allgemeinbildung. In R. Aulke, A. Fischer-Buck, & K. Garnitschnig (Eds.), *Situation – Ursprung der Bildung* (pp. 151–161). Norderstedt: Fischer.

Heymann, H.-W. (2014). Stojanovs Rekonstruktion des Bildungsbegriffs kritisch hinterfragt. *Erwägen – Wissen – Ethik*, 25(2), 247–250.

Horlacher, R. (2016). *The Educated Subject and the German Concept of Bildung*. New York/London: Routledge.

Jablonka, E. (2003). Mathematical literacy. In A. J. Bishop (Ed.), *Second international handbook of mathematics education* (pp. 75–102). Dordrecht: Kluwer.

Jablonka, E., & Gellert, U. (2010). Ideological roots and uncontrolled flowering of alternative curriculum conceptions. In U. Gellert, E. Jablonka, & C. Morgan (Eds.), *Proceedings of the 6th MES conference* (pp. 23–41). Berlin: Freie Universität Berlin.

KMK – Kultusministerkonferenz der Länder (2004). *Bildungsstandards im Fach Mathematik für dem Mittleren Bildungsabschluss*. Neuwied: Luchterhand.

Knauß, F. (2012). Warum unsere Armen nicht arm sind. *Wirtschaftswoche* (09-12-2012). Retrieved from http://www.wiwo.de/politik/deutschland/soziale-ungleichheit-warum-unsere-armen-nicht-arm-sind/7167082.html

Leiss, D., Schukajlow, S., Blum, W., Messner, R., & Pekrun, R (2010). The Role of the Situation Model in Mathematical Modelling. *Journal für Mathematik-Didaktik*, 31(1), 119–141.

Skovsmose, O. (1998). Linking Mathematics Education and Democracy. *ZDM*, 30(6), 195–203.

Sinn, H.-W. (2006). Geld fürs Nichtstun. *Wirtschaftswoche* (12-16-2006). Retrieved from http://www.wiwo.de/politik-weltwirtschaft/geld-fuers-nichtstun-162713/.

Vohns, A. (2013). Relative Armutsgefährdung – nur eine Zahl? *Der Mathematikunterricht*, 59(4), 49–58.

Winter, H. (1990). Bürger und Mathematik. *ZDM*, 22(4), 131–147.

Winter, H. (1995). Mathematikunterricht und Allgemeinbildung. *Mitteilungen der GDM*, (61), 37–46.

Yasukawa, K. (1998). Looking at Mathematics as Technology: Implications for Numeracy. In P. Gates (Ed.), *Proceedings of the 1st MES Conference* (pp. 351–359). Nottingham: Nottingham University.